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2D FE Description of Reinforced Concrete Beams Behaviour

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Abstract: The first series of beams with no shear reinforcement tested by Bresler and Scordelis to investigate the behaviour of reinforced concrete beams in shear is used to calibrate the proposed finite element approach. A non-linear finite element analysis has been established to predict the shear strength of reinforced concrete beams. Based on the smeared crack modelling of quasi-brittle materials, various refinements such as: the shear retention factor, the tension stiffening concept and the dowel action, are introduced. This approach is based on the combination of the behaviour of biaxial description of membranous concrete elements and truss uniaxial reinforcing bars with a full bond between concrete and steel. Beyond the strength of concrete in tension, the tensile capacity of concrete maintained by the reinforcing bars (tension stiffening effect) is assumed. The mathematical relationships of stress-strain materials are displayed in the following section. The procedure is based on incremental finite element analysis and established with 2D description of behaviour using Kupfer failure criteria. The finite element simulation is validated by comparison of numerical results with available experimental data, regarded as a benchmark for various numerical models. Correlation studies between obtained results and experimental values are conducted to assess the validity of the proposed approach.

Key words: Non-linear analysis, finite element method, reinforced concrete beams, smeared crack model, shear strength, failure modes, shear loading

INTRODUCTION

Finite Element (FE) method is our unique tool for the analysis of both simple and complex Reinforced Concrete (RC) structures. The development of computing machines with large capacity of memory has encouraged investigators to reach an advanced level with regards to the concrete state, steel state and concrete-steel interaction. Even since the implementation of the FE method on computers, researchers can applied the method to increasingly problems. Among difficult tasks, the fracturing in concrete and reinforced concrete is most confronted in reinforced concrete structures. In this scope, the fracture mechanics have known a significant progressions and developments. The two most commonly approaches to the FEM analysis of fracture in concrete structures are the discrete crack concept and the smeared crack concept. The analytical models, which require techniques to deal with non-linear problems to predict a more realistic behaviour of reinforced concrete structures has systematically been established over the last years. For its simplicity in numerical implementations and efficiency of results, it is widely applied in most engineering fields.

Experimental and analytical studies predicting the non-linear response of structures through load-deflection, ultimate resistance and failure modes have been performed. Many researches have already tried to predict the shear strength of reinforced concrete beams by using various material models for concrete. Kwak and Filippou (1997) introduced a FE model based on separate material model of concrete and steel bars, Bhatt and Abdel Kader (1998) presented a 2D parabolic isoparametric quadrilateral FE model for analysing the shear strength of RC rectangular beams and recently, Vecchio and Shim (2004) have re-examined the behaviour of Bresler and scordelis beams. On the other hand, Hsu (1987, 1991, 1998) have provided considerable contributions to the shear analysis and mechanical behaviour of RC structures and developed the softened truss model for predicting the strength and the deformation of structures of short beams. In the same context, Rabczuk *et al.* (2005) have already presented a two-dimensional approach based on the fracture theory of reinforced concrete structures.

Various material models have already been published (ACI, 1997) therefore, it is necessary to choose the one that presents an enhanced evaluation of structural behaviour. To reach the performance of the model in non-linear finite element analysis, various phenomena should be considered such as: Strength criterion of concrete, variation of material properties before and after cracking, cracking and crack propagation and bond slip between different materials (Saddique and Abdur Rouf, 2006).

The smeared crack model was used for its simplicity at numerical implementation level. This approach has already been implemented in the developed program conceived in general to the nonlinear behavior analysis of RC structures. The approach is extended for strength decreases and increases under two-dimensional stress states. For reinforced concrete structures, not only the cracking of concrete but also the interaction between concrete and steel can play a significant role in the nonlinear response. This analysis is not aimed to study the interaction between concrete and steel for this concern a perfect bond model is considered.

The computed results of this description have been compared to other numerical and experimental results and the comparison shows a well correlation between the simulated and the experimental data.

