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Mathematical Analysis of the Correction Factor for Analysing Space Frame under the Action of Horizontal Crane Load

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Abstract: This research attempts to analytically investigate, the effect of the space nature of an industrial frame building on its strength. The above subject matter is divided into two parts; the mathematical analysis of the correction factor and design procedures of an industrial frame building considering its space nature. This study examines only the first part of the above subject matter that is the analytical investigation of the effect the space nature of an industrial frame building on its strength in the form of a correction factor. The result obtained shows that frame structures are being over loaded if their space nature is not considered, hence, leading to uneconomic design.

Key words: Space nature, framed building, mathematical analysis, correction factor

INTRODUCTION

A frame structure is a combination of beams, columns, slabs and footings rigidly connected together to form a monolithic entity. Each individual member must be capable of resisting the forces acting on it; hence, the determination of these forces is an essential part of the design process (Mosley and Bungey, 1993).

Rigid-jointed frames are statically indeterminate and basically can be analysed manually using first order approach. However, above method is tend to be tedious, hence, a more simple approximate method or available solutions for specific frames are often been forced to be relied upon (Trahair *et al*., 2001). Also, space framed structure can be analysed by dividing the space frame into a series of plane frames using flexibility or stiffness method (Darkov and Schaposhnikov, 1986).

The division of space frames to series of plane frames reduces the number of equations to be solved at a time to three times the number of joints (3j) (Astill and Martin, 1982). The practical interpretation of this is that one of the transverse plane frames is selected, analysed and designed for and assumed to be typical of all other plane frames that make up the structure. This method is quite appropriate when analyzing and designing for vertical loads (dead and imposed) and horizontal wind load. Since, vertical loads and horizontal wind load are uniformly distributed throughout the whole structure, they will induce uniform stresses and deformation on each plane frame that make up the entire structure (Baikov, 1986).

However, in the case of horizontal load from the crane action, the situation is quite different. This is because in most industrial buildings there is only one crane in a structural block and the action of this crane cannot induce a uniform deformation and stress on all the plane frames that make up that structural block. Therefore, it is not accurate to apply the plane frame method of analysis to an industrial building under the action of horizontal forces due to crane loading.

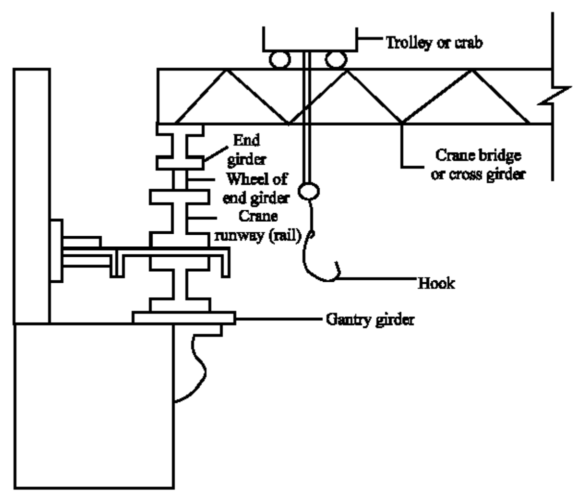

Fig. 1: Cross-sectional view of typical arrangement of crane system

The most common method of analysis of frame structures under the action of horizontal crane load is still the plane frame method, which in the final analysis results in an uneconomic design that does not conform to the economy requirements of the structure. The general typical arrangement of a crane system is shown in Fig. 1 and 2.

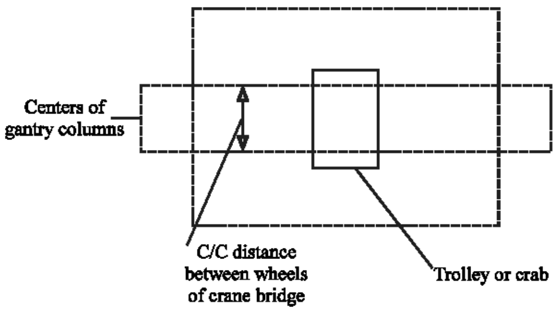

Fig. 2: Plan view of a typical arrangement of a crane system

MATERIALS AND METHODS

The material used in this study is a six panels and single bay space framed structure as shown in Fig. 3 and 4. Stiffness method of structural analysis is used in the computation of the correction factor.

Assumptions in the computation of the correction factor: As explained above, the sudden stopping and starting of the crab and load when moving over the crane girders induce lateral forces. These forces are transferred to the crane girder, then to end girder, to the compression flange of the gantry girder and finally to the gantry columns.

