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The Response of Non-Initially Stressed Euler-Bernoulli Beam with an Attached Mass to Uniform Partially Distributed Moving Loads

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Abstract: The response of non-initially stressed Euler-Bernoulli beam with an attached mass to uniform partially distributed moving loads was examined. The governing partial differential equations were analyzed for both moving force and moving mass in order to determine the dynamic behavior of the system. The response amplitude due to the moving force was greater than that of the moving mass. The response amplitude of the moving force problem with non-initial stressed increased as the mass of the load M was increased.

Key words: Response, non-initially, Euler-Bernoulli beam, stressed, dynamic behavior

INTRODUCTION

All branches of transport system have experienced great advances characterized by increasing higher speeds and weight of vehicles. As a result, structures and media over or in which the vehicles move have been subjected to vibrations and dynamic stresses far larger than ever before.

The vibrations of elastic and inelastic structures under the action of moving loads have been investigated extensively and continued to be studied. In all the studies discussed above it was only the force effects of the moving loads that are taken into consideration.

The structures subjected to moving loads are usually modeled as elastic beams, plates or shells. The problem of elastic beam under the action of the moving loads has been considered under the assumption that the mass of the beams I s smaller than that of the load and obtained an approximate solution of the problem. (Esmailzadeth and Gorashi, 1995).

The response of finite simply supported Euler-Bernoulli beam to a unit force moving at a uniform velocity has been investigated where the effects of the moving force on beams with and without an elastic foundation were analyzed.

In this study, the response of non-initially stressed Euler-Bernoulli beam with an attached mass to uniform partially distributed moving loads is presented. The main objectives are:

- To present the analysis of the dynamic response of a non-initially stressed finite elastic Euler-Bernoulli beam with an attached mass at the end, but arbitrarily supported at the end, to uniform partially distributed moving load.
- To present a very simple and practical analytical-numerical technique for determining the response of beams with non-classical boundary conditions carrying moving mass.

DEVELOPING THE GOVERNING EQUATION

With reference to Fig. 1, a uniform simply supported Euler-Bernoulli beam of finite length L, with an attached mass M_L at x = L is acted upon initially at time t = 0s, by mass M over fixed length $\in$ of the beam with a specified constant velocity v. The load is in contact with the beam throughout the motion.

The governing equations describing the vibration behavior of the uniform non-initially stressed Euler-Bernoulli with an attached mass M_L at the end x = L but traversed by a uniform partially distributed moving mass M

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = F(x,t) \qquad (1)$$

Where, E the is the modulus of elasticity, I is the second moment of area of the beam's cross-sectional, m is the

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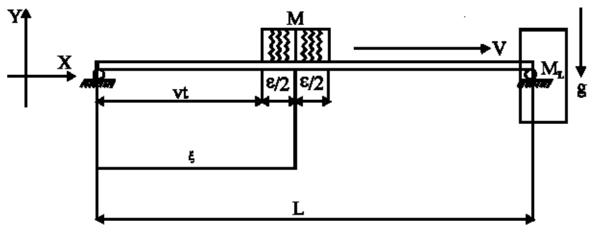

Fig. 1: Diagram of the Euler-Bernoulli Beam with masses

mass per unit length of the beam, y is the deflection of the beam, x is the spatial coordinate, t is the time and F(x, t) is the resultant applied force.

The resultant applied force per unit length F(x, t) is defined as

$$F(x,t)=\frac{1}{\in}\left[-Mg-M\frac{\partial^2 y}{\partial t^2}\right]\left[H\left(x-\zeta_{-}\frac{\in}{2}\right)-H\left(x-\zeta-\frac{\in}{2}\right)\right] \tag{2}$$

M - is the mass of the moving load.
g - is the acceleration due to gravity.
$\in$ - is the fixed length of M.
ζ - is the distance of load M along the length of the beam = vt + $\in$/2.

