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Energy Decay Law of Dusty Fluid Turbulent Flow in a Rotating System

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Abstract: Homogeneous dusty fluid turbulent flow in a rotating system has been considered using 3 and 4 point correlation equations the set of equations is made determinate by neglecting the quintuple correlations in comparison to the 3rd and 4th order correlation terms. Here the correlation equations are converted to spectral form by taking Fourier transforms. Finally, integrating the energy spectrum over all wave numbers, the energy decay law of homogeneous dusty fluid turbulent flow in a rotating system before reaching at the steady state obtained.

Key words: Deissler's method, correlation, dusty fluid, turbulent flow, energy decay law

INTRODUCTION

The motion of dusty fluid occurs in the movement of dust-laden air, in problems of fluidization, in the use of dust in a gas cooling system and in the sedimentation problem of tidal rivers. When the motion is referred to axes, which rotate steadily with the bulk of the fluid, the coriolis force and centrifugal force must be supposed to act on the fluid. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow while the centrifugal force with the potential is incorporated into the pressure. Deissler (1958, 1960) generalized a theory "Decay of homogeneous turbulence for times before the final period. Saffman (1962) derived an equation that described the motion of a fluid containing small dust particles. Dixit and Upadhyay (1989), Kishore and Dixit (1979) and Kishore and Singh (1984) discussed the effect of Coriolis force on acceleration covariance in ordinary and MHD turbulent flows. Shimomura and Yoshizawa (1986) and Shimomura (1986, 1989) also discussed the statistical analysis of turbulent viscosity, turbulent scalar flux and turbulent shear flows, respectively in a rotating system by two-scale Direct-interaction approach. Kishore and Upadhyay (2000) studied the decay of MHD turbulence in a rotating system. Sarker and Islam (2001) also studied the decay of dusty fluid turbulence before the final period in a rotating system using 2 and 3 point correlation equations. By analyzing the above theories we have studied the among decay law of dusty fluid turbulent flow in a rotating system using 3 and 4 point correlation equations.

CORRELATION AND SPECTRAL EQUATIONS

Equations of motion of dusty fluid turbulence in a rotating system at the points p, $p^{/}$, $p^{//}$ and $p^{///}$ are:

$$\frac{\partial u_i}{\partial t}+\frac{\partial(u_i u_m)}{\partial x_m}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_m \partial x_m}-2\varepsilon_{nmi}\Omega_n u_i+f(u_i-v_i) \tag{1}$$

$$\frac{\partial u_j^{/}}{\partial t}+\frac{\partial(u_j^{/} u_m^{/})}{\partial x_m^{/}}=-\frac{1}{\rho}\frac{\partial p^{/}}{\partial x_j^{/}}+\nu\frac{\partial^2 u_j^{/}}{\partial x_m^{/} \partial x_m^{/}}-2\varepsilon_{pmj}\Omega_p u_j^{/}+f(u_j^{/}-v_j^{/}) \tag{2}$$

$$\frac{\partial u_k^{//}}{\partial t}+\frac{\partial(u_k^{//} u_m^{//})}{\partial x_m^{//}}=-\frac{1}{\rho}\frac{\partial p^{//}}{\partial x_k^{//}}+\nu\frac{\partial^2 u_k^{//}}{\partial x_m^{//} \partial x_m^{//}}-2\varepsilon_{qmk}\Omega_q u_k^{//}+f(u_k^{//}-v_k^{//}) \tag{3}$$

$$\frac{\partial u_l^{///}}{\partial t}+\frac{\partial(u_l^{///} u_m^{///})}{\partial x_m^{///}}=-\frac{1}{\rho}\frac{\partial p^{///}}{\partial x_l^{///}}+\nu\frac{\partial^2 u_l^{///}}{\partial x_m^{///} \partial x_m^{///}}-2\varepsilon_{rml}\Omega_r u_l^{///}+f(u_l^{///}-v_l^{///}) \tag{4}$$

Multiplying Eq. 1 by $u_j^{/}$, $u_k^{//}$ $u_l^{///}$, Eq. 2 by u_j, $u_k^{//}$ $u_l^{///}$, Eq. 3 by u_i, $u_j^{/}$ $u_l^{///}$ and Eq. 4 by u_i, $u_j^{/}$ $u_k^{///}$ then adding and taking ensemble average writing in terms of the independent variables r, $r^{/}$ and $r^{//}$ as:

$$
\begin{aligned}
&\frac{\partial}{\partial t}\langle u_i u'_j u''_k u'''_l\rangle - \frac{\partial}{\partial r_m}\langle u_i u_m u'_j u''_k u'''_l\rangle - \frac{\partial}{\partial r'_m}\langle u_i u_m u'_j u''_k u'''_l\rangle - \frac{\partial}{\partial r''_m}\langle u_i u_m u'_j u''_k u'''_l\rangle \\
&+\frac{\partial}{\partial r_m}\langle u_i u'_j u'_m u''_k u'''_l\rangle + \frac{\partial}{\partial r'_m}\langle u_i u'_j u''_k u''_m u'''_l\rangle + \frac{\partial}{\partial r''_m}\langle u_i u'_j u''_k u'''_l u'''_m\rangle \\
&= -\tfrac{1}{\rho}\left(-\frac{\partial}{\partial r_i}\langle p u'_j u''_k u'''_l\rangle - \frac{\partial}{\partial r'_i}\langle p u'_j u''_k u'''_l\rangle - \frac{\partial}{\partial r''_i}\langle p u'_j u''_k u'''_l\rangle + \frac{\partial}{\partial r_j}\langle u_i p' u''_k u'''_l\rangle + \frac{\partial}{\partial r'_k}\langle u_i u'_j p'' u'''_l\rangle + \frac{\partial}{\partial r''_l}\langle u_i u'_j u''_k p'''\rangle\right) \\
&+2\nu\left(\frac{\partial^2\langle u_i u'_j u''_k u'''_l\rangle}{\partial r_m \partial r_m} + \frac{\partial^2\langle u_i u'_j u''_k u'''_l\rangle}{\partial r_m \partial r'_m} + \frac{\partial^2\langle u_i u'_j u''_k u'''_l\rangle}{\partial r_m \partial r''_m} + \frac{\partial^2\langle u_i u'_j u''_k u'''_l\rangle}{\partial r'_m \partial r'_m} + \frac{\partial^2\langle u_i u'_j u''_k u'''_l\rangle}{\partial r'_m \partial r''_m} + \frac{\partial^2\langle u_i u'_j u''_k u'''_l\rangle}{\partial r''_m \partial r''_m}\right) \\
&-2\left(\varepsilon_{nmi}\Omega_n\langle u_i u'_j u''_k u'''_l\rangle + \varepsilon_{pmj}\Omega_p\langle u_i u'_j u''_k u'''_l\rangle + \varepsilon_{qmk}\Omega_q\langle u_i u'_j u''_k u'''_l\rangle + \varepsilon_{rml}\Omega_r\langle u_i u'_j u''_k u'''_l\rangle\right) \\
&+f\left(-\langle v_i u'_j u''_k u'''_l\rangle - \langle u_i v'_j u''_k u'''_l\rangle - \langle u_i u'_j v''_k u'''_l\rangle - \langle u_i u'_j u''_k v'''_l\rangle + 4\langle u_i u'_j u''_k v'''_l\rangle\right)
\end{aligned} \tag{5}
$$

By using $\frac{\partial}{\partial x'_m} = \frac{\partial}{\partial r_m}$, $\frac{\partial}{\partial x''_m} = \frac{\partial}{\partial r'_m}$, $\frac{\partial}{\partial x'''_m} = \frac{\partial}{\partial r''_m}$ and $\frac{\partial}{\partial x_m} = -\frac{\partial}{\partial r_m} - \frac{\partial}{\partial r'_m} - \frac{\partial}{\partial r''_m}$

In order to convert Eq. 5 to spectral form, using nine-dimensional Fourier transforms (Sultana and Sarker, 2004; Sultana *et al*., 2006) we obtain,

Substituting the preceding relations into Eq. 5, we get

$$
\begin{aligned}
&\frac{d}{dt}(\gamma_{ijkl}) + 2\nu\left(k^2 + k_m k'_m + k_m k''_m + k'^2 + k'_m k''_m + k''^2\right)\gamma_{ijkl} = [i(k_m + k'_m + k''_m)\gamma_{imjkl}(k,k',k'') - ik_m\gamma_{jmikl}(-k-k'-k'',k',k'') \\
&-ik'_m\gamma_{kmijl}(-k-k'-k'',k,k'') - ik''_m\gamma_{lmijk}(-k-k'-k'',k,k')] - \frac{1}{\rho}[-i(k_i + k'_i + k''_i)\delta_{jkl}(k,k',k'') + ik_j\delta_{ikl}(-k-k'-k'',k',k'') \\
&+ik'_k\delta_{ijl}(-k-k'-k'',k,k'') + ik''_l\delta_{ijk}(-k-k'-k'',k,k')] - 2\left(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmj}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r\right)\gamma_{ijkl} \\
&+f\left[4\gamma_{ijkl}(k,k',k'') - \gamma_i\delta_{jkl}(k,k',k'') - \gamma_j\delta_{ikl}(-k-k'-k'',k',k'') - \gamma_k\delta_{ijl}(-k-k'-k'',k,k'') - \gamma_l\delta_{ijk}(-k-k'-k'',k,k')\right]
\end{aligned} \tag{6}
$$

To obtain a relation between the terms on the right side of Eq. 6 derived from the quadruple correlation terms, pressure terms, rotational terms and the dust particle terms in Eq. 5, take the divergence of the equation of motion and combine with the continuity equation to give

$$
\frac{1}{\rho}\frac{\partial^2 p}{\partial x_m \partial x_m} = -\frac{\partial^2 (u_m u_n)}{\partial x_m \partial x_n} \tag{7}
$$