THE CONSTITUTIVE MODEL FOR CONCRETE

The constitutive model for concrete is based on the smeared crack concept in tension and a nonlinear with linear softening in compression. The principal object of this study concerns the treatment of fracturing and the propagation of cracks in the continuum. The unloading is purely elastic, meaning no plastic deformations occur. Since structures at monotonic increasing load conditions are of interest, this model is sufficient enough to capture all desired failure mechanisms as will be shown in the applications section.

Various factors complicate the behaviour of concrete such as: Non-linear stress-strain relationship, strain softening and anisotropic stiffness reduction, cracking and propagation, slip between steel and concrete as well as aggregate interlock. All these factors inflict on reinforced concrete structures and the choice of the reinforced concrete model depends on the rigorous selection of among of these parameters. This section presents the concrete model adopted in this study.

Compressive loading: Micromechanical investigations for uniaxial compression tests have shown that the softening behavior of concrete depends on the dimensions of the specimen (Vonk, 1993). The obtained stress-strain curves for specimens of different heights show that smaller higher of the specimen is more ductile in the material behavior. This effect can be explained as: Under uniaxial compression, cracks will evolve perpendicular to the direction of the maximum principal

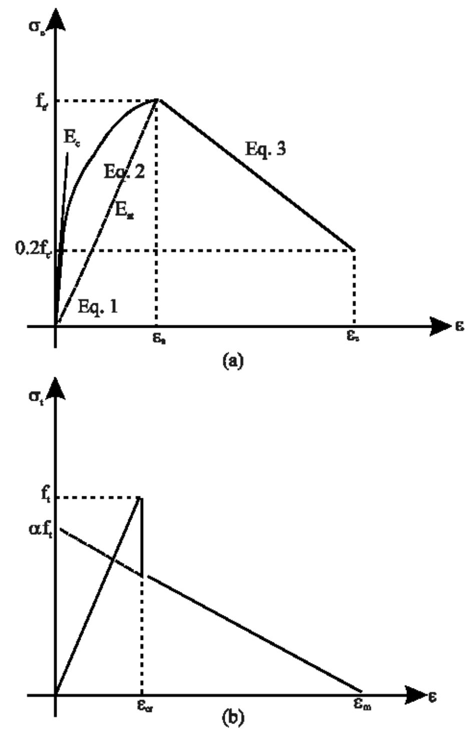

Fig. 1: Stress-strain relationships of concrete: (a) Compression and (b) Tensile

strain. For a uniaxial load, this will be perpendicular to the load direction. The softening behavior is mainly characterized by the height of the specimen and shorter one presents more ductile of the material response.

In general, for the first stages of loading, the behaviour of concrete is assumed an isotropic linear elastic material until the initially yielding surface. Beyond this frontier, the concrete behaves as plastic hardening material and, up to peak stress in the principal axis, the local uniaxial stress-strain relationship follows the constitutive law proposed by Liu *et al*. (1972) (Fig. 1) that is expressed by:

$$\sigma_{ela} = E_c\,\varepsilon \quad (1)$$

$$\sigma_{har} = \frac{E_c\,\varepsilon}{1+(\frac{E_c}{E_{st}}-2)\frac{\varepsilon}{\varepsilon_0}+(\frac{\varepsilon}{\varepsilon_0})^2} \quad (2)$$

Where the initial Young modulus E_c = 5000,

$$\sqrt{f_c'}$$

($f_{c'}$ is the cylinder compression strength), the strain at peak stress

$$\varepsilon_0 = \frac{\sqrt{f_c'}}{2500}$$

and the secant modulus at peak stress

$$E_{sof} = \frac{f_c'}{\varepsilon_0}.$$

In the strain-softening region, Bhatt and Abdel Kader (1998) assumes that the behaviour of concrete depends on the presence of shear reinforcements of beams. In this case, two different considerations were selected, as:

Beams without shear reinforcement: Since the failure mechanism for beams without shear reinforcement is brittle therefore, it is assumed that the stress-strain relationship for concrete has no plastic plateau and the failure is firstly reached at peak stress.

