

## Modelling and Neural Networks Control of Distributed Parameter Bioreactor

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**Abstract:** This study deals with modeling, the estimation the parameters and the controlling the biological process used for the waste water treatment. The mathematical models describing such process are presented in the form of distributed parameters system. The numerical resolution of such system requires the use of the approximation methods. Once the model is established, the concepts of the approach presented in this resaerch aim at modeling the considered process, studying of model identifiability, estimating and controlling the process with neural networks.

**Key words:** Water treatment, distributed parameters system, nonlinear system, modeling, identifiability, linearizing adaptive control, neural networks

### INTRODUCTION

In this study we study dynamic modeling, the analysis and the control of bioprocesses tat use a fixed bed reactor (Fig. 1). This type of engines is used more and more in industrial practices. The fundamental problem, which appears in the control of such process, is the lack of measurements on line, which is always the weak point of controlling in real time of purification processes. In order to perform controlling the process, thus it is necessary to work out of a simulation model useful in real time in order to predict the evolution of the various concentrations.

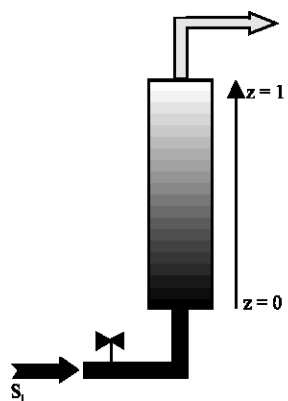
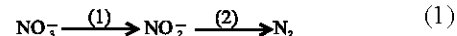


Fig. 1: Diagram of fixed bed reactor

The purpose of denitrification is reducing the nitrates  $\text{NO}_3^-$  in gas nitrogen  $\text{N}_2$  with formation of an intermediate compound, the nitrites  $\text{NO}_2^-$  (Babary *et al.*, 1996).



Denitrification is an anaerobic reaction (reaction took place in a medium deprived of oxygen) catalysed by micro-organisms (of the bacteria whose source of energy is organic carbon).

Our research will be organized in four parts:

- Modeling the biotechnological process (biofiltre).
- Analyzing and identifying of the kinetic parameters and other parameters...
- Estimating and adaptive control of neural networks biotechnological process.
- Results of simulations.

### THE SYSTEM DYNAMICAL MODEL

The denitrification process, which is the subject of our study, is an engine of length  $L$  with fixed granular bed and of ascending type for the liquid. In this type, the micro-organisms are fixed on balls of garnishing, where the substrates can circulate freely.

The mathematical model of the system is described by four (non-linear, nonstationary and of hyperbolic type), partial derivative equations.

$$\begin{cases} \frac{\partial s_1(z,t)}{\partial t} = -\frac{F}{A\epsilon} \frac{\partial s_1(z,t)}{\partial z} + \frac{1}{\alpha_1 \epsilon} (K_1 - 1) \mu_1(s_1, s_3) x(z,t) \\ \frac{\partial s_2(z,t)}{\partial t} = -\frac{F}{A\epsilon} \frac{\partial s_2(z,t)}{\partial z} + \frac{1}{\epsilon} \left[ \frac{(K_1 - 1)}{\alpha_1} \mu_1(s_1, s_3) \right. \\ \left. - \frac{(K_2 - 1)}{\alpha_2} \mu_2(s_2, s_3) \right] x(z,t) \\ \frac{\partial s_3(z,t)}{\partial t} = -\frac{F}{A\epsilon} \frac{\partial s_3(z,t)}{\partial z} + \frac{1}{\epsilon} [K_d \mu_1(s_1, s_3) \\ + K_2 \mu_2(s_2, s_3)] x(z,t) \\ \frac{\partial x(z,t)}{\partial t} = [\mu_1(s_1, s_3) [\mu_2(s_2, s_3) - K_d]] x(z,t) \end{cases} \quad (2)$$

$$\text{With } K_d = (\mu_{li} + \mu_{di}) \frac{x_i(t)}{x_{max}}$$

for  $0 < z \leq 1$

with the following boundary conditions:

