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Contribution to the Development of Approximate Solutions for the Quasi Linear Equations Characterizing the non Permanent Flows in Channels with Removable Bottoms

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Abstract: The numerical models for computing simultaneously the liquid flow and the bed loads in natural channels exist recently. Actually, they have reached such a level of development that they can be used in engineering. These models can simultaneously solve the equations related to the liquid flow, solid transport and the deformation of the bed. We will present in this study approximate solutions using the equations of St-Venant related to the non stationary flows in natural channels with removable bottoms.

Key words: Quasi linear equations, nonpermanent flows, natural channels, transport, deformation

INTRODUCTION

The consist of beds permanently exposed to deformations in length and width due to the flows.

Generally, in a steady regime, the bed can remain stable, if the concentration of the current of water in solid particles does not exceed saturation. As soon as the mode becomes nonpermanent, more or less significant deformations can be noted. Following the erosion of the bed, water will be charged until reaching saturation. This situation gives birth to solid transport by haulage or suspension. At the beginning, erosion is more intense and weakens with time once that the bed reaches its new balanced profile.

Indeed, according to the characteristics of the river (the slope, the grading of the material which constitute it) and of the liquid flow on the one hand and the concentration of this latter in bed load on the other hand, there will be either the deposit of the solid particles, or the erosion of the bed. It is a self controlled system: the current of water deforms the bed; but the change of the bed leads to the variation of the field velocity and influences the flow.

Solid transport by haulage and in suspension depends on the dimensions of the materials of the bed, the flows, the frequencies of these flows and the hydraulic slope. In order to better determine the various aspects of this study and concerning the application of the method of the characteristics for very simple cases of the interaction between the current of water and the bed, we briefly examine certain characteristics of the rivers related to the computing of the nonpermanent flow in the rivers with removable bottoms (Cunge, 1988).

FALLING VELOCITY OF THE SOLID PARTICLES

The problem of the determination of the falling velocity of solid particles is related to the problem of the resistance of the liquid to the movement of these particles. The form of these latter's in the case of the transport of fines in suspension is variable. Nevertheless, the prevalent form is spherical. The solid particles reach their falling limit velocity when the resistance of the liquid becomes equal to their weight (Levy, 1957; Graf, 1971, 1984).

$$F = k\rho\vartheta_0^2\frac{\pi d^2}{4} = (\rho_1 - \rho)g\frac{\pi . d^3}{6} \quad (1)$$

where: ρ_1-density of the solid particles; k-trailing coefficient; ϑ_0 -falling velocity of the particles; d-diameter of the particles; from where:

$$k = \frac{2}{3}(\frac{\rho_1}{\rho} - 1)g\frac{d}{\vartheta_0^2} \quad (2)$$

In addition, the force of resistance:

$$F = 3\rho\pi\upsilon\vartheta_0 d = k\rho\vartheta_0^2\frac{\pi d^2}{4}$$
$$\text{hence}\quad k = \frac{12\upsilon}{\vartheta_0 d} = \frac{12}{Re} \quad (3)$$

Where υ: dynamic viscosity coefficient.

The experiments undertaken by Stokes, Prandtl and Von Karman have shown that the behavior of the

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particles depends on the Reynolds number (Re). The results obtained can be summarized as follows (Reynolds, 1974; Prandtl, 1935):

For Re<1.0; D<0.10 mm, the mode is laminar and the formula of Stokes is checked:

$$\vartheta_0 = \frac{gd^2}{18\upsilon}\left(\frac{\rho_1}{\rho} - 1\right) \quad (4)$$

For $1 \le Re \le 30$, the theory of Prandtl is checked, for whom (smooth bed):

$$\frac{(\rho_1 - \rho)gd}{\rho\vartheta_0^2} = \frac{5.6}{\sqrt{Re}} \quad (5)$$

This zone corresponds to the particles of d = 0.10 ÷ 0.6 mm.