The differential operator $\frac{\partial^2}{\partial t^2}$ according to Adetunde (2003) and Akinpelu (2003) is defined as

$$\frac{\partial^2 y}{\partial t^2}=\frac{\partial^2}{\partial t^2}+2V\frac{\partial^2}{\partial x\partial t}+V^2\frac{\partial^2}{\partial x^2} \tag{3}$$

H (x) it the Heaviside function such that

$$H(x)=\begin{cases}0 & x<0\\ 1 & x>0\end{cases} \tag{4}$$

The governing equations describing the vibrational behavior of a uniform non-initially stressed Euler-Bernoulli beam with an attached mass M_L at the end x = L becomes

$$EI\frac{\partial^4 y}{\partial x^4}+M\frac{\partial^2 y}{\partial t^2}=\frac{1}{\in}\left[-Mg-M\frac{\partial^2 y}{\partial t^2}-2MV\frac{\partial^2 y}{\partial x\partial t}-M\frac{\partial^2 y}{\partial x^2}\right]\left[H\left(x-\zeta_{-}\frac{\in}{2}\right)-H\left(x-\zeta-\frac{\in}{2}\right)\right] \tag{5}$$

Subject to the following boundary conditions:

$$y=0 \text{ at } x=0 \qquad \frac{\partial y}{\partial x}=0 \quad \text{at } x=L \tag{6}$$

$$M\frac{\partial^2 y}{\partial x^2}=0 \text{ at } x=L\text{ , } EI\frac{\partial^3 y}{\partial x^3}-M_L\frac{\partial^2 y}{\partial x^2}=0 \text{ at } x=L \tag{7}$$

The corresponding initial conditions are

$$y(x,0)=0,\ y(x,0)=0 \tag{8}$$

SOLUTION TO THE INITIAL-BOUNDARY VALUE PROBLEM

Assumed a solution in the form of a series (Esmailzadeth and Gorashi, 1995, 1994; Adetunde, 2003; Akinpelu, 2003)

$$y(x,t)=\sum y_i(x)T_i(t) \tag{9}$$

Where $y_i(x)$ are the known eigen functions of the beam, $T_i(t)$ is a function of time (to be determined) satisfying the equaion (Esmailzadeth and Gorashi, 1995; Akinpelu, 2003)

$$y_i^{iv}-\beta_i^4 y_i=0 \tag{10}$$

$$\beta_i^4=\frac{m\lambda_i^2}{EI} \text{ and } \lambda_i=\sqrt{\frac{\beta_i^4 EI}{m}} \tag{11}$$

Where λ_i^2 are natural frequencies

$$y_i(x)=a\sin\beta_i x+b\cos\beta_i x+c\sinh\beta_i x+d\cosh\beta_i x \tag{12}$$

i.e., the solution to Eq. (10) and a-d are constants coefficients.

Assuming that the resultant applied force is

$$F(x,t)=\sum_{i=1}^{\infty}y_i(x)\Psi_i(t) \tag{13}$$

and substituting Eq. 9 into Eq. 2, 5, we have

$$EI\sum_{i=1}^{\infty}y_i^{iv}(x)T_i(t)+m\sum_{i=1}^{\infty}y_i(x)T_i(t)=\frac{1}{\in}\left[-Mg-M\sum_{i=1}^{\infty}y_i(x)T_i(t)-2MV\sum_{i=1}^{\infty}y_i(x)T_i(t)-MV^2\sum_{i=1}^{\infty}y_i(x)T_i(t)\right]\left[\begin{matrix}H\left(x-\zeta+\frac{\in}{2}\right)-\\ H\left(x-\zeta-\frac{\in}{2}\right)\end{matrix}\right]=\sum_{i=1}^{\infty}y_i(x)\psi_i \tag{14}$$