- $s_1(z=0, t) = s_{1, in}(t)$  for  $l = 1, 3$ .
- $s_2(z=0, t) = 0, x(z=0, t) = 0$

and the following initial conditions:

- $s_l(z, t=0) = s_{l,0}(z)$  for  $l = 1, 2, 3$ .
- $x(z, t=0) = x_0(z)$

In the precedent equations,  $z, s_1(z,t), s_2(z,t), s_3(z,t), x(z,t), s_{1, in}(t), s_{3, in}(t)$  represent the variable of space (m), the respective concentrations of nitrate, nitrite, carbon, biomass ( $g\ m^{-3}$ ), the nitrate and carbon concentrations at the entry of the reactor ( $g\ m^{-3}$ );  $F, A, k_{li}, k_D$  and  $\epsilon$  rate of feed at the entry of the engine ( $m^3/h$ ), the section of the biofilter ( $m^2$ ) yield coefficients, porosity,  $\mu_i$  specific rates of growth on nitrate and the nitrite ( $g\ m^{-3}/h$ ).

The specific growth rate expression (the Monod models') is as follows:

$$\mu_i = \eta_g \cdot \mu_{i, max} \cdot \left[ \frac{S_i}{S_i + k_{mi}} \right] \cdot \left[ \frac{S_3}{S_3 + k_{m3}} \right] \quad (3)$$

for  $i = 1, 2$

Where  $\eta_g$  is the factor of correction for the growth in anaerobic mode;  $\mu_{i, max}$  is the maximum specific growth rate of the biomass;  $k_{m1}, k_{m2}, k_{m3}$ , are the of Michaelis-Menten constants' associated, respectively with nitrate, nitrite and carbon.

The model is described by partial derivative equations; the variables of state  $S$  and  $X$  of the system don't depend only on time  $t$  but also on space  $z$ . It is a system with distributed parameters.

There are some methods (Dochain, 1994) intended to transform the distributed parameters system to into localized parameters systems described by an ordinary differential equations.

**Method of orthogonal collocation:** The method of orthogonal collocation (Magnus, 1997) is widely applied method to approximate a partial differential equation system by an ordinary differential system. This method requires the choice of a number  $N$  of internal collocation points, their position  $z_i$  along the engine and the functions of bases make it possible to rebuild the approximated solution to the interpolation points (Bastin and Dochain, 1990).

It is a question of finding starting from the choice of basic functions  $l_i(z)$  an approximate solution  $X(z, t)$  in the form:

$$x(t, z) = \sum_{i=0}^{N+1} l_i(z) \cdot x_i(t) \quad (4)$$

With:

$N$  : Numbers of collocation points,

$x_i(t) = x(z_i, t)$ : value of vector  $X$  at the of collocation points  $z_i$ . This solution not checking exactly the Eq. 2, one defines a residue  $R(x(z,t))$ , the problem consists in minimizing  $R(x(z,t))$ , which is translated in the shape of the scalar product:

$$\langle R(x(z,t)), w_i \rangle = 0 \quad i = 0, 1, 2, \dots, N+1.$$

In Bastin and Dochain (1990), Sylvie (1996) a study is detailed on these problems. An approximate solution of the system with the collocation method by taking the interpolation polynomial of Lagrange like basic function and the zeros of the Jacobi polynomial like collocation points and interpolation (Villadasen and Michelsen, 1978). If the collocation points are confused with the interpolation points, the method is known as of orthogonal collocation.

In Villadasen and Michelsen (1972)  $z_j, j = 1, \dots, N$ , the position of the collocation points is obtained by checking relations of orthogonality, according to the number  $N$  of collocation points and two parameters  $\alpha$  and  $\beta$ .