In the zone of turbulence, two modes are noted and the formulas of Karman can be applied:

*transitory zone: 30 < Re < 400 and d = 0.6÷2.0 mm

$$2\vartheta_0^{1,8} = \frac{gd^{1,2}(\rho_1 - \rho)}{2,2\upsilon^{0,2}\rho} \quad (6)$$

*quadratic zone: Re > 400 and d > d 2.0 mm

$$\vartheta_0 = 1,2\sqrt{gd\left(\frac{\rho_1}{\rho} - 1\right)} \quad (7)$$

It is worth noting that there are other criteria and other formulas for the determination of the falling velocity υ_0. We quoted the preceding formulas as an indication.

Critical velocity and concentration of solid particles: The critical velocity is the minimal velocity which ensures transport in suspension of a given quantity of solid particles. Consequently, the critical velocity varies according to the quantity and the falling velocity of these particles. Its minimal value corresponds to the transitional stage of transport by haulage in transport by suspension. To determine the value of the critical velocity, it is necessary to know the distribution of the solid particles according to the depth as well as to the turbidity of the current of the water (Graf, 1971, 1984).

The experiments carried out by Knoroz have allowed the obtention of the following relation:

$$\mu = \left(\frac{v_{cr} - v_0}{3.5\vartheta_0}\right)^4 \left(\frac{d}{R}\right)^{1.6} \quad (8)$$

Where: μ-average turbidity of the water current. v_{Cr}-critical velocity. v_0 -mean velocity of flow which corresponds to the beginning of the movement of the aggregates by haulage.

Given that $v_{cr} >> v_0$, the relation (8) can be written as:

$$\mu = c\left(\frac{v_{cr}}{\vartheta_0}\right)^4 \left(\frac{d}{R}\right)^{1.6} \quad (9)$$

The coefficient C, following Knoroz is equal to 0.006.

For the critical velocity, Knoroz proposes the following relation:

$$v_{cr} = v_0 + 3.5\vartheta_0\sqrt[4]{\mu}\left(\frac{R}{d}\right)^{0.4} \quad (10)$$

To determine v_0, we can use one of the following formulas of Vélikanov

$$\frac{v_0}{g} = 14d + 5.8mm \quad (11)$$

or of Levy: (Velikanov, 1955)

$$v_0 = 1.4\left[\frac{h}{d}\right]^{1/5}\left[gd\frac{\gamma_1 - \gamma}{\gamma}\right]^{1/2} \quad (12)$$

For the determination of the current of turbidity, there are several formulas. One will quote, as an example, the following relations established by Zamarine (Levy, 1957):

$$\mu = 0.022\left(\frac{v_{cr}}{\vartheta_0}\right)^{3/2}\sqrt{Ri}\text{; If } 0.2 < \vartheta_0 < 0.8 \text{ cms}^{-1} \quad (13)$$

$$\mu = 11v_{cr}\sqrt{Ri\frac{v_{cr}}{\vartheta_0}}\text{; If } 0.04 < \vartheta_0 < 0.2 \text{ cms}^{-1} \quad (14)$$

Knowing the bed load $Qs = \gamma\, bq_s = \gamma bq\mu$ and using the formula (9), we get:

$$Qs = c\frac{v^4}{\vartheta_0^4}\left(\frac{d}{R}\right)^{1.6}\gamma qb = A\gamma qbv^4 \quad (15)$$

where:

$$A = \frac{c}{\vartheta_0^4}\left(\frac{d}{R}\right)^{1.6}$$

liquid q flow; b width of the river; q_S-bed load specific.

EQUATION OF DEFORMATION OF THE BED

The deformation of the bed is due to imbalance between the quantity of solid particles deposited on the section concerned of the channel and that carried out towards the downstream Level. If the rate of the flow increases, there is the erosion of the bed; if it decreases; one notes the deposit of the solid particles. So the equation of deformation of the bed can be obtained by establishing the assessment of the solid particles on the section concerned of the river. This assessment is identical to the equation of continuity of the non stationary flow.

