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On the Effect of Rotatory Inertia and Shear Deformation of Equation of Motion for the Forced Vibration of a Uniform Beam

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Abstract: The effect of rotatory inertia and shear deformation on the natural frequency of a simple supported uniform beam was investigated. The effect of rotatory inertia alone is considered and the resulting equation of motion does not contain any term involving the shear coefficient. If the effect of shear, deformation alone is considered, the resulting equation of motion does not contain the original form $\rho I(\partial^2\phi/\partial t^2)$ and both are disregarded, the resulting equation reduces to the classical equation of motion.

Key words: Inertia, shear, vibration, beam, deformation, natural frequency, gravity

INTRODUCTION

Vibration became an interested topic when the first musical instruments probably whistle or drum were discovered. Since, the scientist applied in genuity and critical investigation to study the phenomenon of vibration. Although, certain very definite rules were observed in connection with the art of music even in ancient times, they can hardly be called a science. Music was highly developed and was much appreciated by the Chinese, Hindus, Japanese and Egyptians as long as 4000 BC from about 300 BC. Stringed instruments such as harps appeared on the wall of Egyptian tombs. In fact, the British museum exhibits a harp with bull headed sound box found on an invalid panel of a royal tomb at Ur from 2600 BC stringed musical instruments probably originate with the hunter's box a weapon favoured by the armies of ancient Egypt. One of most primitive stringed instrument called the Nanga, dating back to 1500 BC can be shown in the British museum. The present system of music is based in ancient Greek civilization. Since, ancient times, both musicians and philosophers sought out the rules and laws of sound production, used them in improving musical instruments and passed them on from generation to generation.

The effect of rotatory inertia and shear deformation has been the concern of several investigators. Among the earliest research in this area of study is the research of Timoshenko (1992), who considered the problem of element of beam theory and the rotatory effect. In his analysis, he considered so many conditions and arrived at reasonable ends.

Furthermore, Kenny (1954) took up the problem of investigating the dynamic response of infinite elastic beam on elastic foundation when the beam is under the influence of a dynamic load moving with constant speed.

Jiya *et al*. (2006) investigated the dynamic analysis of a simple supported Bernoulli-Euler beam subjected to a distributed load.

Ette (2006a-d) researched on the effect of analytical determination of the dynamic load of an imperfect lightly damped spherical cap modulate by a sinusoidally slowly varying dynamic load. Essentially, the formulation is that of an elastic nonlinear oscillatory system, with small perturbation and with coefficient that are harmonically and dynamically slowly varying.

MATERIALS AND METHODS

If the cross sectional dimension are not small compare to the length of the beam, we need to consider the effect of rotatory inertia and shear deformation, the condition has to be considered, i.e., what happen if rotatory inertia and shear deformation is disregarded,

$$\alpha^2 = \frac{EI}{\rho A} r^2 = \frac{1}{A}$$

can be written as:

$$\frac{\alpha^2 \partial^4 \omega}{\partial x^4} + \frac{\partial^2 \omega}{\partial t^2} - r^2\left(1 + \frac{E}{KG}\right)\frac{\partial^4 \omega}{\partial x^4} + \frac{\rho r^2}{KG}\frac{\partial^4 \omega}{\partial t^4} = 0 \quad (1)$$

where:

ρ	=	The mass density of the beam
A	=	Cross sectional area of the beam
G	=	The acceleration due to gravity
C	=	The fundamental constant
L	=	The length of the beam
EI	=	The flexural rigidity of the beam
ω_n	=	The nth natural frequency
K	=	The shear deformation

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Let,
$$\omega(x,t) = C\sin\frac{n\pi x}{L}\text{Cos}\omega_n t \quad (2)$$

Which, satisfies the necessary boundary condition at x = 0, x = l be an assumed solution to Eq. 1.
Then,
$$\frac{\partial^4\omega}{\partial x^4} = -\frac{Cn^4\pi}{l^4}\sin\frac{n\pi x}{L}\text{Cos}\omega_n t \quad (3)$$

$$\frac{\partial^2\omega}{\partial t^2} = -\omega_n^2 C\sin\frac{n\pi x}{L}\text{Cos}\omega_n t \quad (4)$$

$$\frac{\partial^4\omega}{\partial x^2\partial t^2} = -\frac{C\omega_n^2 n^2\pi^2}{l^2}\sin\frac{n\pi x}{L}\text{Cos}\omega_n t \quad (5)$$

$$\frac{\partial^4\omega}{\partial t^4} = \omega_n^4 C\sin\frac{n\pi x}{L}\text{Cos}\omega_n t \quad (6)$$

Putting Eq. 3-6 in Eq. 1, we have

$$\frac{\alpha^2 Cn^4\pi^4}{l^4}\sin\frac{n\pi x}{l}\cos\omega_n t$$
$$-\omega_n^2 C\sin\frac{n\pi x}{L}\cos\omega_n t - r^2\left(1+\frac{E}{KG}\right) \quad (7)$$