A special case of an industrial building under the action of crane load is considered, in which load P be thrust due to sudden starting or stopping (Fig. 3a). The following assumptions are made for the purpose of this computation:

- The joint between column and beam is pin joint
- The stiffness of beam is infinite
- Longitudinal displacement of column is negligible
- The structure rotates about its center of gravity under the action of the crane load

Computation of the correction factor: This analysis is better accomplished by using displacement or stiffness method of analysis. Under the action of the crane load P, the structure deforms as shown in Fig. 3b. In order to prevent that deformation, it is enough to put a pin support at A. This indicates that the structure is kinematically indeterminate to the first degree.

To obtain the displacements Δ_1, Δ_2 and Δ_3, Δ_1 is first calculated by considering plane frame 1 in Fig. 3. Figure 3a shows plane frame 1, exactly as it is in the structural plan of Fig. 2a and this is termed given structure. Figure 3b shows plane frame 1 with restrained support at A and this is termed Restrained structure. The given structure differs from restrained structure by the availability of additional support preventing translation, which is

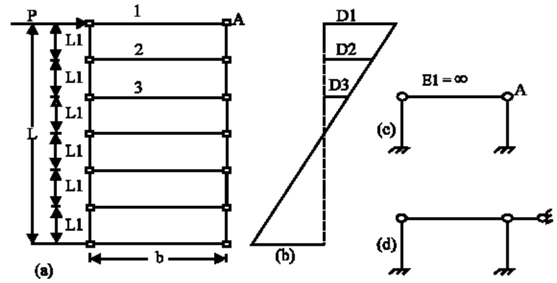

Fig. 3: a) Structural plan of the workshop, b) Displacement diagram, c) Plane frame and d) Restrained plane frame

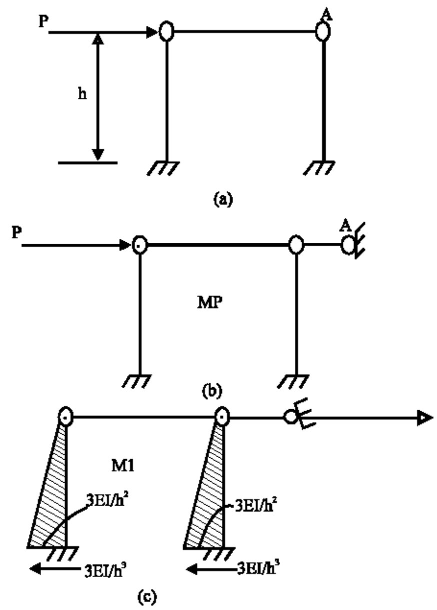

Fig. 4: Plane frame 1

accompanied by a reactive, force. The reactive force can be zero if the restrained support is displaced in such a way that it equals actual linear displacement at the joint. After that, deformation in the restrained structure and its internal forces will be equal to that of the given structure.

The equation of stiffness method can be written as:

$$R_1 = 0 \tag{1}$$

where, R_1 is the reactive force arising from the action of external loading and linear displacement of the joint. Index of the reaction corresponds to the indices of the unknown. The number of equation always corresponds to the number of introduced restrained supports.

Breaking reactive force R_1, into its components gives,

$$R_1 = R_{1P} + R_{11} \tag{2}$$

where,

R_{1P} = The reactive force from the action of the external loading

R_{11} = The reactive force due to the action of linear displacement of joint I, which equal Z_1

The reactive force R_{11} can be replaced with the expression:

$$R_{11} = Z_1\ \Sigma r_{11} \tag{3}$$

where, Σr_{11} is the sum of the reactive forces in the restrained support caused by unit displacement of joint. Substituting for R_{11} in Eq. 2 gives:

$$Z_1\ \Sigma r_{11} + R_{1P} = 0 \tag{4}$$

If we subject plane frame Eq. 1 in Fig. 3a to a displacement Z_1 - Δ_1 = 1 (Fig. 4c),

Then,

$$r_{11}^1 = \frac{3EI}{h^3} + \frac{3EI}{h^3} = \frac{6EI}{h^3} \tag{5}$$

where:

E = Young's Modules

I = Second moment of area

h = Height of frame

From similar triangles, taking Δ_1 = 1, Δ_2 and Δ_3 in Fig. 2b can be calculated as follows:

$$\frac{\Delta_1}{3L_1} = \frac{\Delta_2}{2L_1} \to \frac{1}{3L_1} = \frac{\Delta_2 \to \Delta_2}{2L_1}\ \frac{2}{3}$$

$$\frac{1}{3L_1} = \frac{\Delta_3 \to \Delta_3}{L_1} = \frac{1}{3}$$

If Δ_1 = 1, then

$$r_{11}^1 = \frac{6EI}{h^3}$$

Δ_2 = 2/3, then

$$r_{11}^2 = r_{11}^1 \times \frac{2}{3} = \frac{6EI}{h^3} \times \frac{2}{3} = \frac{4EI}{h^3}$$