Let us examine a section of a bed having a length Δx, a width b and a depth h. Let us suppose that the flow Q is constant and that the flow is gradually varied. This enables us to consider that the flow is telegraphic and that the hydraulic parameters depend only on x and t. This condition is very significant, since by analyzing the local deformations of broad rivers, we must divide the current of the water into a series of filaments of water by the method of Bernadsky (1933).

The dynamic equation of the gradually varied nonpermanent flow is written as follows:

$$\frac{\partial y}{\partial x} = \frac{\partial z}{\partial x} + \frac{\partial h}{\partial x} = -\alpha \frac{\partial}{\partial x}\left(\frac{v^2}{2g}\right) - \frac{v^2}{C^2 R} - \frac{1}{g}\frac{\partial v}{\partial t} \qquad (16)$$

According to Fig. 1, we can write:

$$\left[Qs - \left(Qs + \frac{\partial Qs}{\partial x}\Delta x\right)\right]\Delta t = -\frac{\partial Qs}{\partial x}\Delta x \Delta t$$

This difference must be equal to the weight of the quantity of materials deposited or snatched;

$$\gamma' . \Delta W = \gamma' \Delta z \Delta x b = \gamma' b \frac{\partial z}{\partial t}\Delta t \Delta x$$

$$\text{Thus: } -\frac{\partial Qs}{\partial x}\Delta x \Delta t = \gamma' b \frac{\partial z}{\partial t}\Delta t \Delta x \qquad (17)$$

$$\text{Where } \frac{\partial Qs}{\partial x} + \gamma' b \frac{\partial z}{\partial t} = 0$$

γ'-Specific weight of the solid particles.

According to Eq. 15, the bed load Qs is function of flow rate (v), of the falling velocity (ϑ_0) and of the relative roughness (d\h) of the bed: Qs = $\gamma f(v)bAq$; for each real cases, the values of ϑ_0, v_0 and d\h are given; That's why, we can consider that A is constant. In this case:

$$\frac{\partial Qs}{\partial x} = \gamma A b f'(v)\frac{\partial v}{\partial x} q \qquad (18)$$

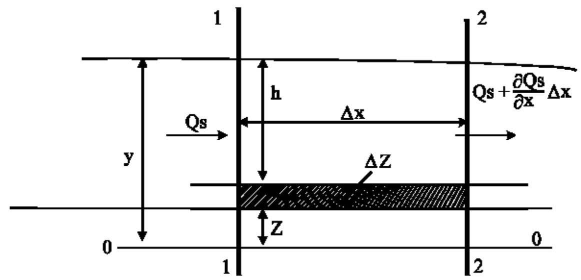

Fig. 1: Plot of calculation of the deformation of the bed

from where:

$$\gamma A f'(v)\frac{\partial v}{\partial x} q = -\gamma' \frac{\partial z}{\partial t} \qquad (19)$$

For the slowly varied flow, the form of the free face is defined either by the back water curve of raising, or by the back water curve of lowering. This is defined by the sign of $\partial v \backslash \partial x$. The value $\partial v \backslash \partial x$ also depends on the form of the bed, determined by the function b = f (x).

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\left(\frac{Q}{bh}\right) = -\frac{Q}{bh}\left(\frac{1}{b}\frac{\partial b}{\partial x} + \frac{1}{h}\frac{\partial h}{\partial x}\right)$$

In the case of a bed with a contracting, the velocity must increase $\partial v \backslash \partial x > 0$ and $\partial b \backslash \partial x < 0$.

If $f(v) = v^4$; $f'(v) = 4v^3$; $f'(v)\frac{\partial v}{\partial x} = -4v^4\left(\frac{1}{b}\frac{\partial b}{\partial x} + \frac{1}{h}\frac{\partial h}{\partial x}\right)$ and the Eq. 19 takes the form:

$$-4\gamma A v^4\left(\frac{1}{b}\frac{\partial b}{\partial x} + \frac{1}{h}\frac{\partial h}{\partial x}\right) q = -\gamma' \frac{\partial z}{\partial t} \qquad (20)$$

Since the values $\partial h \backslash \partial x$ and $\partial b \backslash \partial x$ are negative, it follows that$\partial z \backslash \partial t < 0$; Thus, there will be the erosion of the bed. This erosion is more intense than in the event of absence of contracting.

