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Developing a Computer Based Whole Life-Cycle Cost Model for Buildings (A Fuzzy Set Theory Approach)

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Abstract: The deficiencies of traditional approaches to costing building projects based on initial capital costs have long been recognized. While, recent approaches like Whole Life Cycle Costing (WLCC) offer great promise, data scarcity and uncertainty have been serious bottlenecks to their successful application in the economic evaluation of proposed projects. In spite of this, little has been done to address the problem of data scarcity and uncertainty. This research effort develops a computer based WLCC model capable of mitigating the data scarcity problem. The model was developed by first identifying variables, implementation stages, Cost Breakdown Structure (CBS), discounting and risk analysis techniques that may be used in the successful preparation of WLCC plan. A mathematical model was then formulated using Fuzzy Set Theory (FST) and an algorithm was outlined through, which the model was implemented on the computer. The model accepts 3 point data input and produces a WLCC plan in the format of a specific CBS. It also, produces alpha-cuts for all variables, which enables the assessment and minimization of risk. The main limitations of the model are that; it may not be easily used for the preparation of WLCC analysis at the operating stage of the asset life; the income or benefit generating ability of an asset have not been addressed by the model thereby making it less useful in the preparation of WLCC appraisal. Moreover, non-financial factors (quality variables) of an asset are not accounted for by the model. Despite these limitations the model may be used by Professional cost advisers and Property developers to evaluate alternative designs.

Key words: Whole life cycle costing, fuzzy set theory, cost model, cost breakdown structure, risk analysis

INTRODUCTION

Traditional methods of project cost evaluation were primarily based on initial capital costs. The maintenance, operating and subsequent disposal costs of proposed projects are mostly not considered at the early design stages. Empirical evidence has shown that, the operating cost of a hospital building for instance consume an equivalent of the capital cost every 2-3 years and can continue to do so for 40 years or more and that of a school can consume the equivalent of its capital cost every 4-5 years and remain in service for a century (DPWS, 2001). These suggest that, the traditional approach of costing building projects based on the initial capital cost without considering the occupancy costs do not offer the client best value for money.

As alternative project procurement methods like Private Finance Initiative (PFI), Private Public Financing (PPF) and Build Operate and Transfer (BOT) options are fast evolving in the construction industry, there is a growing pressure on the part of designers to seek for Whole Life-Cycle Cost advice from cost professionals (Grubb, 1998). Property developers have clearly understood the importance of WLCC planning at the conceptualization and design stages of a building project for the purpose of design comparisons and optimization, investment appraisals, maintenance planning and scheduling, service life prediction to mention but a few (Seeley, 1996; Boussabaine and Kirkham, 2004; Flanagan and Jewell, 2005).

In their quest to advice the clients on the basis of WLCC, cost professionals are usually faced with the problems of data scarcity, uncertainty (due to long term forecasts) and the need for assessing non-monetary factors (Kishk and Al-Hajj, 2000a). To overcome these difficulties, several models have been developed by various institutions and professional bodies (Kishk *et al*., 2003). These models have been criticized for their inability to adequately handle decision making and risk analysis in WLCC. The risk analysis techniques that are

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predominantly used are sensitivity analysis and Monte Carlo simulations. While, sensitivity analysis can be effective only when uncertainty in one variable is predominant, Monte Carlo simulation requires probability distribution functions to be developed for every uncertain variable. Such probability functions can only be derived from large set of statistical cost data, which are not likely to be available and even where, available they are unreliable (Flanagan and Jewell, 2005). In the absence of adequate and reliable historical data, other risk analysis techniques need to be examined. Sobanjo (1999) proposed a methodology for handling subjective uncertainty in WLCC analysis based on FST. Kishk and Al-Hajj (2000a) criticized this methodology for assuming that, interest rates and all time horizons were certain and that only Triangular Fuzzy Numbers (TFN) were considered. Kishk and Al-Hajj (2000a, 2000b, 2000c, 2000d), proposed a series of algorithms using FST aimed at overcoming the various limitations of Sobanjo's methodology. The algorithms however, did not consider the various stages of design evolution and did not suggest any CBS. Moreover, the algorithms do not allow the user to select different certainty levels (alpha-cuts) for different input variables. This is clearly, a problem since data certainty and availability depend both on the stage in design and the input variable in question. This study therefore, seeks to develop a computer based WLCC model capable of addressing the aforementioned problems.

MATERIALS AND METHODS

The study begins by identifying all the variables necessary for the computation of WLCC of an asset and the stages, at which WLCC could be prepared as design evolves. The CBS, which forms the structure of the database and output format and the discounting and risk analysis techniques that could be adopted were also, identified. A mathematical model was then formulated using fuzzy extension principle and interval analysis. Finally an algorithm was proposed, based on which the computer model was implemented.