Multiply both sides of the RHS of Eq. (14) by $y_j(x)$ and taking the definite integrals of both sides along the length L of the beam with respect to x, we have

$$-Mg\int_0^\infty y_j(x)\left[H\left(x-\zeta+\frac{\in}{2}\right)-H\left(x-\zeta-\frac{\in}{2}\right)\right]dx - M\sum_{i=1}^\infty T_i(t)\int_0^L y_j(x)y_i(x)H\left(x-\zeta+\frac{\in}{2}\right)-H\left(x-\zeta-\frac{\in}{2}\right)dx - 2MV\sum_{i=1}^\infty T_i(t)$$
$$\int_0^L y_j(x)y_i^I(x)\left[H\left(x-\zeta+\frac{\in}{2}\right)-H\left(x-\zeta-\frac{\in}{2}\right)\right]dx - MV^2\sum_{i=1}^\infty T_i(t)\int_0^L y_j(x)y_i^{II}(x)\left[H\left(x-\zeta+\frac{\in}{2}\right)-H\left(x-\zeta-\frac{\in}{2}\right)\right] = \sum_{i=1}^\infty \Psi_i(t)\int_0^L y_j(x)y_i(x)dx \tag{15}$$

Evaluation the first definite integral in Eq. 15 by carrying out integration by part with respect to x using the following two properties of singularity function

$$\int_{x0}^{x1} x_j(x)\delta(x-x_1) = x_j(x_1), \text{provided } x_0 < x_1 < x_2 \tag{16}$$

Hence

$$\frac{d}{dx}(x-x_1) = \delta(x-x_1) \tag{17}$$

Similar arguments to second, third to fifth definite integral in (15) hence evaluating the integrals using Taylor's series expansion and applying orthogonality properties of the characteristics function yj(x) to the RHS of (15), we finally obtain

$$\begin{aligned}\psi_i(t) = &-Mg\left[y_i(\in)+\frac{\in^2}{24}y_i^{II}(\in)\right] - M\sum_{i=1}^\infty \ddot{T}_i(t)\left\{y_i(\in)y_j(\in)+\frac{\in^2}{24}\left[y_i^{II}(\in)y_j(\in)+2y_i^I(\in)y_i^I(\in)+y_i(\in)y_j^{II}(\in)\right]\right\}\\ &-2MV\sum_{i=1}^\infty \dot{T}_i(t)\left\{y_i^I(\in)y_j(\in)+\frac{\in^2}{24}\left[y_i^{III}(\in)y_j(\in)+2y_i^{II}(\in)y_i^I(\in)+y_i^I(\in)y_j^{II}(\in)\right]\right\}\\ &-MV^2\sum_{i=1}^\infty T_i(t)\left\{y_i^{II}(\in)y_j(\in)+\frac{\in^2}{24}\left[y_i^{iv}(\in)y_j(\in)+2y_i^{III}(\in)y_i^I(\in)+y_i^{II}(\in)y_j^{II}(\in)\right]\right\}\end{aligned} \tag{18}$$

Substituting Eq. 18 into the RHS of Eq. 14 we have

$$\begin{aligned}&EI\sum_{i=1}^\infty y_i^{iv}(x)T_i(t) + m\sum_{i=1}^\infty y_i(x)\ddot{T}_i(t) = \sum_{i=1}^\infty y_i(x)\left\{-Mg\left[y_i(\in)+\frac{\in^2}{24}y_i^{II}(\in)\right]\right.\\ &-M\sum_{i=1}^\infty \ddot{T}_i(t)\{y_i(\in)y_j(\in)+\frac{\in^2}{24}\left[y_i^{II}(\in)y_j(\in)+2y_i^I(\in)y_j^I(\in)+y_i(\in)y_j^{II}(\in)\right]\}\\ &-2MV\sum_{i=1}^\infty \dot{T}_i(t)\left\{y_i^I(\in)y_j(\in)+\frac{\in^2}{24}\left[y_i^{III}(\in)y_j(\in)+2y_i^{II}(\in)y_j^I(\in)+y_i^I(\in)y_j^{II}(\in)\right]\right\}\\ &MV^2\sum_{i=1}^\infty T_i(t)\left\{y_i^{II}(\in)y_j(\in)+\frac{\in^2}{24}\left[y_i^{iv}(\in)y_j(\in)+2y_i^{III}(\in)y_j^I(\in)+y_i^{II}(\in)y_j^{II}(\in)\right]\right\}\end{aligned} \tag{19}$$