**Reduction of the model of denitrification:** With the application of the orthogonal collocation method, the simplified system becomes:

$$\left\{ \begin{aligned} \frac{ds_{1i}(t)}{dt} &= \frac{F}{A\varepsilon} \left( \sum_{j=1}^{N+1} \frac{dl_j(z_i)}{dz} s_{1j}(t) + \frac{dl_0(z_i)}{dz} s_{1, \text{in}}(t) \right) \\ &\quad - \frac{(k_1 - 1)}{\alpha_1 \varepsilon} \mu_{1i} x_1(t) \\ \frac{ds_{2i}(t)}{dt} &= -\frac{F}{A\varepsilon} \sum_{j=1}^{N+1} \frac{dl_j(z_i)}{dz} s_{2j}(t) + \frac{1}{\varepsilon} \left[ \frac{(k_1 - 1)}{\alpha_1} \mu_{1i} \right. \\ &\quad \left. - \frac{(k_2 - 1)}{\alpha_2} \mu_{2i} \right] x_1(t) \\ \frac{ds_{3i}(t)}{dt} &= \frac{F}{A\varepsilon} \sum_{j=1}^{N+1} \frac{dl_j(z_i)}{dz} s_{3j}(t) + \frac{dl_0(z_i)}{dz} s_{1, \text{in}}(t) \frac{1}{\varepsilon} \\ &\quad [k_{1i} \mu_{1i} + k_{2i} \mu_{2i}] x_1(t) \\ \frac{dx_1(t)}{dt} &= [\mu_{1i} + \mu_{2i} - k_{di}] x_1(t) \end{aligned} \right. \quad (5)$$

$$\text{with } k_d = (\mu_{1i} + \mu_{2i}) \frac{x_1(t)}{x_{\max}}$$

With the following initial conditions:

$$\begin{cases} s_l(z, 0) = s_l 0 & \text{for } l = 1, 2, 3 \\ x_1(z, 0) = x_0 \end{cases}$$

and :

$$\begin{aligned} \mu_{1i} &= \eta g \bullet \mu_{1\max} \bullet \left( \frac{S_{1i}}{S_{1i} + k_{m1}} \right) \cdot \left( \frac{S_{3i}}{S_{3i} + k_{m3}} \right) \\ \mu_{2i} &= \eta g \bullet \mu_{2\max} \bullet \left( \frac{S_{2i}}{S_{2i} + k_{m2}} \right) \cdot \left( \frac{S_{3i}}{S_{3i} + k_{m3}} \right) \end{aligned}$$

For the simulation runs the number of the collocation points is chosen as  $N = 4$  and the parameters  $\alpha = 1$ ,  $\beta = 1$  (Sylvie, 1996).

**Denitrification process control:** The adaptive control problem is to keep the sum value, of the concentrations of nitrate and of nitrite equal to its set point ( $y_d$ ) by manipulating (the feeding velocity)  $u(t)$ .

For that, we defines the output variable  $y(z, t)$  such as (Marquardt, 1963):

$$y_L = y(z = z_L, t) = c_1 s_{1L} + c_2 s_{2L} \quad (6)$$

( $c_1 = 0.226$  and  $c_2 = 0.304$ ) are the coefficients of conversion into equivalent nitrite and nitrate concentrations, respectively.

The dynamic equation of the state variable has to control:

$$\begin{aligned} \frac{dy_L}{dt} &= \frac{u}{\varepsilon} \frac{\partial y(z, t)}{\partial z} \Big|_{z=z_L} + (c_2 - c_1) \\ &\quad \frac{k_1 - 1}{\alpha_1 \varepsilon} \mu_{1L} x_{aL} - c_2 \frac{k_2 - 1}{\alpha_2 \varepsilon} \mu_{2L} x_{aL} \end{aligned} \quad (7)$$

**The variable of control is  $u(t)$ :** The variable of control may be feeding velocity of the fluid flow has through the engine  $u(t)$  (Sylvie, 1996; Haag and Queinnec, 2001).