Beams with shear reinforcement: Since the failure mechanism of beams with shear reinforcement is, in general, less brittle than that for the case without shear reinforcement so, it is assumed that the stress-strain relationship for concrete is the same as of beams without shear reinforcement, except that, after the maximum stress is reached, there is a linear descending branch given by:

$$\sigma_{sof} = f_c'(1 - \frac{0.8(\varepsilon - \varepsilon_0)}{\varepsilon_u - \varepsilon_0}) \quad (3)$$

Tensile loading: Initially up to cracking strain ε_{cr} value, the relationship is assumed to be linear until the tensile strength of concrete f_t. As the tensile strength is reached, the concrete behaviour is considered as linear tensile softening, in this study, the smeared crack approach is used and the tension stiffening effect is represented by the proper tensile stress-strain relationship after cracking has been occurred (Fig. 1):

$$\sigma_t = E_c\,\varepsilon \quad (4)$$

$$\sigma_t = 0.50(\frac{\varepsilon_{cr}}{\varepsilon} + \sqrt{\frac{\varepsilon_{cr}}{\varepsilon}})\,f_t \quad (5)$$

In the above Eq. 5, the strain is calculated normal to the crack and it is assumed that if the strain is greater than the steel yield strain, then the tensile stress in concrete is reduced to zero.

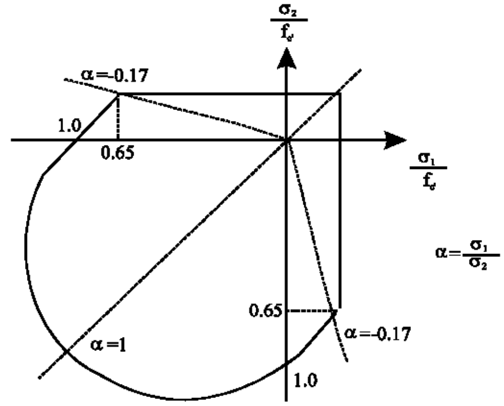

Fig. 2: Biaxial failure surface of concrete

Failure criterion for concrete: Under biaxial stresses combination, the concrete exhibits strength, which differs of that under uniaxial loading conditions. In the biaxial compression state of stress, concrete shows an increase in compressive strength up of 25% via the uniaxial compressive strength. Since only a 2-D state of stresses is being considered for plane stresses and the failure criteria of Kupfer *et al.* (1969) was assumed (Fig. 2). In this context, the linear expression between octahedral normal stress; σ_{oct} and octahedral shear stress; τ_{oct}, is written as follows:

$$\tau_{oct} - b\sigma_{oct} - a = 0 \quad (6)$$

In which

$$\sigma_{oct}\,\frac{\sigma_x + \sigma_y}{3}$$

and

$$\tau_{oct} = \frac{\sqrt{2(\sigma_x^2 + \sigma_y^2 - \sigma_x.\sigma_y + 3\tau_{xy}^2)}}{3}$$

σ_x and τ_{xy} are the Cartesian normal and shear stresses.

A and b are constants that can be determined for each region of behaviour using the limit failure surface in compression-compression stress state and compression-tension stress state.

For biaxial tension stress state, the Rankine failure criterion is assumed as shown in Fig. 2.

Shear retention factor: As well known, the cracking phenomenon reduces the shear modulus G for the concrete integrity. It is assumed that once concrete cracks, the ratio $\beta$is introduced in the computation and its expression depends on the strain normal to the crack as:

$$\beta = 0.40\frac{\varepsilon_{cr}}{\varepsilon_n} \quad (7)$$

Where ε_{cr} and ε_n are the cracking strain and the tensile strain normal to the crack, respectively.

Reinforcing bar modelling: The reinforcing bars are considered as embedded elements, which mean that full bond between steel and concrete is assumed. The bar is assumed to carry axial loads only implying that dowel action of the bar in resisting shear is ignored. For simplicity in computations, it is necessary to idealize the one-dimensional stress-strain curve for the reinforcing bars. In this study, the reinforcing steel is assumed to be a linear elastic, linear hardening material with a yield stressσ_Y.