MODEL DESIGN AND DEVELOPMENT

In order to design a robust WLCC model, certain issues must be addressed. These include the sequence of design evolution to adopt, the cost breakdown structure to adopt, the variables to be included at each stage of design evolution, the risk analysis technique to be used and the formulation of WLCC mathematical model. The following sections discuss these issues in detail.

SEQUENCE OF DESIGN EVOLUTION

The choice of an appropriate sequence of design evolution is very important so as to ensure adaptability, usability and acceptability of the model. In order to achieve the aforementioned, the sequence should support WLCC activities and decisions at the various stages.

Boussabaine and Kirkham (2004) noted that clients and investors could view WLCC process of a building facility as a sequence of investment decisions and that the process contains parallel and interrelated phases. In addition, Flanagan and Jewell (2005) emphasized that the potential for making effective use of WLCC is maximum during the early design stages and the ability to influence cost decreases continually as the project progresses from 100% at the project sanction to typically 20% by the time the construction starts . Moreover, Flanagan *et al*. (1983) showed that the various WLCC activities can be incorporated into the RIBA plan of research. By comparing the WLCC decision phases with the RIBA plan of work, it could be observed that, the 2 can be dovetailed. For instance the Inception and Feasibility stages of the RIBA plan of work could be seen as the Concept decision stage, while the Outline proposals, Scheme design, Detail design, Production information and Bill of Quantities stages could be seen as collectively representing the design decision stage. Due to the popularity of the RIBA plan of work and the fact that both the WLCC activities and decision phases could be dovetailed into it, it was selected as the design sequence in the model.

COST BREAKDOWN STRUCTURE (CBS)

Different CBSs exist in the industry. It is however, necessary to identify a format that is comprehensive, popular and easy to adopt in WLCC computation. Flanagan and Jewell (2005) proposed a CBS that could be utilized in WLCC. The CBS took into cognizance the cost categories of the Building Maintenance Information (BMI), Building Cost Information Service (BCIS), Chartered Institute of Public Finance and Accounting and the Department of Environment. Though, the structure is relatively new, it is a combination of popular and well accepted structures, which makes it comprehensive and useful. The uses of this structure as pointed by Flanagan and Jewell (2005) are that: it can act as a check list of items; it can form the basis of categories for Whole Life Cost Plan or Whole Life Cost Management when used in conjunction with measurement information; cost information can be recorded under the various categories when compiling a Whole Life Cost Analysis. Due to the comprehensiveness and usefulness of this structure it was adopted in the design of the model.

VARIABLES TO BE INCLUDED IN THE MODEL

The variables to be used at each stage of design evolution are dependent upon the CBS adopted and the amount of information available. More information becomes available as the design passes from the inception, through operation and life. Thus, more variables are included in the WLCC computations as design develops. Global variables like discount rate (r), analysis period (n), components life-span (e), inflation rates (i), cost of borrowing (b), Risk Reserves (RR) and level 1 cost variables like Capital Costs (CC), Finance Costs (FC), Operation Costs (OC), Maintenance Costs (MC), Sundries (S), Salvage and Residuals (SR) may be required at all the stages. Each of the cost centers could be further broken down into sub-elements depending upon the design stage and cost information available.

IDENTIFICATION OF RISK ANALYSIS TECHNIQUE

Kishk and Al-Hajj (1999), proposed a framework for WLCC, which evaluates data on the basis of availability, tangibility and uncertainty. The framework utilizes the capabilities of Fuzzy Set Theory (FST), Artificial Neural Networks (ANN), probability and statistics to handle the various tasks associated with WLCC. A thorough examination of the framework reveals the necessity for a feedback loop. Some data after passing through the framework may require further processing. Boussabaine (1997) have shown that, the opacity in the underlying mechanism of an ANN may necessitate further process of the output using FST and Ross (2004) emphasized that, ANN may be used in defining a fuzzy membership function. Figure 1 is a schematic representation of the framework with the feedback incorporated.

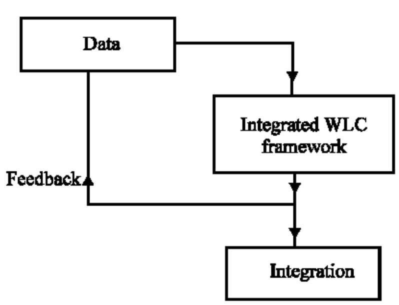

Fig. 1: Schematic representation of WLCC Integrated framework with feedback

By examining the framework, this study identifies FST as the best risk analysis technique that may be employed in handling uncertainty and scarcity in WLCC data.