The principle of the control law is to associate the regulation problem of the variable  $y_L$  at dynamics of the closed loop system represented by the following linear first-order

$$\frac{dy_L}{dt} = \lambda(y_d - y_1) \text{ avec } \lambda > 0 \quad (8)$$

$y_d$  is the desired value of  $y_L$ .

By combination of the Eq. 7 with the 8, it comes:

$$u(t) = \frac{-\lambda(y_d - y_1) + (c_2 - c_1) \frac{k_1 - 1}{\alpha_1 \varepsilon} \hat{\mu}_{1L} \hat{x}_{aL} - c_2 \frac{k_2 - 1}{\alpha_2 \varepsilon} \hat{\mu}_{2L} \hat{x}_{aL}}{\psi} \quad (9)$$

$$\text{With } \psi = \frac{1}{\varepsilon} \frac{\partial y(z, t)}{\partial z} \Big|_{z=z_L}$$

**Estimate of the concentration of biomass:** That  $\xi_1$  the vector of the measurable state variables:

$$\xi_1 = \begin{bmatrix} s_1(z, t) \\ s_2(z, t) \end{bmatrix}$$

And  $\xi_2$  the vector of the non-measurable state variables:  $\xi_2 = x_a(z, t)$

Moreover,

$$K_{c1} = \frac{1}{\varepsilon} \begin{bmatrix} \frac{1}{\alpha_1} (K_1 - 1) & 0 \\ -\frac{1}{\alpha_1} (K_1 - 1) & \frac{1}{\alpha_2} (K_2 - 1) \end{bmatrix} \text{ and } K_{c2} = [1 \ 1]$$

There is a transformation of state:

$$\xi = A_0 \xi_1 + \xi_2 \quad \text{that} \quad A_0 K_{c1} + K_{c2} = 0 \quad \text{if } \det(K_{c1}) \neq 0$$

$$A_0 = [a_1 - a_2] = \begin{bmatrix} \frac{\alpha_1 (k_2 - 1 + \alpha_2)}{k_2 - 2} & \varepsilon \frac{\alpha_2 (k_1 - 1)}{(k_1 - 1)(k_2 - 2)} \end{bmatrix}$$

This transformation leads to an observation dynamic equation of  $\xi$ , independent as  $x_a$ :

$$\frac{\partial \xi(z, t)}{\partial t} = k_d \xi(z, t) + a_1 s_1(z, t) + a_2 s_2(z, t) - \frac{u}{\varepsilon} \left( a_1 \frac{\partial s_1(z, t)}{\partial z} + a_2 \frac{\partial s_2(z, t)}{\partial z} \right) \quad (10)$$

$$\hat{x}_a(z, t) = \xi(z, t) - a_1 s_1(z, t) - a_2 s_2(z, t) \quad (11)$$

**Estimate of the growth kinetics:** For our model  $c_1 = c_2 = 1$  since the nitrite and nitrate concentrations are expressed in equivalent nitrogenizes. Then the control law is written:

$$u(t) = \frac{-\lambda(y_d - y_1) \frac{k_2 - 1}{\alpha_2 \varepsilon} \hat{\mu}_{2L} \hat{x}_{aL}}{\psi} \quad (12)$$

In this case, we will carry out only the estimate of the specific growth rate relating to the reaction of denitrification. For that, we write  $\mu_{2L}$  in the following form:

$$\mu_{2L} = \beta_{2L} s_{2L} \quad (13)$$

$\beta_{2L}$  is unknown parameter which is estimated by the recursive least-squares algorithm at forgetting factor:

$$\begin{aligned} \hat{\beta}_{2L_{t+1}} &= \hat{\beta}_{2L_t} - \gamma_t \frac{k_2 - 1}{\alpha_2 \varepsilon} S_{2L_t} \hat{x}_{aL_t} \Delta t (y_{L_{t+1}} - y_{L_{2in}}) \\ &+ \frac{u_t}{\varepsilon} \Delta t (\Delta s_{1L_t} + \Delta s_{2L_t} + b_{0N+1} (s_{1in} + s_{2in})) \\ &+ \frac{k_2 - 1}{\alpha_2 \varepsilon} \Delta t s_{2L_t} \beta_{2L_t} \hat{x}_{aL_t} \end{aligned} \quad (14)$$