Multiplying Eq. 7 by u'_j, u''_k u'''_l, taking ensemble average and writing the resulting equation in terms of the independent variables r and r', gives

$$
\begin{aligned}
&\frac{1}{\rho}\left(\frac{\partial^2\langle p u'_j u''_k u'''_l\rangle}{\partial r_m \partial r_m} + 2\frac{\partial^2\langle p u'_j u''_k u'''_l\rangle}{\partial r_m \partial r'_m} + 2\frac{\partial^2\langle p u'_j u''_k u'''_l\rangle}{\partial r_m \partial r''_m} + \frac{\partial^2\langle p u'_j u''_k u'''_l\rangle}{\partial r'_m \partial r'_m} + 2\frac{\partial^2\langle p u'_j u''_k u'''_l\rangle}{\partial r'_m \partial r''_m} + \frac{\partial^2\langle p u'_j u''_k u'''_l\rangle}{\partial r''_m \partial r''_m}\right) \\
&= -\left(\frac{\partial^2\langle u_m u_n u'_j u''_k u'''_l\rangle}{\partial r_m \partial r_n} + \frac{\partial^2\langle u_m u_n u'_j u''_k u'''_l\rangle}{\partial r_m \partial r'_n} + \frac{\partial^2\langle u_m u_n u'_j u''_k u'''_l\rangle}{\partial r_m \partial r''_n} + \frac{\partial^2\langle u_m u_n u'_j u''_k u'''_l\rangle}{\partial r'_m \partial r_n} + \frac{\partial^2\langle u_m u_n u'_j u''_k u'''_l\rangle}{\partial r'_m \partial r'_n}\right. \\
&\left. + \frac{\partial^2\langle u_m u_n u'_j u''_k u'''_l\rangle}{\partial r'_m \partial r''_n} + \frac{\partial^2\langle u_m u_n u'_j u''_k u'''_l\rangle}{\partial r''_m \partial r_n} + \frac{\partial^2\langle u_m u_n u'_j u''_k u'''_l\rangle}{\partial r''_m \partial r'_n} + \frac{\partial^2\langle u_m u_n u'_j u''_k u'''_l\rangle}{\partial r''_m \partial r''_n}\right)
\end{aligned} \tag{8}
$$

The Fourier transform of Eq. 8 is

$$-\frac{1}{\rho}\delta_{jkl} = \frac{\left(k_m k_n + k_m k_n' + k_m k_n'' + k_m' k_n + k_m' k_n' + k_m' k_n'' + k_m'' k_n + k_m'' k_n' + k_m'' k_n''\right)\gamma_{mnjkl}}{\left(k^2 + 2k_m k_m' + 2k_m k_m'' + k'^2 + 2k_m' k_m'' + k''^2\right)} \tag{9}$$

Equation 6 and 9 are the spectral equations corresponding to the 4 point correlation equations. The spectral equations corresponding to the three point correlation equations are:

$$\begin{aligned}
&\frac{d}{dt}(k_k\beta_{iik}) + 2\nu(k^2 + k_l k_l' + k'^2)k_k\beta_{iik} = ik_k(k_l + k_l')\beta_{ilik}(\underline{k}, \underline{k}') - ik_k k_l \beta_{ilik}(-\underline{k} - \underline{k}', \underline{k}') \\
&-ik_k k_l' \beta_{klii}(-\underline{k} - \underline{k}', \underline{k}) - \frac{1}{\rho}\left[-ik_k(k_i + k_i')\alpha_{ik}(\underline{k}, \underline{k}') + ik_k k_i \alpha_{ik}(-\underline{k} - \underline{k}', \underline{k}') + ik_k k_k' \alpha_{ii}(-\underline{k} - \underline{k}', \underline{k})\right] \\
&-2k_k\left[\varepsilon_{mli}\Omega_m + \varepsilon_{nli}\Omega_n + \varepsilon_{qlk}\Omega_q\right]\beta_i\beta_i'\beta_k'' + Rfk_k
\end{aligned} \tag{10}$$

Where:

$$R\beta_i\beta_i'\beta_k'' = 3\left\langle \beta_i\beta_i'\beta_k'' \right\rangle - \left\langle \gamma_i\beta_i'(\underline{k})\beta_k''(\underline{k}') \right\rangle - \left\langle \gamma_i\beta_i'(-\underline{k}-\underline{k}')\beta_k''(\underline{k}') \right\rangle - \left\langle \gamma_k\beta_i'(-\underline{k}-\underline{k}')\beta_i''(\underline{k}) \right\rangle, \text{(say)}$$

R is an arbitrary constant and

$$-\frac{1}{\rho}\alpha_{ik} = \frac{k_l k_m + k_l' k_m + k_l k_m' + k_l' k_m'}{k^2 + 2k_l k_l' + k'^2}\beta_{lmik} \tag{11}$$

Here the spectral tensors are defined by

$$\left\langle u_i u_j'(\underline{r}) u_k''(\underline{r}') \right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \beta_{ijk}(\underline{k}, \underline{k}')\exp[i(\underline{k}.\underline{r} + \underline{k}'.\underline{r}')]d\underline{k}\,d\underline{k}' \tag{12}$$

$$\left\langle u_i u_l u_j'(\underline{r}) u_k''(\underline{r}') \right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \beta_{iljk}(\underline{k}, \underline{k}')\exp[i(\underline{k}.\underline{r} + \underline{k}'.\underline{r}')]d\underline{k}\,d\underline{k}' \tag{13}$$

$$\left\langle p u_j'(\underline{r}) u_k''(\underline{r}') \right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha_{jk}(k, k')\exp[i(\underline{k}.\underline{r} + \underline{k}'.\underline{r}')]dkdk' \tag{14}$$