FORMULATION OF WLCC MATHEMATICAL MODEL

Discount rate: According to Flanagan and Jewell (2005), discount rates may be expressed either in Nominal (Market) terms where, the effects of general price inflation and real earning power of money invested over time are reflected, or in Real terms where only the earning power of money is reflected and the effects of inflation are not included. Considering the nature of the Nigerian economy, characterized by persistent rise in the prices of goods and services, the effect of inflation should be accounted for in the discount rate.

Seeley (1996) suggested the use of the formula:

$$r = \{(1+b)/(1+i)\} - 1$$

where:

r = The discount rate
b = The borrowing rate
I = The inflation rate

Simplifying the equation yields:

$$r = \frac{1+b}{1+i} - 1 \quad (1)$$

Flanagan and Jewell (2005) noted that, this formula should only be used where b > i. Where, b< i, they suggested the use of the formula

$$r = \frac{1+i}{1+b} - 1 \quad (2)$$

COST VARIABLES

All the costs associated with WLCC could be broken-down into four broad groups. These are; initial costs, annual costs, periodic (non-annual) costs and Future one-off costs (e.g. terminal cost). In order to adjust for time value of money, each cost category will be discounted using the following discounting technique:

Initial costs: These are in present worth, thus, they will not require any discounting.

Annual costs: These will be discounted to the present using the present-worth of an annuity factor (PWA).

$$PWA = \frac{A-1}{rA} \quad (3)$$

where, Ais the future worth factor and is defined by:

$$A = (1+r)^n \quad (4)$$

and n is the analysis period.

Periodic costs: These will be discounted to the present by considering the number of times (k) the cost will be incurred and the frequency at which the cost will be incurred; that is the component's life-cycle (e). Mathematically,

$$k = \mathrm{Int}(n/e) \quad (5)$$

and the future worth factor A of the total replacement cost is defined as:

$$A' = (1+r)^e \left\{ \frac{(1+r)^{ke} - 1}{(1+r)^e - 1} \right\} \quad (6)$$

The Amortization Factor (AF) is defined as:

$$AF' = \frac{A'r}{A'-1} \quad (7)$$

Future one-off costs: These will be discounted to the present using the Present Worth (PW) factor, which is defined as:

$$PW = \frac{1}{A} \quad (8)$$

FUZZIFICATION OF THE EQUATIONS

Cost information could be expressed using different types of Triangular Fuzzy Numbers such as rectangular, triangular, trapezoidal, Bayesian etc fuzzy numbers. TFN was used in the Fuzzification of the equations for the following reasons:

- Quantitative variables like costs, inflation rate, life expectancy, analysis period, borrowing rate etc must have lower and upper limits. Outside these limits the variables cannot assume any value. This suggests that, the membership functions for these variables are close ended.
- Each variable may take any value between the two limits. This suggests that the variables are continuous within those limits.
- A value with highest likelihood of occurrence could be identified within the limits, which suggest that their membership function has a convex shape.
- Triangular membership function has been identified as the most popular and easy to use (Ross, 2004).

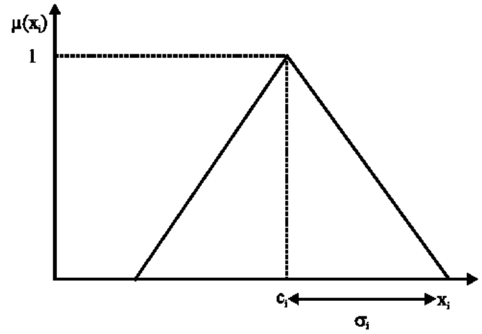

Fig. 2: Fuzzy triangular membership function

Using three-point data to represent the lower, likely and upper values of all the independent variables in the model, a triangular membership function, as shown in Fig. 2 could be defined for each variable.
Defined as:

$$\mu(x_i) = \begin{cases} \max\{0, 1 + \frac{x_i - c_i}{1.0\sigma_i}\} & \text{if } x_i \le c_i \\ \max\{0, 1 + \frac{c_i - x_i}{1.0\sigma_i}\} & \text{otherwise} \end{cases}$$