Modelling and analysis method: Finite element method is used to solve the differential equations governing the behaviour of the body. The material models described above were used in the finite element non-linear analysis program developed by the author Khalafallah (2003) that was based on 2D isoparametric quadrilateral membranous elements.

Then, an incremental-iterative strategy to describe the cracking description of concrete is applied. The stresses and strains are evaluated at four Gaussian points, while the stress reaches the tensile strength of the concrete, it is assumed that the first crack opens and this procedure is carried out for every point of integration.

When the tensile stress is still below the ultimate tensile strength, the Young's modulus is unchanged but beyond this limit, a softening modulus is incorporated in the analysis to describe the behaviour of concrete in the softening region.

As mentioned above, this study aims the 2D behaviour of the reinforced concrete structures, in which the cracking has taken in higher consideration. The cracking plays an important role and it is the responsible on the global nonlinear response.

As mentioned above, this procedure is already implemented in the developed program conceived, in general manner, for the nonlinear analysis of the reinforced concrete behaviour:

$$K^{t+1}\Delta u^{t+1} = \Delta P^{t+1} \quad (8)$$

Where:

$$K^{t+1}, \Delta u^{t+1} \text{ and } \Delta P^{t+1}$$

are the stiffness matrix, the increment displacement vector and the external force vector, respectively. Since, $K^{t+\Delta t}$ depends on the current and stress state using nonlinear material behaviour.

The displacement vector in the next increment of loading can be computed by:

$$u_n^{t+\Delta t} = u_{n-1}^{t+\Delta t} + \Delta u_n^{t+\Delta t} \quad (9)$$

A total stress-strain relationship is used (Eq.1 and 2) and the actual stress state is determined by the actual strain:

$$\sigma^{t+\Delta t} = f(\varepsilon^{n+1}) \quad (10)$$

The treatment of the cracking phenomenon is then become necessary while the stress in gauss point reaches the tensile strength of concrete.

NUMERICAL PROBLEMS

The experimental results for reinforced concrete beams tested by Bresler and Scordelis (1963) are widely used to check the ability of many analytical models. For this reason, the first series of beams tested without shear reinforcement is considered in this study, which is constituted of three beams differed in amount of longitudinal reinforcement, span length, cross section dimensions and mechanical properties of concrete, have been considered. The detail of cross-section and material properties are summarized in Table 1 and 2, respectively. All beams were subjected to monotonic loading and the increment of load is initially applied of 40 KN per load stage.

The analysis of the examples was performed considering all RC beams discretized for 55 membranous finite elements and 10-truss element for representing concrete and reinforcing bars, respectively. The finite element used in the analysis was the eight-node

Table 1: Cross-section details of Bresler-Scordelis beams

Beam number	B (mm)	H (mm)	D (mm)	L (mm)	Span (mm)
OA1	310	556	461	4100	3660
OA2	305	551	466	5010	4570
OA3	307	556	462	6840	6400

Table 2: Material properties of Bresler-Scordelis beams

Material	Concrete		Reinforcing bars			
Beam Number	f'_c (MPa)	f_t (MPa)	σ_y (MPa)	f_u (MPa)	E_s (MPa)	A_s (mm^2)
OA-1	22.60	3.97	325	430	218.000	2580
OA-2	23.70	4.34	345	542	218.000	3225
OA-3	27.60	4.14	555	933	218.000	3870