A relation between β_{iljk} and γ_{ijkl} can be obtained by letting $r'' = 0$ in Eq. 12 and comparing the result with Eq. 13,

$$\beta_{iljk}(\underline{k}, \underline{k}') = \int_{-\infty}^{\infty} \gamma_{ijkl}(\underline{k}, \underline{k}', \underline{k}'')d\underline{k}'' \tag{15}$$

The spectral equation corresponding to the 2 point correlation equation in presence of dusty fluid in a rotating system is

$$\begin{aligned}
&\frac{d}{dt}\phi_{i,i} + (2\nu k^2 - Qf + 2\varepsilon_{mki}\Omega_m + 2\varepsilon_{nki}\Omega_n)\phi_{i,i} \\
&= ik_k\phi_{iki}(\underline{k}) - ik_k\phi_{iki}(-\underline{k})
\end{aligned} \tag{16}$$

where, $\phi_{i,i}$ and ϕ_{iki} are defined by

$$\left\langle u_i u_j'(\underline{r})\right\rangle = \int_{-\infty}^{\infty} \phi_{ij}(\underline{k}) \exp(i\underline{k}\cdot\underline{r}) d\underline{k} \tag{17}$$

and

$$\left\langle u_i u_k u_j'(\underline{r})\right\rangle = \int_{-\infty}^{\infty} \phi_{ikj}(\underline{k}) \exp(i\underline{k}\cdot\underline{r}) d\underline{k} \tag{18}$$

The relation between ϕ_{ikj} and β_{ijk} obtained by letting $\underline{r}' = 0$ in Eq. 13 and comparing the result with Eq. 18 is

$$\phi_{ikj}(\underline{k}) = \int_{-\infty}^{\infty} \beta_{ijk}(\underline{k},\underline{k}') d\underline{k}' \tag{19}$$

SOLUTION

Equation 9 shows that if the terms corresponding to the quintuple correlations are neglected, then the pressure force terms also must be neglected. Thus neglecting first and second terms on the right side of Eq. 6, the equation can be integrated between t_1 and t to give

$$\gamma_{ijkl} = (\gamma_{ijkl})_1 \exp\left[\left\{-2\nu(k^2 + k_m k_m' + k_m k_m'' + k'^2 + k_m' k_m'' + k''^2) + 2\left(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmj}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r\right) - sf\right\}(t - t_1)\right] \tag{20}$$

Where:

$$\begin{aligned} S\gamma_{ijkl} &= 4\gamma_{ijkl}(\underline{k},\underline{k}',\underline{k}'') - \gamma_i\delta_{jkl}(\underline{k},\underline{k}',\underline{k}'') - \gamma_j\delta_{ikl}(-\underline{k}-\underline{k}'-\underline{k}'',\underline{k}',\underline{k}'') \\ &-\gamma_k\delta_{ijl}(-\underline{k}-\underline{k}'-\underline{k}'',\underline{k},\underline{k}'') - \gamma_l\delta_{ijk}(-\underline{k}-\underline{k}'-\underline{k}'',\underline{k},\underline{k}') \end{aligned} \tag{21}$$

is an arbitrary constant and (γ_{ijkl}) is the value of γ_{ijkl} at $t = t_1$. The quantity (γ_{ijkl}) can be considered also as the value of γ_{ijkl} at small values of k, k' and k'', at least for times when the quintuple correlations are neglected.

Equation 15 and 20 can be converted to scalar form by contracting the indices i and j, as well as k and l. Substitution of Eq. 11, 15 and 20 into the three point scalar Eq. 10 results in