Finally the Eq. 19 can be written as:

$$\frac{\gamma'}{\gamma}\frac{\partial z}{\partial t} b = 4 A v^4\left(\frac{1}{b}\frac{\partial b}{\partial x} + \frac{1}{h}\frac{\partial h}{\partial x}\right) Q \qquad (21)$$

In the case of a bed with widening and if there is a river flow ($h>h_0$ where: h_0-depth corresponding to the uniform permanent flow), the formation of a back water curve of raising occurs, in other words velocity decreases; which means that and, therefore; consequently, there will be the deposit of the solid particles.

Let us examine the two following cases:

Case of the flow of water in a short section, when losses by friction can be neglected;

Case of the flow slowly varied in a long level, where the forces of friction play a considerable role.

In the first case, the equation of deformation of the bed can be solved in a simple way without additional conditions if the shape of the curve of the free face is given.

In the second case, it is necessary to take into account the forces of friction and the equation of deformation of the bed must be solved using Eq. 16.

We will examine the two following cases.

DEFORMATION OF A SECTION ON A SHORT DISTANCE

Larger = C te: Let us study the deformation of a short section of a channel under the conditions of a one-dimensional flow. Let us use again the equation of deformation of the bed:

$$\gamma A f'(\gamma)\frac{\partial v}{\partial x}q = -\gamma'\frac{\partial z}{\partial t} \qquad (22)$$

Let us pose $f(v) = v^4$; $f'(v) = 4v^3 = \frac{4q^3}{h^3}$ (23)

Let us consider, at first approximation, that the water level in this section is constant with respect to time, but is variable with length: y = f (x); consequently:

$$y = h + z = f(x); \frac{\partial y}{\partial t} = 0$$

In other words y = f (x,t); the variation of y in time can be neglected without any particular influence on the precision of the calculations, but the variation in space cannot be neglected. By taking into account what proceeds, one can take:

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\left(\frac{q}{h}\right) = -\frac{q}{h^2}\frac{\partial h}{\partial x}; \quad \frac{\partial z}{\partial t} = -\frac{\partial h}{\partial t} \qquad (24)$$

Upon substitution of the expressions obtained (23) and (24) into Eq. 22, we get:

$$4\frac{\gamma A}{\gamma'}\frac{q^5}{h^5}\frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} = 0; \quad \text{or}: \frac{A_1}{h^5}\frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} = 0 \qquad (25)$$

With $A_1 = \frac{4\gamma A q^5}{\gamma'}$

The solution of the Eq. 25 is equivalent to the solution of the system:

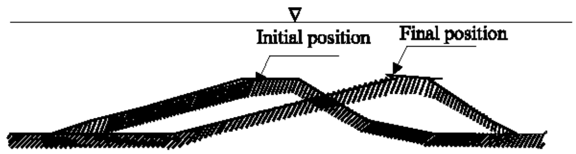

Fig. 2: Diagram of deformation of the bed

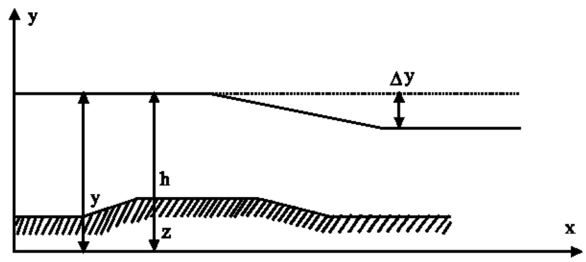

Fig. 3: Diagram of calculation

$$\frac{dx}{\frac{A_1}{h^5}} = \frac{dt}{1} = \frac{dh}{0} \qquad (26)$$

$$\text{Thus}: dx = \frac{A_1}{h^5}dt; \quad \text{Where}: \quad x - x_0 = \frac{A_1}{h^5}t \qquad (27)$$

$$h = \psi\left(\frac{A_1 t}{h^5} - x\right) \qquad (28)$$

where: **Ψ**: function defined by the initial conditions and h = y-z.