Which give:

$$\frac{\rho r^2}{KG}\omega_n^4 - \omega_n^2\left[1+\frac{r^2n^2\pi^2}{l^2}+\frac{r^2E}{KG}\frac{n^2\pi^2}{l^2}\right]+\frac{\alpha^2Cn^4\pi^4}{l^4} = 0 \quad (8)$$

Equation 8 is a quadratic equation in ω_n^2 there are two values of ω_n that satisfy Eq. 8. The smaller value corresponds to a bending deformation mode, while the larger one corresponds to the shear deformation mode.
Let,
$$\frac{\rho r^2}{KG} = a_1, 1+\frac{r^2n^2\pi^2}{l^2}+\frac{r^2n^2\pi^2}{l^2}\frac{E}{KG} = a_2, \frac{\alpha^2Cn^4\pi^4}{l^4} = a_3 \quad (9)$$

Putting Eq. 9 in 8:

$$-\omega_n^2 a^2 + a_3 = 0,$$
$$\omega_n = \sqrt{\frac{a_2+\sqrt{a_2^2-4a_1a_3}}{2a}} \text{ or } \sqrt{\frac{a_2-\sqrt{a_2^2-4a_1a_3}}{2a}} \quad (10)$$

$$\omega_n = \sqrt{\frac{\frac{1+r^2n^2\pi^2}{l^2}+\frac{r^2n^2\pi^2}{l^2}\frac{E}{KG}+\sqrt{\frac{1+r^2n^2\pi^2}{l^2}+\frac{r^2n^2\pi^2}{l^2}-\frac{4\rho r^2}{KG}\frac{\alpha^2Cn^4\pi^4}{l^4}}}{2\left(\frac{\rho r^2}{KG}\right)}} \quad (11)$$

RESULTS AND DISCUSSION

If the effect of rotatory inertia alone is considered. The resulting equation of motion does not contain any term involving the shear coefficient K hence, the Eq. 12 becomes:

$$EI\frac{\partial^4\omega}{\partial x^4}+\rho A\frac{\partial^2\omega}{\partial t^2}-\rho A\frac{\partial^4\omega}{\partial x^4\partial t^2} = 0 \quad (12)$$

Putting Eq. 3-6 in Eq. 12, we have
Recall that:
$$\alpha = \frac{EI}{\rho A}, r^2 = \frac{I}{A}$$

Then, let $\in I = \alpha\rho A$

$$\omega_n^2 = \frac{\alpha^2n^4\pi^4}{l^4\frac{\left(1+r^2n^2\pi^2\right)}{l^2}} \quad (13)$$

If the effect of shear deformation alone is considered, the resulting equation of motion does not contain the terms originating from:
$$\rho I\frac{\partial^2\phi}{\partial t^2}$$

thus, the equation of motion becomes:

$$EI\frac{\partial^4\omega}{\partial x^4}+\rho A\frac{\partial^2\omega}{\partial t^2}-\frac{\in I\rho}{KG}\frac{\partial^4\omega}{\partial x^4\partial t^2} = 0 \quad (14)$$

Putting Eq. 3-6 in Eq. 14, we have

$$\omega_n^2 = \frac{\alpha^2n^4\pi^4}{l^4\left[\frac{1+r^2n^2\pi^2}{l^2}\frac{E}{KG}\right]} \quad (15)$$

If both the effect of rotatory inertia and shear deformation are disregarded the equation reduces to the classical equation of motion i.e.:

$$EI\frac{\partial^4\omega}{\partial x^4}+\rho A\frac{\partial^2\omega}{\partial t^2} = 0 \quad (16)$$

Putting Eq. 3-6 in Eq. 16, we have:

$$\omega_n^2 = \frac{\alpha^2n^4\pi^4}{l^2}$$

CONCLUSION

By using an assumed solution and a modified integral method. In this study, we have developed an analytical solution of the frequency of a vibrating beam with both rotary and shear deformation considered. It is therefore, generally observed that for a vibrating beam to be balance and to avoid unnecessary resonance effect, rotatory inertia and shear deformation must be properly considered.

REFERENCES

Ette, A.M., 2006a. On the dynamic stability of a quadratic-cubic model structure pressurized by a slowly varying load. J. Nig. Assoc. Maths Physics, 10: 185-196.

Ette, A.M., 2006b. On the dynamic buckling of highly damped cylindrical shells modulated by a periodic load. J. Nig. Assoc. Maths Physics, 10: 327-344.

Ette, A.M., 2006c. Asymptotic solution on the dynamic buckling of a column stressed by a dynamically slowly varying load. J. Nig. Assoc. Maths Physics, 10: 197-202.

Ette, A.M., 2006d. Perturbation analysis on the dynamic buckling of a lightly damped spherical cap modulated by slowly varying sinusoidal load. J. Nig. Assoc. Maths Physics, 10: 315-326.

Jiya, M. *et al*., 2006. Dynamic analysis of a Bernoulli-Euler beam via the Laplace transformation technique. J. Nig. Assoc. Maths Physics, 10: 203-210.

Kenny, J., 1954. Steady state vibration of a beam on an elastic foundation of a moving load. J. Applied Mech., 6: 359-364.