Considering Eq. 10 and 11, then Eq. 19 becomes

$$\begin{aligned}&\sum_{i=1}^{\infty} y_i(x)\left\{m\ddot{T}_i(t)+m\lambda_i^2T_i(t)+Mg\left[y_i(\in)+\frac{\in^2}{24}+y_i^{II}(\in)\right]\right\}+\\&M\sum_{i=1}^{\infty}\ddot{T}_i(t)\left\{y_i(\in)y_j(\in)+\frac{\in^2}{24}\left[y_i^{II}(\in)y_j(\overline{\in})+2y_i^{I}(\in)y_j^{I}(\overline{\in})+y_i(\in)y_j^{II}(\overline{\in})\right]\right\}+\\&2MV\sum_{i=1}^{\infty}\dot{T}_i(t)\left\{y_i^{I}(\in)y_j(\in)+\frac{\in^2}{24}\left[y_i^{III}(\in)y_j(\overline{\in})+2y_i^{II}(\in)y_j^{I}(\overline{\in})+y_i^{I}(\in)y_j^{II}(\overline{\in})\right]\right\}+\\&MV^2\sum_{i=1}^{\infty}T_i(t)\left\{y_i^{II}(\in)y_j(\in)+\frac{\in^2}{24}\left[y_i^{iv}(\in)y_j(\overline{\in})+2y_i^{III}(\in)y_j^{I}(\overline{\in})+y_i^{II}(\in)y_j^{II}(\overline{\in})\right]\right\}=0\end{aligned} \quad (20)$$

The Eq. 20 must be satisfied for arbitrary yi(x) and this possible only when the expression in the curl bracket is equal to zero. Hence

$$\begin{aligned}&m\ddot{T}_i(t)+m\lambda_i^2T_i(t)+Mg\left[y_i(\in)+\frac{\in^2}{24}+y_i^{II}(\in)\right]+\\&M\sum_{i=1}^{\infty}\ddot{T}_i(t)\left\{y_i(\in)y_j(\in)+\frac{\in^2}{24}\left[y_i^{II}(\in)y_j(\overline{\in})+2y_i^{I}(\in)y_j^{I}(\overline{\in})+y_i(\in)y_j^{II}(\overline{\in})\right]\right\}\\&+2MV\sum_{i=1}^{\infty}\dot{T}_i(t)\left\{y_i^{I}(\in)y_j(\in)+\frac{\in^2}{24}\left[y_i^{III}(\in)y_j(\overline{\in})+2y_i^{II}(\in)y_j^{I}(\overline{\in})+y_i^{I}(\in)y_j^{II}(\overline{\in})\right]\right\}\\&-MV^2\sum_{i=1}^{\infty}T_i(t)\left\{y_i^{II}(\in)y_j(\in)+\frac{\in^2}{24}\left[y_i^{iv}(\in)y_j(\overline{\in})+2y_i^{III}(\in)y_j^{I}(\overline{\in})+y_i^{II}(\in)y_j^{II}(\overline{\in})\right]\right\}=0\end{aligned} \quad (21)$$

$i = 1, 2, 3, \ldots\ldots$

The system of Eq. 21 is a set of coupled ordinary second order differential equations and it is easily observed that a numerical approach is required to solve it.

The eigen functions $y_i(x)$ for the present configuration is

$$Y_i(x)=\sin\frac{a_ix}{L}+\beta_i\sinh\frac{a_ix}{L} \quad (22)$$

$i = 1, 2, 3, \text{------}n$

Where $\beta_i=\dfrac{\sin a_i}{\sinh a_i}$,

a_1 are the roots of the simply supported transcendental frequency equation

$$\text{Cos}a_i\sinh a_i-\text{Sin}a_i\cosh a_i-\frac{2\lambda_i^2M_L}{EIa_i^3}\sin a_i\sinh a_i=0 \quad (23)$$