$$\begin{aligned} k_k\beta_{iik} &= (k_k\beta_{iik})_0 \exp\left[\left\{-2\nu(k^2 + k_l k_l' + k'^2) + 2\left(\varepsilon_{mli}\Omega_m + \varepsilon_{nli}\Omega_n + \varepsilon_{qlk}\Omega_q\right) - Rf\right\}(t - t_0)\right] \\ &+ \frac{\pi^{\frac{3}{2}}[a]_1}{\nu}\left\{\omega^{-1}\exp\left[-\omega^2\left(\frac{3}{4}k^2 + \frac{1}{2}k_l k_l' + \frac{3}{4}k'^2\right) + \left\{2\left(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r\right) - Sf\right\}(t - t_1)\right]\right. \\ &+ 2\left(\frac{1}{4}k^2 + \frac{1}{2}k_l k_l' + \frac{1}{4}k'^2\right)^{\frac{1}{2}} \exp[-\omega^2(k^2 + k_l k_l' + k'^2) + \left\{2\left(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r\right) - Sf\right\}(t - t_1)] \\ &\times \int_0^{\omega\left(\frac{1}{4}k^2 + \frac{1}{2}k_l k_l' + \frac{1}{2}k'^2\right)^{\frac{1}{2}}} \exp\left(x^2\right)dx\Big\} + \frac{\pi^{\frac{3}{2}}[b]_1}{\nu}\{-\omega^{-1}\exp[-\omega^2(\frac{3}{4}k^2 + k_l k_l' + k'^2) + \left\{2\left(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qlk}\Omega_q + \varepsilon_{rml}\Omega_r\right)\right. \\ &\qquad -Sf\}(t - t_1)] \\ &+ k\exp[-\omega^2(k^2 + k_l k_l' + k'^2) + \left\{2\left(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r\right) - Sf\right\}(t - t_1)]\int_0^{\frac{1}{2}\omega k} \exp\left(x^2\right)dx\} \\ &+ \frac{\pi^{\frac{3}{2}}[c]_1}{\nu}\{-\omega^{-1}\exp[-\omega^2(k^2 + k_l k_l' + \frac{3}{4}k'^2) + \left\{2\left(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r\right) - Sf\right\}(t - t_1)] \\ &+ k'\exp\left[-\omega^2(k^2 + k_l k_l' + k'^2) + \left\{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\right\}(t - t_1)\right]\int_0^{\frac{1}{2}\omega k'} \exp\left(x^2\right)dx\} \end{aligned} \tag{22}$$

Where:

$$\omega = [2\nu(t - t_1)]^{\frac{1}{2}}$$

In order to simplify the calculations, we shall assume that $[a]_1 = 0$; that is, we assume that a function sufficiently general to represent the initial conditions can be obtained by considering only the terms involving $[b]_1$ and $[c]_1$.

The substitution of Eq. 19 and 22 in Eq. 16 and setting $E = 1\pi k^2 \phi_{i,i}$ results in

$$\frac{dE}{dt} + (2\nu k^2 + 2\varepsilon_{mki}\Omega_m + 2\varepsilon_{nki}\Omega_n - Qf)E = W \tag{23}$$

Where:

$$\begin{aligned}
W = {} & k^2 \int_{-\infty}^{\infty} 2\pi i[k_k\beta_{iik}(k,k') - k_k\beta_{iik}(-k,-k')]_0 \exp[\{-2\nu(k^2 + k_1k_1' + k'^2) + 2(\varepsilon_{mli}\Omega_m + \varepsilon_{nli}\Omega_n + \varepsilon_{qlk}\Omega_q) - Rf\}(t - t_0)]dk' \\
& + k^2 \int_{-\infty}^{\infty} \frac{2\pi^{\frac{5}{2}} i}{\nu}[b(k,k') - b(-k,-k')]_1\{-w^{-1}\exp[-w^2(\frac{3}{4}k^2 + k_1k_1' + k'^2) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\}(t - t_1)] \\
& + k\exp[-w^2(k^2 + k_1k_1' + k'^2) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\}(t - t_1)]\int_0^{\frac{1}{2}wk}\exp(x^2)dx\}dk' \\
& + k^2 \int_{-\infty}^{\infty} \frac{2\pi^{\frac{5}{2}} i}{\nu}[c(k,k') - c(-k,-k')]_1\{-w^{-1}\exp[-w^2(k^2 + k_1k_1' + \frac{3}{4}k'^2) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\}(t - t_1)] \\
& + k'\exp[-w^2(k^2 + k_1k_1' + k'^2) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmi}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\}(t - t_1)]\int_0^{\frac{1}{2}wk'}\exp(x^2)dx\}dk'
\end{aligned} \tag{24}$$

The quantity E is the energy spectrum function, which represents contributions from various wave numbers or eddy sizes to the total energy. W is the energy transfer function, which is responsible for the transfer of energy between wave numbers.

In order to find the solution completely and following Deissler (1960), we assume that

$$(2\pi)^2 i\left[k_k\beta_{iik}(\underline{k},\underline{k}') - k_k\beta_{iik}(-\underline{k},-\underline{k}')\right]_0 = -\beta_0(k^4k'^6 - k^6k'^4) \tag{25}$$

For the bracketed quantities in Eq. 24, we let

$$\frac{4\pi^{\frac{7}{2}}}{\nu} i\left[b(\underline{k},\underline{k}') - b(-\underline{k},-\underline{k}')\right]_1 = \frac{4\pi^{\frac{7}{2}}}{\nu} i\left[c(\underline{k},\underline{k}') - c(-\underline{k},-\underline{k}')\right]_1 = -2\gamma_1(k^6k'^8 - k^8k'^6) \tag{26}$$

where the bracketed quantities are set equal in order to make the integrands in Eq. 24 antisymmetric with respect to k and k'.