$$\gamma_t = \frac{\gamma_{t-1}}{\sigma + \gamma_{t-1}^2 \Delta t^2 \frac{(k_2 - 1)^2}{\alpha_2^2 \varepsilon^2} s_{2L_t}^2 \hat{x}_{aL_t}^2} \quad 0 \leq \sigma \leq 1 \quad (15)$$

**Neural inverse model control:** The purpose of the neural networks hereafter is to provide at every moment the control increment  $\Delta u(t) = u(t) - u(t-1)$  (Fig. 2). The future variations of the output  $\Delta y(t+1)$  will make it possible to carry out a one step ahead prediction (Mokhtari and Marie, 1998; Ray, 1981).

The training of the network is carried out with Matlab neural NetToolbox; it's consists in modifying, with each step of training, the weights and skews in order to

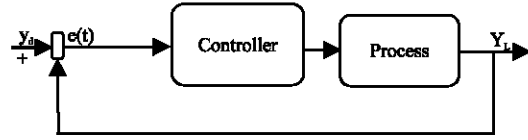


Fig. 2: Functional scheme of the control strategy

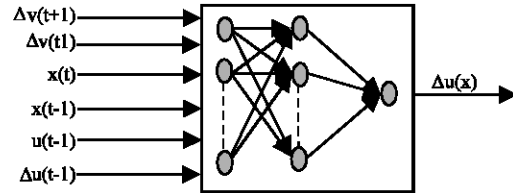


Fig. 3: The neural network controller

minimize the sum of the errors squares at outlet. The retropropagation method is based on the technique of the gradient. The weights and skews are initialized with random values. Of uniform distribution -0.5 and 0.5, the adaptation gain is selected equal to 0.8. We chose neural network architecture, in the sense of feedforward data processing, as follows:

Seven cells in the input layer that distributes the input data to the processors in the next layer.

Six cells in the hidden layer where the nonlinear behavior comes from and 1 cell in the output layer that transmits the response of the network to real world Fig. 3. (Mokhtari and Marir, 1998).

## RESULTS OF DIGITAL SIMULATIONS

Simulations are carried out for the initial conditions following:

$\forall_i = 1, \dots, N+1$

- $s_1(z_i, t = 0) = 16.93 \text{ g[N]}/\text{m}^3$
- $s_2(z_i, t = 0) = 0 \text{ g[N]}/\text{m}^3$
- $s_3(z_i, t = 0) = 101.5 \text{ g[N]}/\text{m}^3$
- $x_a(z_i, t = 0) = 525 \text{ g[N]}/\text{m}^3$

The rate of flow of the fluid is equal to:

- $u = 9 \text{ m h}^{-1}$
- $Yh_1 = 0.51, Yh_2 = 0.42, \mu_{1max} = 0.35 \text{ h}^{-1}, \mu_{2max} = 0.27 \text{ h}^{-1}$
- $K_1 = 1.5 \text{ g[N]}/\text{m}^3, K_2 = 1 \text{ g[N]}/\text{m}^3$
- $K_3 = 40 \text{ g[N]}/\text{m}^3$
- $\eta g = 0.8, \varepsilon = 0.52, x_{amax} = 675 \text{ g[DCO]}/\text{m}^3$
- $\lambda = 4, \sigma = 0.98, \gamma_0 = 10$

Figure 4 shows that the output process (concentration in substrate of the effluent) reached the set

point at the end of a few days at the price of a light oscillation in the transitional stage (Fig. 5). As we considered in the calibration, it is noticed that the output cascade converges nevertheless towards the instruction.