We obtain the set of exact governing differential equation for the vibration of the beam by employing Eq. 22 and evaluating the exact values of the integral in Eq. 15 and we finally obtain

$$
\begin{aligned}
& m\ddot{T}_i(t) + m\lambda_i^2 T_i(t) = \left\{ -\frac{2MgL}{\in a_j}\left[\frac{\sin\varsigma}{L} a_i \sin\frac{\in a_i}{2L} + \beta_i \sinh\frac{\varsigma}{L} a_i \sinh\frac{a_i \in}{2L}\right]\right\} + \\
& \sum_{i=1}^{\infty} \ddot{T}_i(t) \left\{ \frac{ML}{\in\left(a_i^2 + a_j^2\right)}\left[\cos(a_i - a_j)\frac{\varsigma}{L}\sin(a_i - a_j)\frac{\in}{2L}\right] - \left[\frac{ML}{\in\left(a_i^2 + a_j^2\right)}\left[\cos(a_i + a_j)\frac{\varsigma}{L}\sin(a_i + a_j)\frac{\in}{2L}\right]\right] + \right. \\
& \frac{MLB_i}{\in\left(a_i^2 + a_j^2\right)}\left[a_i\left[\cos(1 - a_j)\frac{\varsigma}{L}\sin(1 - a_j)\frac{(-\in)}{2L} + \cos(1 + a_i)\frac{\varsigma}{L}\sin(1 + a_i)\frac{(-\in)}{2L}\right]\right] + \\
& a_j\left[\cos(a_i - a_j)\frac{\varsigma}{L}\sin(a_i - a_j)\frac{\in}{2L} + \cos(a_i + a_j)\frac{\varsigma}{L}\sin(a_i + a_j)\frac{\in}{2L}\right] + \frac{\beta_i^2}{\in(a_i - a_j)}\left[\cosh(a_i - a_j)\frac{\varsigma}{L}\sinh(a_i - a_j)\frac{\in}{2L}\right] \\
& \left. \frac{\beta_i^2}{\in(a_i + a_j)}\left[\cosh(a_i + a_j)\frac{\varsigma}{L}\sinh(a_i + a_j)\frac{\in}{2L}\right]\right\} + \\
& 2MVa_i \sum_{i=1}^{\infty} T_i(t) \left\{ \frac{L}{\in\left(a_i - a_j\right)}\left[\cos(a_i - a_j)\frac{\varsigma}{L}\sin(a_i - a_j)\frac{\in}{2L}\right] - \left[\frac{L}{\in\left(a_i + a_j\right)}\left[\cos(a_i + a_j)\frac{\varsigma}{L}\sin(a_i + a_j)\frac{\in}{2L}\right]\right] + \right. \\
& \frac{LB_i}{\in\left(a_i^2 + a_j^2\right)}\left[a_i\left[\cos(1 - a_j)\frac{\varsigma}{L}\sin(1 - a_j)\frac{(-\in)}{2L} + \cos(1 + a_i)\frac{\varsigma}{L}\sin(1 + a_i)\frac{(-\in)}{2L}\right] + \right. \qquad (24) \\
& \left. a_j\left[\cos(1 - a_j)\frac{\varsigma}{L}\sin(1 - a_j)\frac{\in}{2L} + \cos(1 + a_j)\frac{\varsigma}{L}\sin(1 + a_j)\frac{\in}{2L}\right]\right] + \\
& \left. \frac{\beta_i^2 L}{(a_i + a_j)}\left[\cosh(a_i + a_j)\frac{\varsigma}{L}\sinh(a_i - a_j)\frac{\in}{2L}\right] - \frac{\beta_i^2 L}{(a_i - a_j)}\left[\cosh(a_i - a_j)\frac{\varsigma}{L}\sinh(a_i - a_j)\frac{\in}{2L}\right]\right\} - \\
& MV^2 a_i^2 \sum_{i=1}^{\infty} T_i(t) \left\{ \frac{L}{\left(a_i - a_j\right)}\left[\cos(a_i - a_j)\frac{\varsigma}{L}\sin(a_i - a_j)\frac{\in}{2L}\right] + \left[\frac{L}{\left(a_i + a_j\right)}\left[\cos(a_i - a_j)\frac{\varsigma}{L}\sin(a_i - a_j)\frac{\in}{2L}\right] + \right.\right. \\
& \frac{LB_i}{\in\left(a_i^2 + a_j^2\right)}\left[a_j\left[\cos(1 - a_j)\frac{\varsigma}{L}\sin(1 - a_j)\frac{(-\in)}{2L} + \cos(1 + a_i)\frac{\varsigma}{L}\sin(1 - a_i)\frac{(-\in)}{2L}\right] + \right. \\
& \left. a_i\left[\cos(1 - a_i)\frac{\varsigma}{L}\sin(1 - a_i)\frac{(-\in)}{2L} + \cos(1 - a_i)\frac{\varsigma}{L}\sin(1 - a_i)\frac{\in}{2L}\right]\right] + \\
& \left. \frac{\beta_i^2 L}{(a_i + a_j)}\left[\cosh(a_i + a_j)\frac{\varsigma}{L}\sinh(a_i + a_j)\frac{\in}{2L}\right] - \frac{L\beta_i^2}{(a_i - a_j)}\left[\cosh(a_i - a_j)\frac{\varsigma}{L}\sinh(a_i - a_j)\frac{\in}{2L}\right]\right\}
\end{aligned}
$$