The Eq. 27 gives the translation velocity of a constant depth along the channel during the erosion of the bed. It is noted that value $x - x_0$ is proportional to t, but the scaling factor $A_1\backslash h^5$ is larger, when the depth is smaller; consequently, a bed with a soft initial profile, is transformed gradually into undulations with soft upstream slopes and stiff downstream slopes (Fig. 2).

By using the formula (27), we can easily calculate the deformation of the bed.

The problem arises as follows: the form of the initial bed is given by $z_0 = f(x)$ (Fig. 3); the level of the free face is known and is taken constant for all the period of deformation; we need to calculate the deformation of the bed. To solve this problem, we proceed as follows:

- One varies x_{0i}, each one of these values corresponds to $h_I = yi_I\text{-}z_{0i}$.
- One calculates the value $x_I = f(t)$; by the formula (27) Since x_I is a linear function of t, the results of calculation can be represented graphically. On Fig. 4, are represented lines with angular coefficients $A_1\backslash h^5$ corresponding to the given values of x_{0i} and h_I.

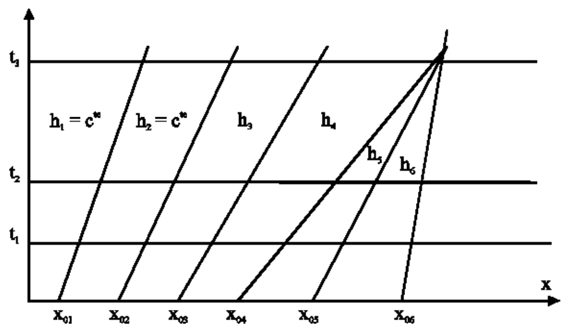

Fig. 4: Diagram of calculation x = f (t, h_1)

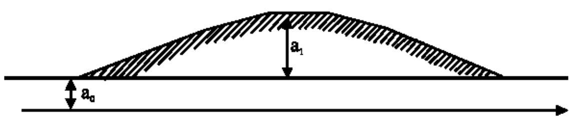

Fig. 5: Initial form of the bed of the channel

- By plotting horizontal straight lines, corresponding to times of interest t_I, one obtains at the points of intersection of these lines with lines $x_I = f(t_I)$ the values x_I and h_I, consequently, we can build the deformation of the bed of the channel at the end of each interval of time.

It is not difficult to also obtain an analytical solution for this case; to this end, it is enough to know only the function which defines the initial form of the bed. Let us take, as an example, an initial form of the bed with a cosine law (Fig. 5).

$$z_0 = a_0 + a_1 \cos\frac{2\pi x_0}{\lambda};$$

where: λ-wave length; a_1- its amplitude.

Initial depth: $h = y - z_0 = y - a_0 - a_1 \cos\frac{2\pi x_0}{\lambda};$

At time t: h = y - z; by replacing x_0 by its expression in the formula (27), we find:

$$z = a_0 + a_1 \cos\frac{2\pi}{\lambda}\left[x - \frac{A_1 t}{(y-z)^5}\right] \qquad (29)$$

So y = Cte, the Eq. (29) can be solved compared to x = f (t,z):

$$x = \frac{\lambda}{2\pi}\text{Arccos}\left(\frac{z-a_0}{a_1}\right) + \frac{A_1 t}{(y-z)^5} \qquad (30)$$

By analyzing the formula (30), we can see that the displacement of a certain point with a height z_I is directly