By substituting Eq. 27 and 26 in Eq. 24 remembering that

$$d\underline{k}' = -2\pi k'^2 d(\cos\theta)dk'$$

$k_l\ k_l^{/} = kk^{/}\cos\theta$, ($\theta$ is the angle between vectors k $\underline{k}$ and $\underline{k}^{/}$) and carrying out the integration with respect to θ, we get

$$W = \int_0^{\infty}\Bigg[\frac{\beta_0(k^4k^{/6} - k^6k^{/4})kk^{/}}{2\nu(t-t_0)}.\{\exp[\{-2\nu(k^2 + kk^{/} + k^{/2}) + 2(\varepsilon_{mli}\Omega_m + \varepsilon_{nlj}\Omega_n + \varepsilon_{qlk}\Omega_q) - Rf\}(t-t_0)]$$

$$-\exp[\{-2\nu(k^2 - kk^{/} + k^{/2}) + 2(\varepsilon_{mli}\Omega_m + \varepsilon_{nlj}\Omega_n + \varepsilon_{qlk}\Omega_q) - Rf\}(t-t_0)] - \gamma_1\frac{(k^6k^{/8} - k^8k^{/6})kk^{/}}{\nu(t-t_1)}$$

$$\times\Bigg((\omega^{-1}\exp[-\omega^2(\frac{3}{4}k^2 + kk^{/} + k^{/2}) + \{2(\varepsilon_{nmi}\Omega_m + \varepsilon_{pmj}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\}(t-t_1)]$$

$$-\ \omega^{-1}\exp[-\omega^2(\frac{3}{4}k^2 - kk^{/} + k^{/2}) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmj}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\}(t-t_1)]$$

$$+\ \omega^{-1}\exp[-\omega^2(k^2 + kk^{/} + \frac{3}{4}k^{/2}) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmj}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\}(t-t_1)]$$

$$-\ \omega^{-1}\exp[-\omega^2(k^2 - kk^{/} + \frac{3}{4}k^{/2}) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmj}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\}(t-t_1)]$$

$$+\ \{k\exp[-\omega^2(k^2 - kk^{/} + k^{/2}) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmj}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\}(t-t_1)]$$

$$-k\exp[-\omega^2(k^2 + kk^{/} + k^{/2}) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmj}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\}(t-t_1)]\}\int_0^{\frac{1}{2}\omega k}\exp(x^2)dx$$

$$+\ \{k^{/}\exp[-\omega^2(k^2 - kk^{/} + k^{/2}) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmj}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\}(t-t_1)]$$

$$-\ k^{/}\exp[-\omega^2(k^2 + kk^{/} + k^{/2}) + \{2(\varepsilon_{nmi}\Omega_n + \varepsilon_{pmj}\Omega_p + \varepsilon_{qmk}\Omega_q + \varepsilon_{rml}\Omega_r) - Sf\}(t-t_1)]\}.\int_0^{\frac{1}{2}\omega k^{/}}\exp(x^2)dx\Bigg)\Bigg]d\underline{k}^{/}$$

(27)

Where:

$$\omega = \left[2\nu(t-t_1)\right]^{\frac{1}{2}}$$

The integrand in this equation represents the contribution to the energy transfer at a wave number k, from a wave number $k^{/}$. The integral is the total contribution to W at k, from all wave numbers. Carrying out the indicated integration with respect to $k^{/}$ in Eq. 27, results in

$$W = W_\beta + W\gamma \qquad (28).$$

Where:

$$W_\beta = -\frac{\left(\frac{\pi}{2}\right)^{\frac{1}{2}}\beta_0}{256\nu^{\frac{15}{2}}(t-t_0)^{\frac{15}{2}}}\exp\left[\left(-\frac{3}{2}\varepsilon^2\right)\left(105\varepsilon^6 + 45\varepsilon^8 - 19\varepsilon^{10} - 3\varepsilon^{12}\right) + \left\{2(\varepsilon_{mli}\Omega_m + \varepsilon_{nlj}\Omega_n + \varepsilon_{qlk}\Omega_q) - Rf\right\}(t-t_0)\right]$$

(29)

and

$$W_\gamma = -\frac{\gamma_1}{\nu^{10}(t-t_1)^{10}}\left[\frac{\pi^{\frac{1}{2}}}{16}\exp\left[(-\eta^2)\left(\frac{3}{128}\eta^{16}+\frac{3}{8}\eta^{14}+\frac{21}{64}\eta^{12}-\frac{105}{16}\eta^{10}-\frac{945}{128}\eta^{8}\right)+\{2(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r)-Sf\}(t-t_1)\right]\right.$$
$$+\frac{2\pi^{\frac{1}{2}}}{\sqrt{3}}\exp\left[\left(-\frac{4}{3}\eta^2\right)\left(\frac{160}{19683}\eta^{16}+\frac{40}{729}\eta^{14}-\frac{14}{27}\eta^{12}-\frac{455}{162}\eta^{10}-\frac{35}{18}\eta^{8}\right)+\{2(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r)-Sf\}(t-t_1)\right]$$
$$-\frac{\left(\frac{\pi}{2}\right)^{\frac{1}{2}}}{16}\exp\left[\left(-\frac{3}{2}\eta^2\right)\int_0^{\frac{\eta}{\sqrt{2}}}\exp(y^2)dy\left(\frac{3}{64}\eta^{17}+\frac{3}{4}\eta^{15}+\frac{21}{32}\eta^{13}-\frac{105}{8}\eta^{11}-\frac{945}{32}\eta^{9}\right)+\{2(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r)\right.$$
$$\left.-Sf\}(t-t_1)\right]+\frac{\pi^{\frac{1}{2}}}{2}\exp\left[\left\{(-\frac{3}{2}\eta^2)(5.386\eta^8+9.118\eta^{10}+3.1017\eta^{12}+0.1793\eta^{14}-0.03106\eta^{16}-0.004942\eta^{18}\right.\right.$$
$$-3.615\times10^{-4}\eta^{20}-1.890\times10^{-5}\eta^{22}-7.561\times10^{-7}\eta^{24}-2.447\times10^{-8}\eta^{26}-6.64\times10^{-10}\eta^{28}-1.55\times10^{-11}\eta^{30}..........)$$
$$\left.\left.+\{2(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r)-Sf\}(t-t_1)\right]\right] \tag{30}$$

