

Analyze of Contact Pressures at the Soil-Foundation Interface

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Abstract: The main objective of this research is to study the validity of the simplifying assumptions which admit that the distribution of contact pressures under a shallow foundation is uniform or linear. This stress distribution depends on the mode of loads transfer, the foundation geometry, the soil type and its characteristics. In this research, a study of the distribution of contact pressures under a rigid foundation was undertaken by using analytical and numerical method. The latter one is based on the finite element-limit elements. The two methods showed different distribution of stress distribution. The contact pressures over the entire length of the foundation are uniform when calculated by the analytical method. On the other hand, the numerical method gives a variable distribution with a maximum at the edges. The pressures seem to be very important in the central zone up to 70% when evaluated by the first method.

Key words: Contact pressures, shallow foundation, interaction soil-structure, element boundary, functions of green, finite element, settlement

INTRODUCTION

The distribution of interface contact pressures is essential for the design of any foundation and the assessment of its related settlement. However, due to the highly hyper static character of the soil-foundation system its evaluation steel up to now difficult to establish. Most of models experimental work carried out in laboratory use the assumption that a soil is an elastic homogeneous semi-infinite medium.

In the design of foundation, various methods were elaborated using the elasticity theory in a plane deformation (Krol, 1971; Costet and Sanglerat, 1975; Leonards, 1982; Simvoulidi, 1987; Beer, 1948) to calculate the distribution of contact pressures at the interface soil-foundation.

Most of these analytical methods are usually limited to the calculation of contact pressures related to problems of simple geometry foundation and semi-infinite homogeneous soil profile. When the foundation has an irregular form or the ground has a vertical or a horizontal heterogeneity, the solution is much complicated.

During the last decades, a number of new methods more appropriate to the three-dimensional analysis of soil-structure interaction were developed. These methods make use of GREEN functions which enable to calculate

the required displacement. All these displacements constitute the solution which is formulated in terms of limit integral equation in a discretized soil foundation medium where the GREEN function are known for each element. These is done by taking into account the soil structure conditions (Chow and Schmid, 1990; Leung *et al.*, 1990; Boumekik, 1985). The functions of GREEN are calculated in the discretized medium by Kausel-Peek method (1974).

The present research uses this approach to calculate the contact pressures to the interface soil-foundation.

DETERMINATION OF CONTACT PRESSURES UNDER THE FOUNDATION

The present numerical method is based on coupling Elements finish-Elements boundaries on a rigid rectangular foundation (Fig.1). The foundation lay in a homogeneous soil of height H with a rigidity modulus G , a density ρ and a Poisson's ratio (ν) limited by a bedrock. This layer is then subdivided in a multi-layer surface of ground. The foundation is subjected to a static vertical load.

The determinations of the stress at the interface soil-foundation, taking into account the interaction of the two mediums, need to establish a function of stress versus displacements and check the limit conditions. This equation is as follow:

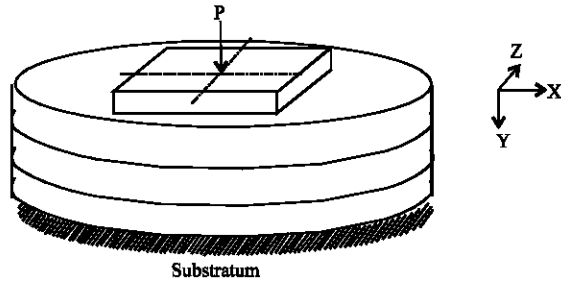


Fig. 1: Model of calculation

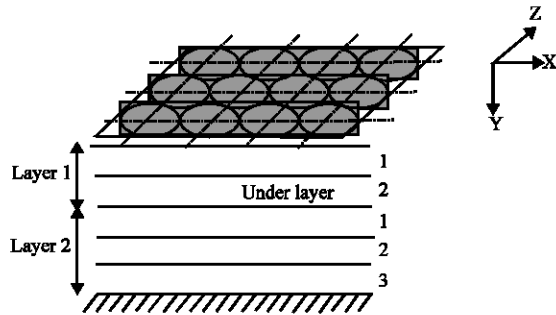


Fig. 2: Vertical and horizontal discretization

$$u_i = \int U_{ij} \cdot p_j \quad (1)$$

Where,

p and u are respectively the stress and displacements under the foundation and U the function of influence giving the interdependence of the latter.