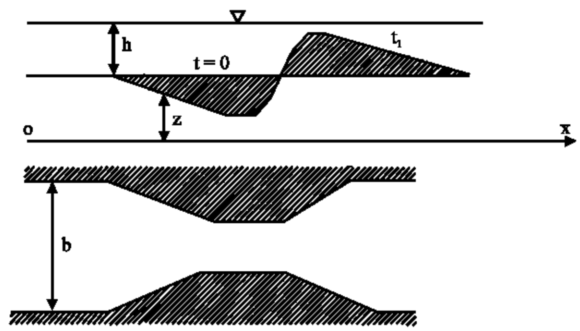

Fig. 6: Diagram of deformation of the bottom of the section with contracting

proportional to time t and inversely proportional to $(y - z_I)^5$; The larger z_I is, the more the second term of the right part of the equation is large. That means that the top of the dune moves more quickly than the remainder of its points, which shows that with time, the profile of the bed will take a form with upstream slopes softer than downstream slopes.

If we also takes into account the variation of y = f(x), it will be more convenient to use the formula (29); by considering that x_I is given, we determine t = f(z) as:

$$t = \left[x - \frac{\lambda}{2\pi}\text{Arccos}\left(\frac{z-a_0}{a_1}\right)\right]\frac{(y-z)^5}{A_1} \qquad (31)$$

where x and y - constant values and z variable.

Width of the variable bed: Let us consider now that the bed of the channel narrows then widens and then find the deformation of this section (Fig. 6); let us also suppose that the banks are stable.

To analyze the deformation of the bed, we use Eq. 17; Replacing

$$\frac{\partial z}{\partial t} = -\frac{\partial h}{\partial t} \quad \text{and} \quad Qs = \gamma A Q v^4,$$

$$\text{one finds}: 4\gamma A v^3 Q\frac{\partial v}{\partial x} = \gamma' b\frac{\partial h}{\partial t}$$

$$4\gamma A\frac{Q^4}{\omega^3}\frac{\partial}{\partial x}\left(\frac{Q}{\omega}\right) = \gamma'\frac{\partial \omega}{\partial t}; \; -4\gamma A\frac{Q^5}{\omega^5}\frac{\partial \omega}{\partial x} = \gamma'\frac{\partial \omega}{\partial t};$$

$$\frac{A_2}{\omega^5}\frac{\partial \omega}{\partial x} + \frac{\partial \omega}{\partial t} = 0 \qquad (32)$$

where $$A_2 = \frac{4\gamma A Q^5}{\gamma'}$$

$$\frac{dx}{\frac{A_2}{\omega^5}} = \frac{dt}{1} = \frac{d\omega}{0} \quad (33)$$

two equations are obtained:

$$x - \frac{A_2 t}{\omega^5} = C_1 \quad (34)$$

$$\omega = f(x, t) = C_2 \quad (35)$$

Since b = f(x), by using the Eq. 34, one finds ω = f(x, t). Consequently, we have reached the preceding solution, with only one difference; the required function is ω (wet cross section) instead of the depth h.

By applying the same process, one finds:

$$x = x_{0i} + \frac{A_2 t}{\omega_i^5} \quad (36)$$

Knowing the initial profile of the bottom z_0 = f(x) and the form of the bed in plan b = f(x) and by supposing that b with a given section is constant (case where the banks are stable), we find on the graph x = f(t) a series of straight lines; each one of them corresponds to a value ω_i, this last value is obtained by using the initial conditions expressed by x_{0i} (Fig. 7).

By plotting horizontal straight lines for the intervals of times considered t_i, with their intersection with the lines x = f (t), one obtains the distances for which the cross section will be equal ω_i at the time t.

Since the width of the bed in these sections is known, we can easily determine the depth of the water channel, i.e., the new coast of the bed. So one plots the deformed profile of the river.