Where:

$$\eta = \nu^{\frac{1}{2}}(t-t_1)^{\frac{1}{2}}\,k \;\text{ and }\; \varepsilon = \nu^{\frac{1}{2}}(t-t_0)^{\frac{1}{2}}\,k$$

The quantity W_β is the contribution to the energy transfer arising from consideration of the 3 point correlation equation; W_γ arises from consideration of the 4 point equation. Integration of Eq. 28 over all wave numbers shows

$$\int_0^\infty W dk = 0$$

that indicating the expression for W satisfies the conditions of continuity and homogeneity.

In order to obtain the energy spectrum function E, we integrate Eq. 23 with respect to time. This integration results in

$$E = E_j + E_\beta + E_\gamma \tag{31}$$

Where:

$$E_j = \frac{J_0\varepsilon^4}{3\pi\nu^2(t-t_0)^2}\exp[-2\varepsilon^2+\{2(\varepsilon_{mki}\Omega_m+\varepsilon_{nkj}\Omega_n)-Qf\}(t-t_0)] \tag{32}$$

$$E_\beta = \frac{(2\pi)^{\frac{1}{2}}\beta_0}{256\nu^{\frac{15}{2}}(t-t_0)^{\frac{13}{2}}}\exp[(-\tfrac{3}{2}\varepsilon^2)\left(-15\varepsilon^6-12\varepsilon^8+\frac{7}{3}\varepsilon^{10}+\frac{16}{3}\varepsilon^{12}-\frac{32}{3\sqrt{2}}\varepsilon^{13}\exp(-\frac{\varepsilon^2}{2})\int_0^{\frac{\varepsilon}{\sqrt{2}}}\exp(y^2)dy\right)$$
$$+\{2(\varepsilon_{mli}\Omega_m+\varepsilon_{nlj}\Omega_n+\varepsilon_{qlk}\Omega_q)-Rf\}(t-t_0)] \tag{33}$$

$$
\begin{aligned}
E_\gamma = & -\frac{\gamma_1}{\nu^{10}(t-t_1)^9}\{\frac{\pi^{\frac{1}{2}}}{32}\exp[(-\eta^2)\Big(\frac{189}{64}\eta^8+\frac{1029}{256}\eta^{10}+\frac{287}{256}\eta^{12}+\frac{95}{512}\eta^{14}+\frac{71}{512}\eta^{16}-\frac{71}{512}\eta^{18}\exp(-\eta^2)[Ei(\eta^2)-0.5772]\Big)\\
& +\{2(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r)-Sf\}(t-t_1)]\\
& +\left(\frac{\pi}{3}\right)^{\frac{1}{2}}\exp[(-\frac{4}{3}\eta^2)(\frac{7}{9}\eta^8+\frac{497}{324}\eta^{10}+\frac{1001}{1458}\eta^{12}+\frac{761}{4374}\eta^{14}+\frac{1963}{19683}\eta^{16}-\frac{3926}{59049}\eta^{18}\exp(-\frac{2}{3}\eta^2)[Ei(\frac{2}{3}\eta^2)-0.5772]\Big)\\
& +\{2(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r)-Sf\}(t-t_1)]\\
& +\frac{\pi^{\frac{1}{2}}}{2}\exp[(-\frac{3}{2}\eta^2)\Big(0.2307\eta^{10}+0.3632\eta^{12}+0.1502\eta^{14}+0.04463\eta^{16}-0.01326\eta^{18}\exp(-\frac{1}{2}\eta^2)[Ei(\frac{1}{2}\eta^2)-0.5772]\\
& +2.459\times10^{-3}\eta^{18}+2.935\times10^{-4}\eta^{20}+2.846\times10^{-5}\eta^{22}\\
& +2.52\times10^{-6}\eta^{24}+1.69\times10^{-7}\eta^{26}+1.25\times10^{-8}\eta^{28}+5.80\times10^{-10}\eta^{30}+4.00\times10^{-11}\eta^{32}+\cdots\cdots\Big)\\
& +\Big\{2(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r)-Sf\Big\}(t-t_1)]\\
& +\frac{1}{2}\pi^{\frac{1}{2}}\exp[(-\frac{3}{2}\eta^2)\Big(1.077\eta^8+2.414\eta^{10}+1.408\eta^{12}+0.4416\eta^{14}+0.1898\eta^{16}\\
& -0.0899\eta^{18}\exp(-\frac{1}{2}\eta^2)[Ei(\frac{1}{2}\eta^2)-0.5772]+6.575\times10^{-4}\eta^{18}+3.271\times10^{-5}\eta^{20}+1.270\times10^{-6}\eta^{22}\\
& +4.03\times10^{-8}\eta^{24}+1.08\times10^{-9}\eta^{26}+2.50\times10^{-11}\eta^{28}+5.09\times10^{-13}\eta^{30}+\cdots\cdots\Big)+\\
& \{2(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r)-Sf\}(t-t_1)\Big]\}
\end{aligned}
\tag{34}
$$