ECONOMIC VARIABLES

Using fuzzy extension principle and interval analysis on Eq. 1 and 2, the lower and upper bounds for the discount rate becomes:

$$[r_l, r_u] = \frac{[b_l, b_u] - [i_l, i_u]}{1 + [i_l, i_u]} = \frac{[(b_l - i_u), (b_u - i_l)]}{[(1+i_l), (1+i_u)]} = \left[\left(\frac{b_l - i_u}{1+i_u} \right), \left(\frac{b_u - i_l}{1+i_l} \right) \right] \quad (9)$$

where, $i < b$ and $b_l > b_u$

$$[r_l, r_u] = \frac{[i_l, i_u] - [b_l, b_u]}{1 + [b_l, b_u]} = \frac{[(i_l - b_u), (i_u - b_l)]}{[(1+b_l), (1+b_u)]} = \left[\left(\frac{i_l - b_u}{1+b_u} \right), \left(\frac{i_u - b_l}{1+b_l} \right) \right] \quad (10)$$

Where, $i < b$ and $b_l > b_u$

The subscripts l and u represents the lower and upper bounds of the variables in question.

Given any membership value (α), the lower and upper values that corresponds to the α-cuts for the variables b, i, n and e are:

$$[b_{\alpha l}, b_{\alpha u}] = [(b\alpha - b_l(\alpha - 1)), (b\alpha - b_u(\alpha + 1))] \quad (11)$$

$$[i_{\alpha l}, i_{\alpha u}] = [(i\alpha - b_l(\alpha - 1)), (i\alpha - i_u(\alpha + 1))] \quad (12)$$

$$[n_{\alpha l,}n_{\alpha u}] = [(n\alpha - n_l(\alpha-1)),(n\alpha - n_u(\alpha+1))] \quad (13)$$

$$[e_{\alpha l,}e_{\alpha u}] = [(e\alpha - e_l(\alpha-1)),(e\alpha - e_u(\alpha+1))] \quad (14)$$

DISCOUNTING FACTORS

Using fuzzy extension principle and interval analysis on Eq. 3-8, the lower and upper bounds of the equations at any point on the membership function are outlined:

$$[PWA_l, PWA_U] = \left[\left(\frac{A_l - 1}{r_u A_u}\right), \left(\frac{A_u - 1}{r_l A_l}\right)\right] \quad (15)$$

$$[A_l, A_u] = [(1+r_l)^{n_l}, (1+r_u)^{n_u}] \quad (16)$$

$$[k_l, k_u] = [\mathrm{Int}(n_l / e), \mathrm{Int}(n_u / e)] \quad (17)$$

$$A'_l = (1+r_l)^e \left\{\frac{(1+r_l)^{k_l e} - 1}{(1+r_u)^e - 1}\right\} \quad (18)$$

$$A'_u = (1+r_u)^e \left\{\frac{(1+r_u)^{k_u e} - 1}{(1+r_l)^e - 1}\right\} \quad (19)$$

$$[AF'_l, AF'_u] = \left[\left(\frac{A'_l r_l}{A_u - 1}\right), \left(\frac{A'_u r_u}{A_l - 1}\right)\right] \quad (20)$$

$$[PW_l, PW_u] = \left[\frac{1}{A_u}, \frac{1}{A_l}\right] \quad (21)$$

COST VARIBLES

Inception and feasibility stage: Whole Life-cycle Cost (WLCC) could be expressed as an aggregation of all the level one cost variables. Thus,

$$WLCC = CC + FC + OC + MC + S + SR + RR \quad (22)$$

Let, the annual finance cost, annual operational cost, annual maintenance cost, annual sundries expenses and the present equivalent for the salvage or residual value be represented by AFC, AOC, AMC, AS and PSR, respectively. Then

$$WLCC = CC + PWA(AFC + AOC + AMC + AS) + PW(PSR) + RR \quad (23)$$

Given any value of alpha (α), the lower and upper values that correspond to the α-cuts for the variables CC, AFC, AOC, AMC, AS, PSR and RR are:

$$[CC_{\alpha l,}CC_{\alpha u}] = [(CC\alpha - CC_l(\alpha-1)), (CC\alpha - CC_u(\alpha+1))] \quad (24)$$

$$[AFC_{\alpha l,}AFC_{\alpha u}] = [(AFC\alpha - AFC_l(\alpha-1)), (AFC\alpha - AFC_u(\alpha+1))] \quad (25)$$

$$[AOC_{\alpha l,}AOC_{\alpha u}] = [(AOC\alpha - AOC_l(\alpha-1)), (AOC\alpha - AOC_u(\alpha+1))] \quad (26)$$

$$[AMC_{\alpha l,}AMC_{\alpha u}] = [(AMC\alpha - AMC_l(\alpha-1)), (AMC\alpha - AMC_u(\alpha+1))] \quad (27)$$

$$[AS_{\alpha l,}AS_{\alpha u}] = [(AS\alpha - AS_l(\alpha-1)), (AS\alpha - AS_u(\alpha+1))] \quad (28)$$

$$[PSR_{\alpha l,}PSR_{\alpha u}] = [(PSR\alpha - PSR_l(\alpha-1)), (PSR\alpha - PSR_u(\alpha+1))] \quad (29)$$

$$[RR_{\alpha l,}RR_{\alpha u}] = [(RR\alpha - RR_l(\alpha-1)), (RR\alpha - RR_u(\alpha+1))] \quad (30)$$