if Δ_3 = 1/3 then

$$r_{11}^3 = \frac{6EI}{h^3} \times \frac{1}{3} = \frac{2EI}{h^3} \text{ therefore,}$$

$$\sum r_{11} = r_{11}^1 + r_{11}^2 + r_{11}^3$$

$$= \frac{6EI}{h^3} + \frac{6EI}{h^3} + \frac{4EI}{h^3} + \frac{2EI}{h^3}$$

$$\sum r_{11} = \frac{12EI}{h^3}$$

From Fig. 3b

$$R_{1p} = -P$$

Substituting for Σr_{11} and R_{1p} in Eq. 4.

$$Z_1 x\ \frac{12EI}{h^3} - P = 0$$

$$Z_1 = \frac{Ph^3}{12EI} \tag{6}$$

Now, considering the situation when plane frame Eq. 1 acts alone.

$$r_{11} = \frac{6EI}{h^3};\ R_{1P} = -P \tag{7}$$

Substituting, for these values in Eq. 4 yields.

$$Z_1^1 \frac{6EI}{h^3} - P = 0$$

$$Z_1^1 = \frac{Ph^3}{6EI} \tag{8}$$

$$C = K\frac{Z_1^1}{Z_1} = \frac{Ph^3}{6EI} \times \frac{12EI}{Ph^3} \times K = 2.0K \tag{9}$$

where,

C = Correction factor

K = Coefficient, which takes into consideration the effect of the flexibility of beams

A general formula for C can now be derived. Let n be equal to the half number of total steps, That is

$$n = \frac{w}{2} \tag{10}$$

where, w is number of steps in the structural layout.

$$\frac{\Delta_1}{nL_1} = \frac{\Delta_2}{(n-1)L_1} \to \frac{1}{nL_1} = \frac{\Delta_2}{(n-1)L_1} \to \Delta_2 = \frac{(n-1)L_1}{nL_1}$$

$$\text{then } \Delta_2 = \frac{n-1}{n} \text{ gives}$$

$$r_{11}^2 = r_{11}^1 \times \frac{n-1}{n} = \frac{6EI}{h^3} \times \frac{n-1}{n}$$

$$r^2{}_{11} = (n-1)\ \frac{6EI}{nh^3}$$

and if $\Delta_3 = \frac{n-2}{n}$ yields

$$r^3_{11} = \frac{6EI}{h^3} \times \frac{n-2}{n} = (n-2)\,\frac{6EI}{nh^3}$$

$\Delta_n = 1/n$ gives:

$$r^n_{11} = \frac{6EI}{h^3} \times \frac{1}{n} = \frac{6EI}{nh^3}$$

$$_{i=1}{}^{n}\sum r_{11} = \frac{6EI}{h^3} + (n-1)\frac{6EI}{nh^3} + (n-2)\frac{6EI}{nh^3} + \cdots + \frac{6EI}{nh^3}$$

$$= \frac{6EI\,(n + (n-1) + (n-2) + \ldots\ldots + 1)}{nh^3}$$

Let, $Sn = [n + (n-1) + (n-2) + \ldots\ldots\ldots + 1]$

$$_{i=1}{}^{n}\sum r_{11} = \frac{6EI\,Sn}{nh^3}$$

Substituting for $_{i-1}{}^{n}\Sigma r_{11}$ in Eq. 4 gives:

$$Z_1 \times \frac{6EI\,Sn}{nh^3} - P = 0$$

$$Z1 = P \times \frac{nh^3}{6EI\,Sn} = \frac{nPh^3}{6EI\,Sn}$$

$$C = \frac{KZ^1_1}{Z_1} = K \times \frac{Ph^3}{6EI} \times \frac{6EI\,Sn}{nPh^3}$$

$$C = \frac{KSn}{n} \qquad (10)$$

RESULTS AND DISCUSSION

The correction factor is the ratio of the Elastic Reaction of the space structure to that of the plane structure. The amount of the horizontal force acting on each plane frame of a space structure is directly proportional to its elastic reaction. From Eq. 10, it can be seen that the higher the number of steps the lower the elastic reaction.

Elastic reaction is maximum when a plane frame is acting alone, since it carries the load alone. Therefore, the factor indicates by how much the force acting on the space structure should be reduced when modeling it as a plane frame.

The method reflects approximately what actually happen in space frame under horizontal force and hence, provides economic design.

CONCLUSION

The above analysis shows that in the simple method of analysis of space frame under horizontal forces, by using a plane frame, the structure is heavily loaded and therefore, give uneconomic design. The correction factor attempts to redress this overloading.

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