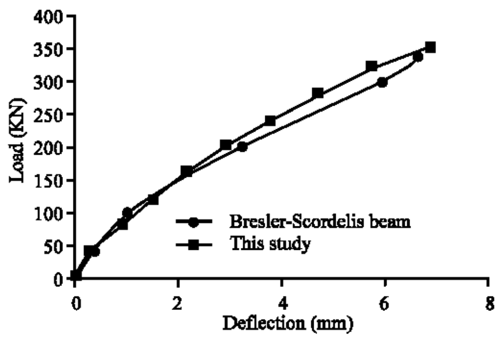

Fig. 3: Load-deflection of AO-1 beam

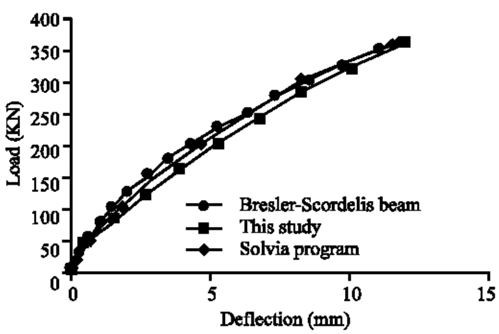

Fig. 4: Load-deflection of AO-2 beam

serendipity elements with reduced point's number of integration (2×2).

The load-deflection responses plots are given in Fig. 3-5 and it is well noted that reinforcing bars influence on the mode failure of beams. The failure was sudden and brittle for OA-1 beam and becomes more ductile with the increase of reinforcing bars amount. At the ultimate load of each beam, the maximum deflection of OA-3 is higher of 4.66% time and 2.67% time for OA-1 beam and OA-2 beam, respectively. The experimental and analytical load-deflection curves (Bresler and Scordelis, 1963;

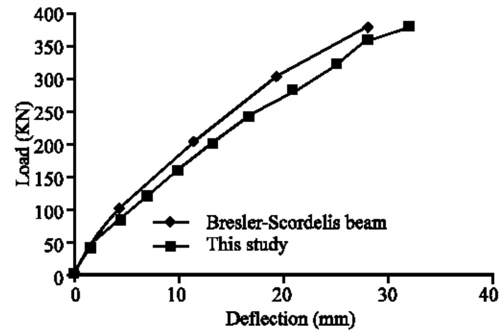

Fig. 5: Load-deflection of AO-3 beam

www.lrz-muenchen.de/services/software/fem/solvia/nonlin99.pdf) are compared (Fig. 3-5) which indicate a good agreement. The follow-up of the numerical simulation allows that the failure occurred by crushing of concrete in the compressive zone near the load point, which results from the development of diagonal cracks and also by fracture along tensile reinforcement near the middle of the beam.

For a general manner, it seems that the distribution of normal stresses σ_{xy}, σ_{yy} and shear stress σ_{xy} are identically for the three beams (Fig. 6-8). The cracking map and high tensile stresses can be observed in the bottom at mid-span of beams and it becomes more pronounced from OA-1 up to OA-3. In the same, in the vicinity of loaded point, the compressive phenomenon is remarkable and appreciable, respectively.

Addicting, it is seen that the three beams having the same failure mode that failed in shear in diagonal tension failure type mode. The failure, as described by Bresler and Scordelis (1963), was sudden and occurred as a result of the longitudinal splitting in the compression zone near the load point and also by horizontal splitting along tensile reinforcement near the end of the beam.

The behaviour of beams is governed by the combination of the yielding of the steel reinforcing and the tensile concrete cracking. The first beam (OA-1) failed by the yielding of the reinforcement steel following the initial tensile cracking of concrete. On the other hand, the OA-2 and OA-3 beam failed by crushing of concrete from the near loaded point of force without the steel yielding is achieved.