Note for the case $a_i = a_j$ we replace the expression involving $\frac{1}{a_i - a_j}$ by $\frac{a_i \in}{2L}$

To solve Eq. 24, recourse can be made to a numerical method, but 2 interesting cases are to be tackled.

Case I: The moving force non-initially stressed Euler-Bernoulli beam: A moving force problem is one in which the inertia effects of the moving load are neglected and only the force effects are retained. This is done in Eq. 24 by neglecting all the terms on the right hand side of the later except the first term in the first curly bracket i.e., by neglecting all the terms apart from the first term on the right hand side of Eq. 24.

Case II: The moving mass non-initially stressed Euler-Bernoulli beam: This is the case in which both the inertia effect as well as the force effect are taken into consideration. The entire Eq. 24 is the moving mass problem.

To obtain results, an approximate Central difference formulas were used, for the derivatives in Eq. 24 for both cases (cases I and II). Thus, for N modal shapes, Eq. 24 are transformed to a set of N linear algebraic equations, which were solved for each interval of time. Regarding the definition of approximation involved, in order to ensure the stability and convergence of the solution, sufficiently small time steps have been utilized.

Computer program was developed and the following numerical data which are the same as those in reference (Esmailzadeth and Gorashi, 1995, 1994;

Table 1: Variation of the lateral displacement, Ұ (x, t) of the non-initially-stressed simply supported Euler-Bernoulli beam carrying a lumped mass as its end x = L and traversed by moving force. For t = 0.5 s = 0.1 and various values of M

Length of the beam X (m)	Y_F (x,t) For M = 7.04 kg m^{-1}	Y_F (x,t) For M = 8.0 kg m^{-1}	Y_F (x,t) For M = 10 kg m^{-1}
1.4644	-2.6068E-03	-2.9622E-03	-3.7028E.03
2.2788	-7.9777E.03	-9.0645E.03	-1.1133F-02
4.2481	-1.1664E.02	-1.3255E.02	-1.6569E.02
5.8575	-1.0316E.02	-1.1723E.02	-1.4654E.02
7.1215	-5.6476E.03	-6.4177E.03	-1.3315E.02
8.5353	-8.4752E.04	-9.6310E.04	-1.2039E.03
9.9501	-7.4436E.04	-8.4588E.04	-1.0573E.03

Table 2: Variation of the lateral displacement, Y_F (x, t) of the non-initially-stressed simply supported Euler-Bernoulli beam carrying a lumped mass at its end and traversed by moving force. For t = 1.0 s ε = 0.1 m and different values of M

Length of the beam X (m)	Y_F (x,t) For M = 7.04 kg m^{-1}	Y_F (x,t) For M = 8.0 kg m^{-1}	Y_F (x,t) For M = 10 kg m^{-1}
1.4644	-2.6069E-03	-2.9624E-03	-3.7031E-03
2.2788	-7.9784E-03	-9.0664E-03	-1.1333E-02
4.2481	-1.1667E-02	-1.3251E-02	-1.6574E-02
5.8575	-1.0322E-02	-1.1729E-02	-1.4666E-02
7.1215	-5.6546E-03	-6.4259E-03	-8.0325E-03
8.5353	-8.5203E-04	-9.6823E-04	-1.2103E-03
9.9501	-7.3755E-04	-8.3813E-04	-1.0476E-03