The analytical solution can be obtained in the following way: At time t = 0, the value $\omega_0 = bh_0$ = f(x) is known; consequently, one can find the relation between C_1 and C_2.

$$C_1^0 = x_0 \ ; \ C_2^0 = \omega_0 = bh_0 \quad (37)$$

This relation can be expressed graphically while reporting on the x-axis $C_1^0 = x_0$ and the y-axis $C_2^0 = \omega_0$

Let us examine, as an example the particular case studied by Exner (1925):

h_0 = Cte and b = a(c + cosαx); in this case:
$C_2 = h_0 a (c + \cos\alpha C_1)$ (38)

While replacing in (38) the expressions of C_1 and C_2 in (34) and (35), one obtains:

$$\omega = ha(c + \cos\alpha x) = h_0 a\left[c + \cos\alpha\left(x - \frac{A_2 t}{\omega^5}\right)\right] \quad (39)$$

The value of h can be determined by the method of successive approximation by using the expression:

$$\left(\frac{h}{h_0} - 1\right)c + \frac{h}{h_0}\cos\alpha x = \cos\alpha\left(x - \frac{A_2 t}{b^5 h^5}\right) \quad (40)$$

In this case, one must give the values of h and x to calculate t One notes that if cosαx<0 (section with contracting), the depth increases with the increase of t; when cosαx>0, the depth decreases. Consequently, erosion is more intense on the sections of contracting, whereas on the parts of the bed with widening, there are significant deposits.

The analysis made previously is accurate for beds of low widths. In the opposite case, the examination of the deformation of the bed becomes particularly more complicated, because related velocity varies not only in length but also in width.

Numerical application: Let suppose that the initial form of the bed is given according to the law: $z_0 = a_0 + a_1 \cos\frac{2\pi x_0}{\lambda}$

with a_0 = 2 m ; a_1 =1.0 m ; λ= 40 m; x_1 = - 20 m; y = 5.95 m; A = 0.000004; γ' = 1.1 T m^{-3}; q = 6 m^3/s. The aim is to study the deformation of the bed on a section with a length equal to λ.

Solution: $A_1 = \frac{4Aq^5\gamma}{\gamma'} = \frac{4.4.10^{-6}.6^5.1}{1,1} \cong 0.11$;

The parameters of the initial form of the bed are calculated as follows

The computations are shown in Table 1.

The diagram of calculation obtained is represented on Fig. 8.

Time t is calculated by the formula:

$$t = (x - x_0)\frac{(y - z)^5}{A_1}$$

Table 1: The parameters of the initial form of the bed

x_I	z_{0i}	x_i	z_{0i}
x_1 = -20	z_{01} = 1.0	x_6 = 5	z_{06} = 2.7
x_2 = -15	z_{02} = 1.3	x_7 = 10	z_{07} = 2.0
x_3 = -10	z_{03} = 2.0	x_8 = 15	z_{08} = 1.3
x_4 = - 5	z_{04} = 2.7	x_9 = 20	z_{09} = 1.0
x_5 = 0	z_{05} = 3.0		

Table 2: The peak of the dune moves with the section x = + 5 m After 10155 sec

N° Section	x_I (m)	x_0 (m)	x-x_0 (m)	x_{xi} (m)	z_0 (m)	h = y-z_{0i} (m)	t (s)	Erosion (m)	Deposit (m)
2	-15	-20	5	1.3	1.0	4.95	135084	0.30	-
3	-10	-20	10	2.0	1.0	4.95	270168	1.00	-
3	-10	-15	5	2.0	1.3	4.65	98819	0.70	-
4	-5	-10	5	2.7	2.0	3.95	43708	0.70	-
5	0	-10	10	3.0	2.0	3.95	87416	1.00	-
5	0	-5	5	3.0	2.7	3.25	16481	0.30	-
6	+5	0	5	2.7	3.0	2.95	10155	-	0.30
7	+10	+5	5	2.0	2.7	3.25	16481	-	0.70
8	+15	+10	5	1.3	2.0	3.95	43708	-	0.70
9	+20	+15	5	1.0	1.3	4.65	98819	-	0.30