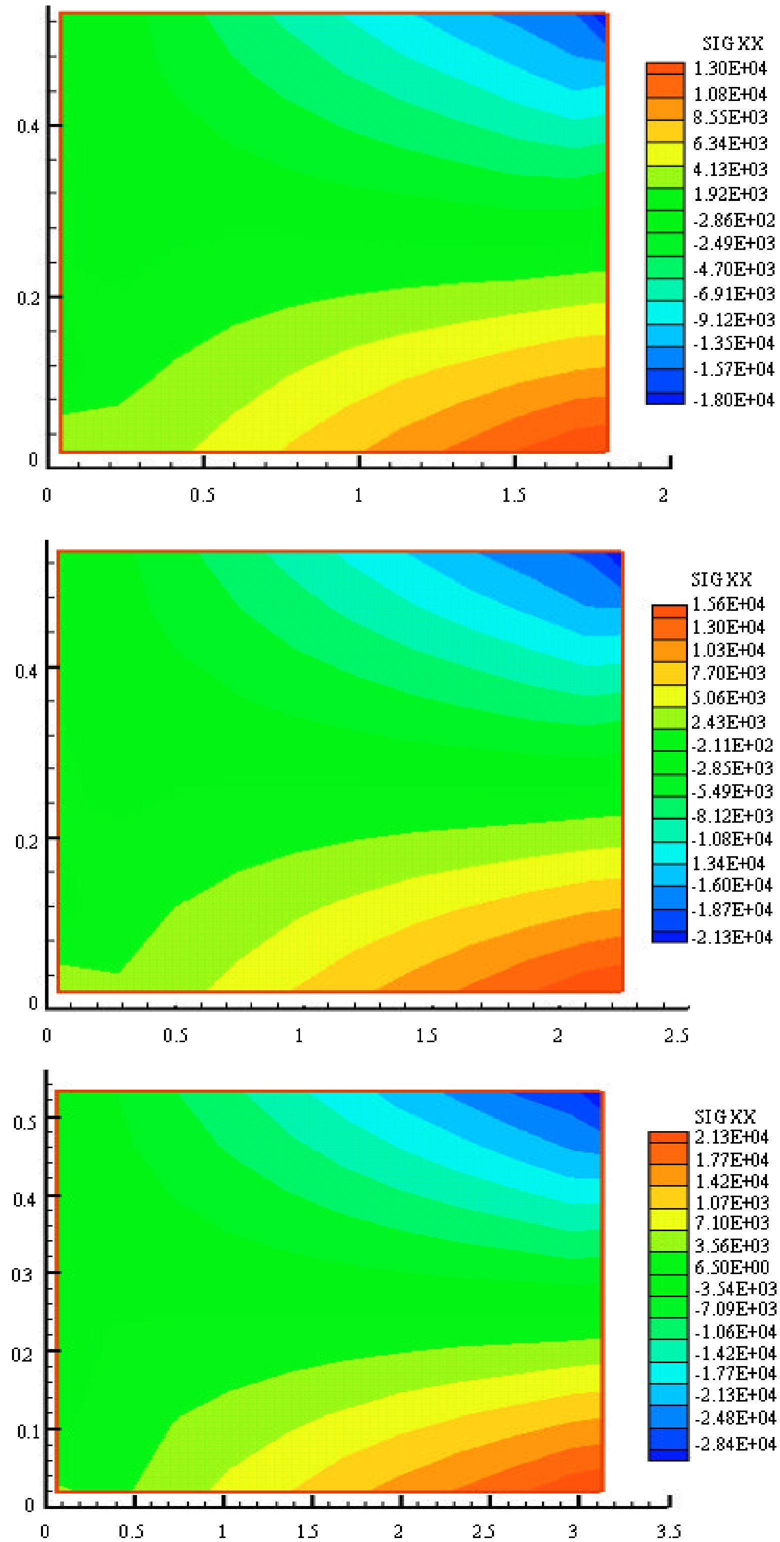

Fig. 6: The normal stress σ_{xx} of beams

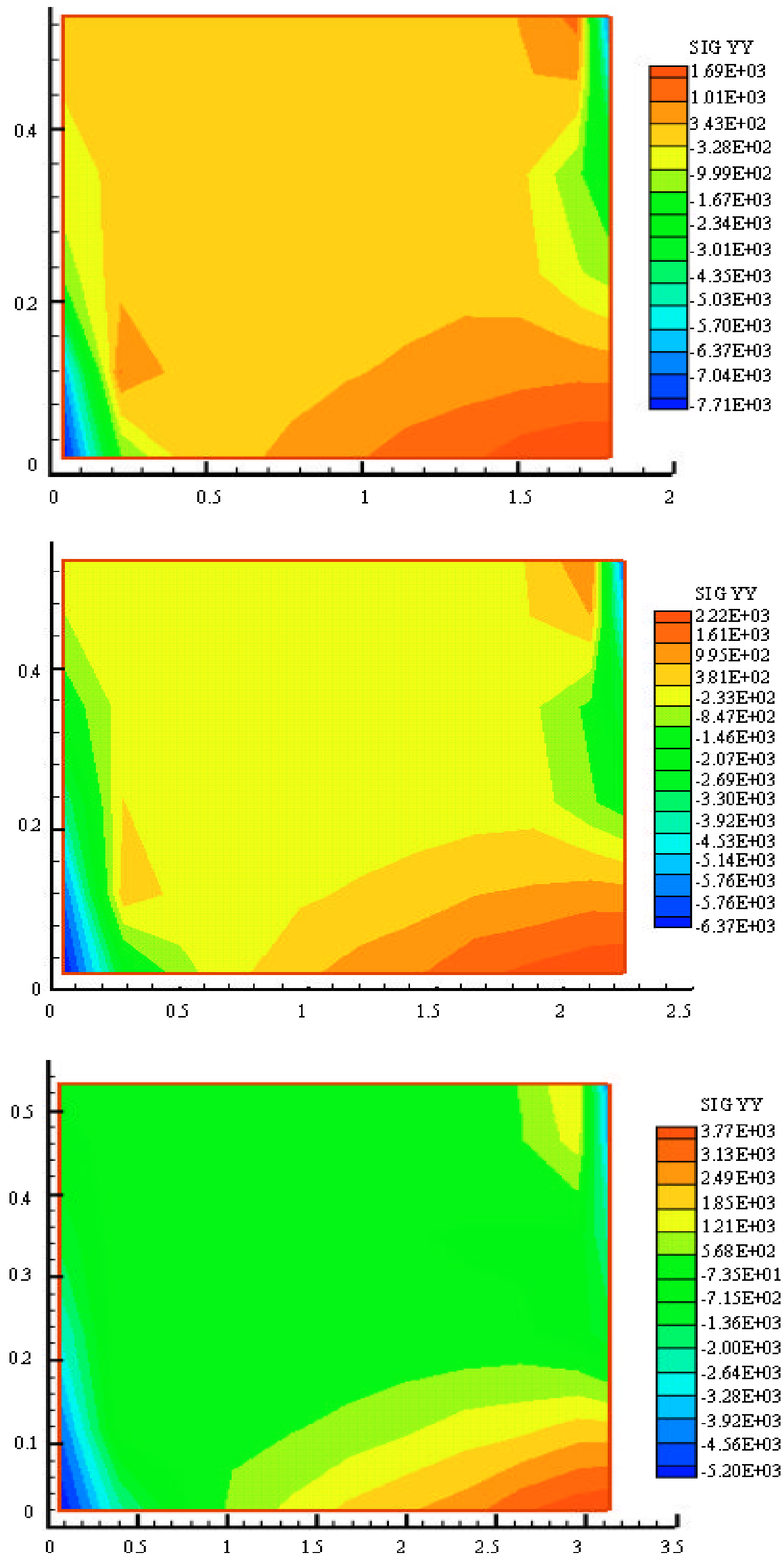

Fig. 7: The normal stress σ_{yy} of beams

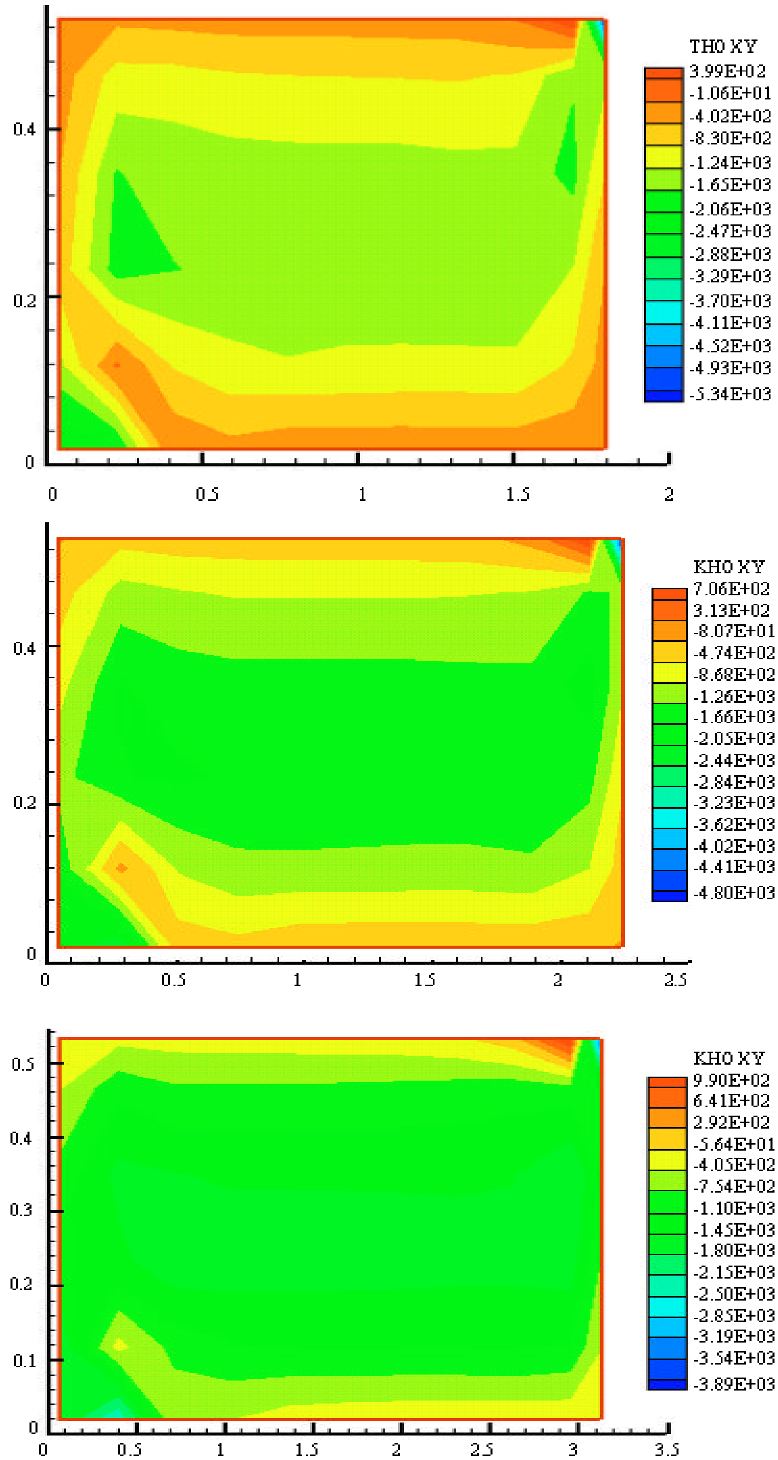

Fig. 8: The shear stress σ_{xy} of beams

CONCLUSION

The results computed on the behaviour of classical beams shown in Fig. 3-6 clearly indicate that the present 2D finite element model can be used to predict with satisfactory the main results of reinforced concrete beams: The ultimate load, the load-deflection curve and the mode of failure.

From experimental and obtained results, the following conclusions can be inspired:

- The correlation observed between experimental and numerical results leads that the proposed approach reproduces faithfully Bresler-Scordelis results.
- The developed FE program is applied for simulating the shear behaviour of RC beams. Since the comparison of obtained results indicates that main reinforcement, strengths of concrete, stress yielding of steel may affect the ductility of RC beams.
- The failure mode is sudden and brittle for lower ratio of reinforcement and becomes more highly as soon as the reinforcement ratio increases.
- The premature failure in the computation due to the crushing of concrete begins in the load application zone.
- Non-linear finite element analysis can be useful tool in investigating local and global responses of reinforced concrete structures.

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