Table 3: Variation of the lateral displacement, Y_F (x, t) of the non-initially-stressed simply supported Euler-Bernoulli beam carrying a lumped mass at end x = L and traversed by moving force. For ε = 0.1 m and different values of t = 0.5 s, t = 1.0 s, t = 1.5 s and M = 7.04 kg m^{-1}

Length of the beam X (m)	Y_F (x,t) For M = 7.04 kg m^{-1}	Y_F (x,t) For M = 8.0 kg m^{-1}	Y_F (x,t) For M = 10 kg m^{-1}
1.4644	-2.6068E-03	-2.6069E-03	-2.6075E-03
2.2788	-7.9777E-03	-7.9784E-03	-7.9795E-03
4.2481	-1.1664E-02	-1.1667E-02	-1.1669E-02
5.8575	-1.0316E-02	-1.0322E-02	-1.0323E-02
7.1215	-5.6476E-03	-5.6546E-03	-5.6556E-02
8.5353	-8.4752E-04	-8.5203E-04	-8.5240E-04
9.9501	-7.4436E-04	-7.3755E-04	-7.3722E-04

Table 4: Variation of the lateral displacement, Y_M (x, t) of the simply supported non-initially-stressed simply supported Euler-Bernoulli beam carrying a lumped mass at x = L traversed by moving mass. For t = 0.5 s and ε = 0.1 and different values of M

Length of the beam X (m)	Y_F (x,t) For M = 7.04 kg m^{-1}	Y_F (x,t) For M = 8.0 kg m^{-1}	Y_F (x,t) For M = 10 kg m^{-1}
1.4644	-8.3230E-07	-8.3550E-07	-8.3330E-07
2.2788	-8.8780E-07	-8.8220E-07	-8.8600E-07
4.2481	-1.7828E-06	-1.1141E-06	-1.0011E-06
5.8575	-1.2603E-06	-1.2606E-06	-1.2604E-06
7.1215	-9.4010E-07	-9.4090E-07	-9.1100E-07
8.5353	-3.2500E-07	-3.2410E-07	-3.2470E-07
9.9501	-4.3400E-07	-4.3780E-07	-4.3690E-07

Adetunde, 2003; Akinpelu, 2003) were used for the purpose of comparisons E = 2.07*10^{11}N m^{-2} I = 1.04 * 10-6 m4, V = 12 km h^{-1}, m = 70 kg, g = 9.8 m s^{-1}2, M = 7.04, 8 and 10 kg m^{-1}, t = 0.5, 1.0, 1.5 sec, L = 10, ∈ = 0.1 and 1.0 m.

Hence we have the Table 1-7 of results.

Table 5: Variation of the lateral displacement, Ym (x, t) of the simply supported non-initially-stressed Euler-Bernoulli beam carrying a lumped mass at x = L and traversed by moving mass. For ε = 0.1 m and different values of t = 0.5 s, t = 1.5 s and M = 7.04 kg m^{-1}

Length of the beam X (m)	Y_F (x,t) For M = 7.04 kg m^{-1}	Y_F (x,t) For M = 8.0 kg m^{-1}	Y_F (x,t) For M = 10 kg m^{-1}
1.4644	-8.3230E-07	-1.6125E-06	-2.4184E-06
2.2788	-8.8780E-07	-1.7199E-06	-2.5791E-06
4.2481	-1.7828E-06	-3.4535E-06	-5.1794E-06
5.8575	-1.2603E-06	-2.4416E-06	-3.6615E-06
7.1215	-9.401E-07	-1.8212E-06	-2.7314E-06
8.5353	-3.2500E-07	-6.2970E-07	-9.4420E-06
9.9501	-4.3400E-07	-8.4560E-07	-1.2683E-06