The quantity E_j is the energy spectrum function for the final period, where, E_β and E_γ are the contributions to the energy spectrum arising from consideration of the three and four point correlation equations, respectively.

Equation 31 can be integrated over all wave numbers to give the total turbulent energy

$$\frac{1}{2}\langle u_i u_i\rangle = \int_0^\infty E dk \tag{35}$$

Hence,

$$
\begin{aligned}
\frac{\langle u_i u_i\rangle}{2} = & \frac{j_0^{\frac{14}{9}}\nu^{\frac{5}{9}}}{\beta_0^{\frac{5}{9}}}\left[\frac{1}{32(2\pi)^{\frac{1}{2}}}T^{-\frac{5}{2}}\exp[\{Qf-2(\varepsilon_{mki}\Omega_m+\varepsilon_{nkj}\Omega_n)\}(t-t_0)]\right.\\
& +0.2296T^{-7}\exp[\{Rf-2(\varepsilon_{mli}\Omega_m+\varepsilon_{nlj}\Omega_n+\varepsilon_{qlk}\Omega_q)\}(t-t_0)]\\
& +6.18\frac{\gamma_1\nu^{\frac{5}{9}}J_0^{\frac{5}{9}}}{\beta_0^{\frac{14}{9}}}\left(\frac{t-t_1}{t-t_0}\right)^{-\frac{19}{2}}T^{-\frac{19}{2}}\exp[\{Sf-2(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r)\}(t-t_1)\Big]
\end{aligned}
\tag{36}
$$

Where:

$$
\begin{aligned}
\langle u^2\rangle = & C_1T^{-\frac{5}{2}}\exp[\{Qf-2(\varepsilon_{mki}\Omega_m+\varepsilon_{nkj}\Omega_n)\}(t-t_0)]+C_2T^{-7}\exp\Big[\{Rf-2(\varepsilon_{mli}\Omega_m+\varepsilon_{nlj}\Omega_n+\varepsilon_{qlk}\Omega_q)\}(t-t_0)\Big]\\
& +C_3T^{-\frac{19}{2}}\left(\frac{t-t_1}{t-t_0}\right)^{-\frac{19}{2}}\exp[\{Sf-2(\varepsilon_{nmi}\Omega_n+\varepsilon_{pmj}\Omega_p+\varepsilon_{qmk}\Omega_q+\varepsilon_{rml}\Omega_r)\}(t-t_1)\Big]
\end{aligned}
\tag{37}
$$

Where:

$$\frac{t-t_1}{t-t_0} = 1 - \left(\frac{\gamma_1 \nu^{\frac{5}{9}} J_0^{\frac{5}{9}}}{\beta_0^{\frac{14}{9}}} \right)^{1/9} \left[\frac{(t_1 - t_0)\nu^{\frac{94}{81}} J_0^{\frac{13}{81}}}{\beta_0^{\frac{4}{81}} \gamma_1^{\frac{1}{9}}} \right] \frac{1}{T} \text{ and } T = \frac{\nu^{\frac{11}{9}} J_0^{\frac{2}{9}} (t - t_0)}{\beta_0^{\frac{2}{9}}}$$

and C_1, C_2, C_3 are arbitrary constants.

RESULTS

In Eq. 44 we obtain the among decay law of dusty fluid turbulent flow in a rotating system before reaching at the steady state. The Eq. 36 shows that turbulent energy decays more rapidly in an exponential manner than the energy decay for non-rotating clean fluid. Thus the terms associated with the higher order correlations die out faster than those associated with the lower order ones.

If the system is non-rotating, we put Ω's = 0, the Eq. 37 becomes

$$\left\langle u^2 \right\rangle = AT^{-\frac{5}{2}} \exp\{Qf(t-t_0)\} + BT^{-7} \exp\{Rf(t-t_0)\} + C\left(\frac{t-t_1}{t-t_0}\right)^{-\frac{19}{2}} T^{-\frac{19}{2}} \exp\{Sf(t-t_1)\} \quad (38)$$

Again if the fluid is clean, we put f = 0, then Eq. 38 becomes

$$\left\langle u^2 \right\rangle = C_1 T^{-\frac{5}{2}} + C_2 T^{-7} + C_3 \left(\frac{t-t_1}{t-t_0}\right)^{-\frac{19}{2}} T^{-\frac{19}{2}}$$

which is obtained earlier by Deissler (1960).

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