OUTLINE PROPOSAL AND SCHEME DESIGN STAGE

At this stage the cost elements could be further broken down to level two at the outline proposal and level three at the scheme design stage. The WLCC mathematical model could be further expanded to take into cognizance the summation of various sub-elements to arrive at the total for an element. Thus:

$$WLCC = \sum_{j=1}^{m} CC_j + PWA\left(\sum_{j=1}^{p} AFC_j + \sum_{j=1}^{q} AOC_j + \sum_{j=1}^{t} AMC_j + \sum_{j=1}^{v} AS_j\right) + PW\left(\sum_{j=1}^{w} PSR_j\right) + RR \quad (31)$$

where, m, p, q, t, v and w represent the number of sub-elements under each cost centre.

DETAIL DESIGN AND PRODUCTION INFORMATION

At these stages, specific information on components would have been available. It will therefore, be logical to consider component life cycle in the computation of maintenance cost.

The maintenance cost could be divided into periodic replacement cost and the routine maintenance cost. To formulate the periodic replacement cost, Let, RC be the component replacement cost, TRC be the present equivalent of the total replacement cost and ATRC be the annual equivalent of the total replacement cost. From Eq. 6 and 7,

$$TRC = RC \times A' \quad (32)$$

Given any value of alpha (α), the lower and upper values that correspond to the α-cuts for the variable RC are:

$$ATRC = RC \times AF' \quad (33)$$

The Annual equivalent of the total Maintenance Cost (AMC) is given by:

$$[RC_{l,} RC_u] = [(RC\alpha - RC_l(\alpha - 1)), (RC\alpha - RC_u(\alpha + 1))] \quad (34)$$

where ARM is the annual routine maintenance.

COMPUTER IMPLEMENTATION

Due to the flexibility and robustness of spreadsheet software (MS Excel), it was used as the interface for implementing this model. The algorithm that was followed in the implementation of the model is described as follows:

- Input general project information like project title, location, description etc.
- Input three point data for inflation rate, cost of borrowing and analysis period.
- Define the membership functions for cost of borrowing, inflation rate and analysis period by generating their values at various α-cuts using Eq. 11-13, respectively.
- Define membership functions for the discount rate using Eq. 9 or 10.
- Define membership functions for the present worth of an annuity factor, future worth factor and the present worth factor using Eq. 15, 16 and 21, respectively.
- Input three point data for Capital Cost, Annual Finance Cost, Annual Operating Cost, Annual Maintenance Cost, Annual Sundries, Salvage and Residual Values, Risk reserves and define their membership functions using Eq. 24-30, respectively.
- Define the membership functions for the discounted values of the cost categories in 6 above by applying the α-cut values of the relevant discount factor.
- Input three point data for components life cycle and components replacement costs and define their membership functions using Eq. 14 and 34, respectively.
- Define membership function for number of replacements using Eq. 17.
- Define membership function for future worth factor and amortization factor to be used in discounting the replacement cost using Eq. 18-20.
- Define membership functions for Total replacement cost and Annual total replacement costs using Eq. 32 and 33.
- Define the membership function for Annual routine maintenance using Eq. 35 and 36.
- Define membership function for Annual maintenance cost using Eq. 37
- Define membership function for WLCC by substituting the α-cut values for the discounted cost categories in 8 and/or 15 above in Eq. 23 or 31.
- Define certainty level by selecting a specific α-cut for each input variable defined above and recompute the WLCC using the ranges of values obtained.

CONCLUSION

- A WLCC model was developed in order to overcome the problem of data scarcity and uncertainty in WLCC computations.
- The integrated WLCC framework proposed by Kishk and Al-Hajj (1999) was used in the design and development of the model. Going by this framework, FST appeared be the best option considering the nature of our construction industry and hence, it was selected as the risk analysis tool for the model.
- The model could be used at the various stages of the RIBA plan of research.
- The model produces its output in the format of the CBS proposed by Flanagan and Jewell (2005).
- The model prepares an estimate of a WLCC of a proposed project based on 3 point data input of the various cost variables.
- Membership function for the various cost and economic variables used in the model are developed to assist the user in the selection of an appropriate alpha-cut (degree of certainty) that may be used in the interpretation of the result.

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