Table 6: Variation of the lateral displacement, Ym (x, t) of the simply supported non-initially-stressed Euler-Bernoulli beam for different values of ε = 0.1 and 1 m against time t, and M = 7.04 kg m^{-1}

Time t(s)	Ym(X,t) For ε = 0.1 m	Ym(X,t) For ε = 1.0 m
0.48	-8.3230E-07	-1.2731E-06
0.86	-8.8780E-07	-1.3278E-06
1.29	-1.7828E-06	-1.17409E-06
2.15	-1.2603E-06	-1.5588E-06
2.15	-9.4010E-07	-1.5137E.03
2.58	-3.2500E-07	-3.1861E-07
3.01	4.3400E-07	5.8588E-07

Table 7: Variation of the deflection, Y_F (x, t) of the simply supported non-initially-stressed Euler-Bernoulli beam carrying a lumped mass at x = L and traversed by moving mass. For ε = 0.1 m, t = 1.0 s and different values of M

Length of the beam X (m)	Y_F (x,t) For M = 7.04 kg m^{-1}	Y_F (x,t) For M = 8.0 kg m^{-1}	Y_F (x,t) For M = 10 kg m^{-1}
1.4644	-1.6125E-06	-1.6145E-06	-1.6187E-06
2.2788	-1.7199E-06	-1.7163E-06	-1.7089E-06
4.2481	-3.4535E-06	-3.4582E-06	-3.4682E-06
5.8575	-2.4416E-06	-2.4417E-06	-2.4421E-06
7.1215	-1.8212E-06	-1.8218E-06	-1.8228E-06
8.5353	-6.2970E-07	-6.2910E-07	-6.3310E-07
9.9501	-8.4560E-07	-8.4650E-07	-8.4830E-07

DISCUSSION

The dynamic response of non initially stressed finite elastic Euler Bernoulli Beam with an attached mass at the end x = L but arbitrarily supported at the end x = 0 to uniform partially distributed moving load have been analyzed.

Table 1 and 2 were presented for the case I (i.e., moving force problem). In particular in Table 1 are shown the deflections of the system for t = 0.5 sec, **ε** = 0.1 m and various of M. Table 2 contains similar results but for t = 1.0 sec, **ε** = 0.1 m and various values of M.

Table 3 and 4 shown the variation of lateral displacement of the non-initially stressed Euler Bernoulli beam carrying a lumped mass at x = L and traversed by a moving mass (i.e., case II problem) for (i) t = 0.5 sec, **ε** = 0.1 m, (ii) t = 1.5 sec, **ε** = 0.1 m.

Table 5 shows the variations of displacement of the non-initially stressed simply supported Euler Bernoulli beam carrying a lumped mass at x = L and traversed by a moving force (Case I) for **ε** = 0.1 m, at different value of time t = 0.5, 1.0 and 1.5 sec, when M = 7.04 kg.

Table 6 shows the variations of deflection of the simply supported non-initially stressed Euler Bernoulli beam carrying a lumped mass at x = L and traversed by a moving mass (Case II) for ε = 0.1m, at different value of time t = 0.5, 1.0 and 1.5 sec, when M = 7.04 kg.

Table 7 shows the variation of deflection of the simply supported non-initially stressed Euler-Bernoulli beam for different values of ε = 0.1 and ε = 1.0 m, when M = 7.04 kg.

CONCLUSION

From Table 1 and 2 it is clear that the response amplitude of the moving force problem of non-initially stressed increases at mass of the load M increases.

From Table 3 and 4 it was observed that the response amplitude of the deflection increases as M increases.

Table 5 depicted the behaviour of the deflection as a function of time t, clearly it was shown that the response amplitude increases as time t increases. Furthermore Table 6 depicted the various value of the deflection of the beam against time t, for ε = 0.1 and ε = 1.0 m. It was found that the amplitude deflection decreases as ε increases.

Comparing Table 1-4, it was observed that the response amplitude due to the moving force are greater than those due to the moving mass for non initially stressed Euler Bernoulli beam.

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