As a long as the medium is continuous, the determination of the stress remains very difficult if not impossible if the ground presents stratifications. This is due primarily to the conditions at the mixed boundaries of the equations of elasticity.

However if the medium, where the foundation is laid (Boumekik, 1985) is discretized on the one hand in the horizontal plane with square elements which can be replaced without much error by discs (elements boundaries) and on the other hand in the vertical plane in layers which in their turn will be subdivided in under layers (finite elements) (Fig. 2), it is then possible to solve the problem algebraically by considering that the variation of displacement is a linear function of displacements at the interfaces and consequently to obtain the influence functions (functions of Green):

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \text{influences functions}$$

These functions represent the displacements in the x, y, z directions in any point of the soil mass due to units

loads distributed on discs in any point of the soil mass. Therefore, it is possible to determine the flexibility matrix of the foundation starting from the of interaction soil-structure considerations.

The vertical distributed loads t_{yi} on the discs and displacements of the ground can be related by the following equation:

$$u = F_s \cdot t \quad (2)$$

From which the distributed loads are calculated as:

$$t = F_s^{-1} \cdot u \quad (3)$$

Where:

F_s : Represent the flexibility matrix of the discretized ground with dimension $[N \times N]$ where N is the total number of discs where the pressures are required.

t : Represent the vector of the distributed loads applied to each disc of dimensions $(N \times 1)$.

u : Vector of displacement resulting from dimensions $(N \times 1)$.

However, the presence of the foundation imposes on the discs a displacement compatible with the body rigid movement. This condition of compatibility is expressed by the relationship:

$$u = R \cdot \Delta_y \quad (4)$$

Where, R is a matrix of dimension $(N \times 1)$ which depends only on the geometrical characteristics of the foundation and Δ_y is a vector giving the vertical displacement of the foundation. The Eq. 3 and 4 lead to:

$$t = F_s^{-1} \cdot R \cdot \Delta_y \quad (5)$$

From the Eq. 5, constraint p_i on a disc I can be obtained by:

$$p_i = \frac{t_i}{\pi \cdot r^2}$$

r : Half diameter of the disc.

The representation of this value according to axes x and z makes it possible to know the mode of distribution of the stresses under the foundation.

APPLICATION

Numerical method and analytical stiffness K method (Krol, 1971; Beer, 1948; Ugural, 1982) have been used to study a rectangular foundation ($L = 4$ m, $b = 2$ m and $h =$

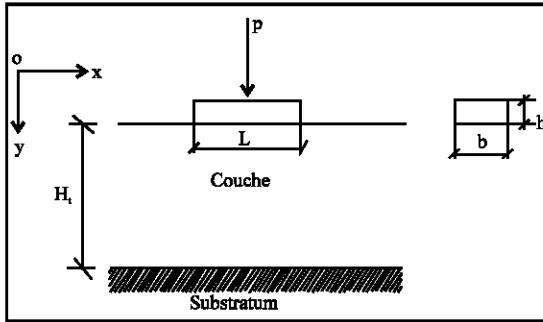


Fig. 3: Schematic view of the foundation

1.3 m) in a homogenous elastic and isotropic semi infinite soil. This foundation is subjected to a vertical concentrated load $P = 30$ tons (Fig. 3).

An elaborated numerical computer program for homogenous and heterogenous soils has been used to evaluate the contact pressure distribution of this weightless foundation. This program was originally written by Kausel (1974) and Boumekik (1985) using several subroutines and later on developed by Sahal (1994) and Mendjel (1997).