This proves that the action of integration of the error at the entry of the regulator is actually carried out thanks.

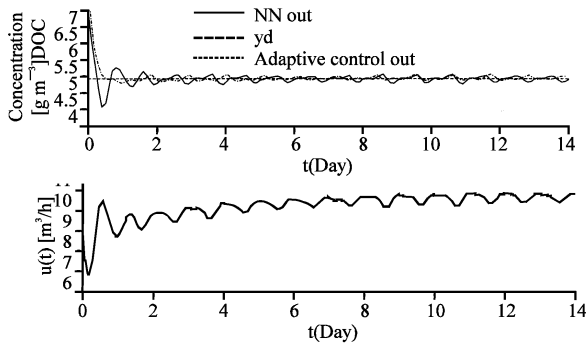


Fig. 4: Evolution of output  $Y_L$  and control evolution

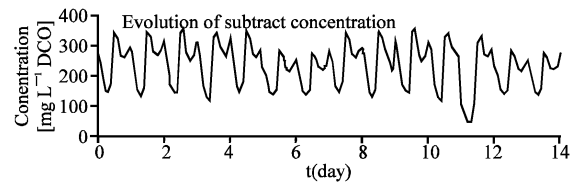


Fig. 5: Evolution of the substrate input

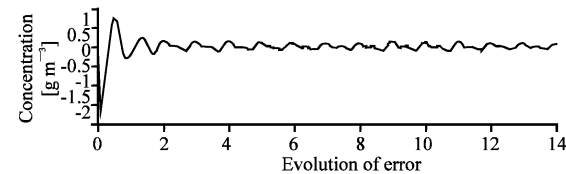


Fig. 6: Evolution of error

to the estimator of  $\hat{\mu}_2$  intended to gauge the outputs of the cascade (Fig. 6).

The estimated values of the biomass concentration  $\hat{x}_a$  using the asymptotic observer converges quickly, towards the effective concentrations in the effluent as shown in Fig. 7. The results of simulation are satisfactory; they show that the regulator functions well and present of good performances from the point of view of the regulation.

## CONCLUSION

In this research, the matter flows and transforms in the sewer collector were modeled by a piston engine in which the substrate is degraded by the micro-organisms that multiply. We, obtain in any rigour, a system with hyperbolic distributed parameters governed by a system of differential equations to the partial derivative. These stages of modeling, analysis and identification were of primary importance for the development of estimate algorithms. The objective of our study being to regulate the sum of the substrate concentrations to a standard fixed at exit of the biofilter while acting on the fluid flow rate. As the kinetic term of the process is related to the variation of the operating conditions, an algorithm of adaptive control was adopted.

We studied the possibility of considering an improvement of the control process structure by implementing a control of the bioreactor by neural networks.

We showed that it is possible to establish a controller by neural inverse model in the process to ensure the waste water treatment in the biofilter.

The simulations carried out thanks to data (I.N.S.A. of Toulouse) show that the adopted regulator presents of the satisfactory performances.

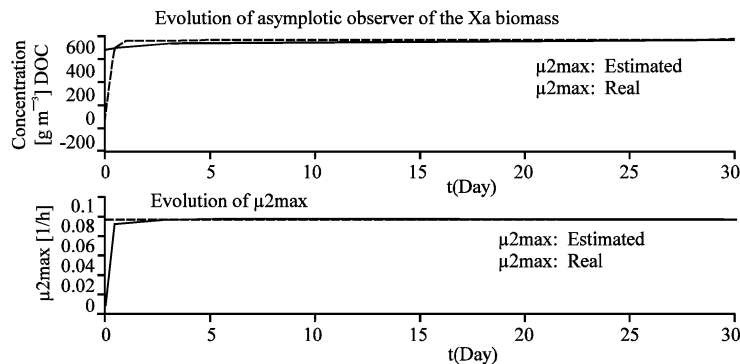


Fig. 7: Evolution of estimated biomass and estimate  $\mu_{2max}$

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