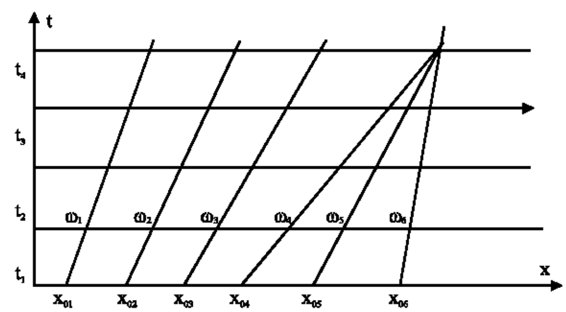

Fig. 7: Diagram of calculation of the deformation of the bed x = f(t, h_I)

It arises from the Table 2 that after 10155 seconds, the peak of the dune moves with the section x = + 5 m. Here, one notes a deposit a 0. 30 m height.

To know what occurs at the same moment at section x = 0, where is initially situated the crest, one was must solve the following system with two unknown factors z and x_0:

$$\left.\begin{array}{l} t = \dfrac{(y-z)^5}{A_1}(x - x_0) = 10155 \\ x_0 = \dfrac{\lambda}{2\pi}\arccos(\dfrac{z - a_0}{2}) \end{array}\right\} \Rightarrow \begin{array}{l} z = 2.83 \text{ m. h} = 3.\ 12 \text{ m} \\ \text{and } x_0 = -3.77 \text{ m} \end{array}$$

Therefore, as in point x = 0, we will have erosion with a depth of: 3.00 - 2.83 = 0.17 m.

To illustrate the phenomenon of deformation of the bed during the intervals of time t_1, t_2, t_3...., we will carry out the necessary calculations in Table 3.

On Fig. 8, the phenomenon of deformation of the bed at the time t_1, t_2, t_3...., is represented according to the data of Table 2.

These same results can be obtained graphically by plotting the horizontal straight lines corresponding to the times t_1, t_2, t_3.... and by using Fig. 7; the points of intersection of these lines with lines x_I = f(t) yield the distances x_I, for which within the interval t_I, the depth will be equal to h_I. From the Fig. 8 we can see

Table 3: Necessary calculations to illustrate the phenomenon of deformation of the bed

$x = x_0 + \frac{A_1}{h^5}t$					
X_0 (m)	T (s) h(m)	10.000	20.000	30.000	40.000
-20	4.95	-19.63	-19.26	-18.90	-18.52
-15	4.65	-14.49	-13.99	-13.48	-12.98
-10	3.95	-8.86	-7.71	-6.57	-5.42
-5	3.25	-1.97	1.07	4.10	7.13
0	2.95	4.92	9.85	14.77	19.69
+5	3.25	8.03	11.07	14.10	17.13
+10	3.95	11.14	12.29	14.58	14.57
+15	4.65	15.50	16.01	16.52	17.02

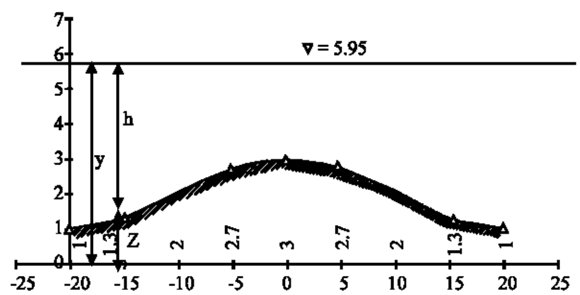

Fig. 8: Diagram of the initial bed situation

discontinuities which start to occur after 27 sec approximately.

CONCLUSION

The computing of the no stationary flow in channels with removable bottoms is a very complex problem. It remains an unexplored field for a thorough research. From the analysis of simple cases the following conclusions can be drawn:

The complexity of the problem lies particularly in the development of an analytical relation of the form of the bed of the channel, which often requires the recourse to graphical and analytical processes

The example of the studied problem requires thorough knowledge as regards to the resolution of quasi linear equations.

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