The soil characteristics were chosen as follow:

- Soil elasticity modulus $E_s = 1000 \text{ kgf cm}^{-2}$.
- Poisson's ratio ($\nu = 0,3$).
- Rigidity modulus $G = E_s / 2(1 + \nu) = 384,62 \text{ kgf cm}^{-2}$.
- Unite weight of the ground $\rho = 18 \text{ Kg m}^{-3}$.

To simulate the semi-space, the substratum is placed at a depth equal to 10 times the width of the foundation that is to say $H_i = 20$ m. With a good approximation, the semi-space conditions reached.

An interface soil-foundation discretization in plan of 10 times 5 elements was used in the study (Fig. 4). Although this number is limited, from the point of view of the author the results obtained are well within the accepted values. However, for more accurate results a higher number of elements should be used.

For the determination of the pressures of contact by the analytical stiffness K method and although the foundation is a rigid one, this latter is taken as beam of finite dimensions with $a.L = 0.98$ (a limit value between a rigid and a flexible foundation (Ugural, 1982)). The 0.98 value will certainly help to have a good idea of the magnitude if errors that may accrue.

The modulus of subgrade reaction of the soil was expressed by Beer (1948):

$$K = 1,33 \frac{E_s}{\sqrt[3]{b^2 l_e}}$$

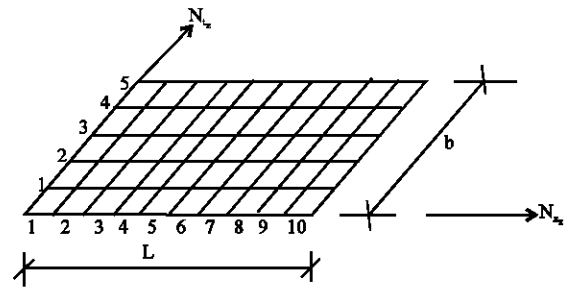


Fig. 4: Horizontal discretization

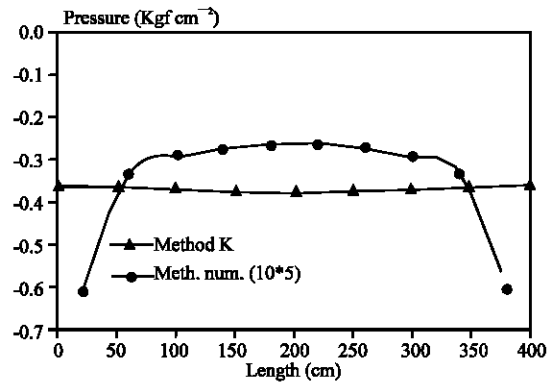


Fig. 5: Results of analytical and numerical methods

Where:

E_s : Modulus of elasticity of the soil,
 b : Width of the foundation,
 I : Moment of inertia of the foundation
 And:

$$a = 4 \sqrt{\frac{K \cdot b}{4 \cdot E \cdot I}} = \frac{1}{l_e}$$

K : Modulus of subgrade reaction of the ground.
 b : Width of the foundation.
 E : Modulus of elasticity of material of the foundation.
 I : Moment of inertia of the foundation.
 l_e : Elastic length.

The results are shown by the Fig. 5.

CONCLUSION

In this study, analytical and numerical methods were used to calculate the contact pressures of the ground-rigid foundation surface contact (interface). In the analytical study, the coefficient of stiffness K method from the elasticity theory in plane deformations was used. In the numerical study, contact pressures were calculated using the finite element-limit elements coupled method.

The study has shown that the contact pressures over the entire length of the foundation are uniform when calculated by the analytical method. On the other hand, the numerical method gives a variable distribution with a maximum at the edges. The pressures seem to be very important in the central zone up to 70% when evaluated by the first method.

The shape of the curve obtained by the numerical method analysis is in good agreement with the Boussinesq-schleifer theory which has been confirmed by some experimental work carried out by A. Fœppl, F. Emperger, C. Walterback in laboratory.

From this study, it can be concluded that the analytical method over estimate the magnitude of a rigid foundation pressure distribution on the soil and therefore the simplifying assumptions which admit that the distribution of contact pressures under a shallow foundation is uniform or linear seem to be unrealistic.

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