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The Response of Initially Stressed Euler-Bernoulli Beam with an Attached Mass to Uniform Partially Distributed Moving Loads

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Abstract: An investigation into the response of initially stressed Euler Bernoulli Beam with an attached mass to uniform partially distributed moving load is carried out. The resulting coupled partial differential Eq is solved using finite difference method. Graphs were presented for the results obtained. It was found that the response amplitude increases as mass of the load (M) increases under a moving force problem, and also that the response amplitude increases with an increase in the mass of the load (M) for various values of time t and ε.

Key words: Investigation, Euler Beroulli Beam, unipoem, graphs, amplitude

INTRODUCTION

The problem of determining the dynamic response of elastic structures subjected to moving loads have long been of theoretical and practical interest in the field of applied mathematics, physics and engineering .The practical importance of this area of studies is that the initially stressed beams are commonly incorporated in the design of aero planes. Advances in technology have accelerated utilization of such initially stressed structural elements. In general an aircraft is subjected to a wide range of temperature variation during flight, which may cause considerable tensile or comprehensive pre stresses in the beams when they are fixed in the plane direction. It is therefore, of technological interest to investigate to what extent the dynamic response of the beam is affected by moving loads.

The problem is well studied, including several solutions of the continuous foundation case (Priece, *et al*., 2000; Priece and Smith, 2001) corresponding to the limiting case of a discrete foundation with very small support spacing. Staddler and Shreeues (1970), Mead (1970) have considered energy propagation in aero space systems of this type, which generally are concerned with moving loads. Chang *et al*. (2001) and McGhie (1990) have previously describes and analyzed a dynamic model of a long beam on discrete elastic supports applied to the problem of vibration transmission.

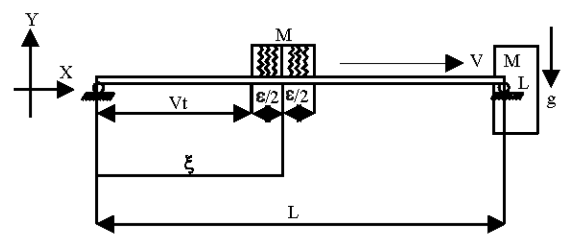

Fig. 1: The mathematical model of the problem

The approach of the study is to develop a theory for the dynamic response of a finite initially stressed Bernoulli beam which carries a lumped mass at one of its ends, to a distributed moving load. The goal is to extend the analysis of the dynamic response of a finite initially-stressed Bernoulli beam which carries a lumped mass at the end X = L but arbitrary supported at the end X = 0 to a uniformly partially distributed moving load and to pretend a very simple as well as practical analytical-numerical technique to determine the response of beam with non-classical boundary conditions.

Model development: With reference to Fig. 1, it is assumed we have a uniform simply supported initially stressed Euler-Bernoulli beam carrying a mass M. The load is assumed to start entering the beam of length L, from the left hand support at t = 0 and advancing uniform along a beam with a constant speed V .The mass is also assumed to be uniformly distributed over a fixed length ∈ of the beam.

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The governing equation of the initially stressed euler-bernoulli beam: The equation of motion that governs the response of initially stressed Euler-Bernoulli beam carrying an attached mass to uniform partially distributed load is

$$\frac{EI\partial^4 Y(x,t)}{\partial^4 x}+\frac{m\partial^2 y(x,t)}{\partial^2 t}-N\frac{\partial^2 y(x,t)}{\partial^2 x}=P(x,t) \quad (1)$$

Where E is the young modulus of elastic, I is the second moment of area for beam's cross- section, m is the mass per unit length, p(x, t) is the external load, y(x, t) is the deflection of the beam, t is the time, x is the spatial coordinate, EI is the constant flexural stiffness and N is the initially-stressed constant.

As an external load, we considered a uniform partially distributed moving load which travels with a constant velocity v is expressed as

$$P(x,t)=\frac{1}{\varepsilon}\,[-Mg-M\Delta y(x,t)]\,[H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi\frac{-\varepsilon}{2})] \quad (2)$$

Where

$$\Delta=\frac{\partial^2}{\partial^2}+\frac{2v\partial^2}{\partial x\partial t}-v^2\frac{\partial^2}{\partial x^2} \quad (3)$$

M is the mass of the load, g is the acceleration due to gravity, H (x) is the Heaviside function such that

$$H(x)\begin{cases}0 & x<0\\ 1 & x>0\end{cases} \quad (4)$$

In view of Eq. 3, 2, becomes

$$P(x,t)=\frac{1}{\varepsilon}\,[-Mg-M\frac{\partial^2}{\partial^2}y(x,t)-2Mv\frac{\partial^2}{\partial x\partial t}(x,t) + Mv^2\frac{\partial^2}{\partial x^2}y(x,t)]\,[H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi\frac{-\varepsilon}{2}] \quad (5)$$

hence Eq. 1 become

$$\frac{EI\partial^4 y(x,t)}{\partial x^4}+\frac{m\partial^2 y(x,t)}{\partial t^2}-N\frac{\partial^2 y(x,t)}{\partial x^2}= \frac{1}{\varepsilon}[-Mg-M\frac{\partial^2 y(x,t)}{\partial t^2}-2v\frac{\partial^2 y(x,t)}{\partial x\partial t}+ v^2\frac{\partial^2}{\partial x^2}y(x,t)]\,[H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi-\frac{\varepsilon}{2})] \quad (6)$$

The associated boundary conditions are

$$\frac{\partial^2}{\partial x^2}y(x,t)=0 \text{ at } x=L \quad (7)$$

$$EI\frac{\partial^3}{\partial y^3}y(x,t)-\frac{\partial^2}{\partial t^2}y(x,t)=0 \text{ at } x=L \quad (8)$$

The corresponding initial conditions are

$$y(x,0)=0,\ \frac{\partial y}{\partial x}(x,t)=0 \text{ at } t=0 \quad (9)$$

Where

M_L is the attached mass at x =L

Reduction of governing equation to a system ofordinary differential equation: The traverse displacement and external applied force may be expressed as

$$y(x,t)=\sum_{k=1}^{\infty}\phi_k(t)y_k(x) \quad (10)$$

$$P(x,t)=\sum_{k=1}^{\infty}\psi_k(t)y_k(x) \quad (11)$$

Where ϕ_k and Ψ_k are the unknown functions of time t which have to be determined, $y_k(x)$ is the known eigen function of free vibration of beam.

Substituting Eq. 10 and 11 into Eq. 6 and multiply by $y_p(x)$ gives

$$\frac{1}{\varepsilon}[-Mg\,y_p(x)-My_p(x)\sum_{k=1}^{\infty}\ddot{\phi}_k(t)y_k(x)-2MVy_p(x) \sum_{k=1}^{\infty}\ddot{\phi}_k(t)y'_k(x)-MX^2y_p(x)\sum_{k=1}^{\infty}\ddot{\phi}_k(t)y''k(x)\,[H(x -\xi+\frac{\varepsilon}{2})-H(x-\xi-\frac{\varepsilon}{2})]==y_p(x)\sum_{k=1}^{\infty}\psi_k(t)y_k(x) \quad (12)$$

Taking the definite integrals of both sides of Eq. 12 along the length of the beam with respect to x we have

$$\frac{Mg}{\varepsilon}\int_0^L y_p(x)[H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi+\frac{\varepsilon}{2})]dx- \frac{M}{\varepsilon}\sum_{k=1}^{\infty}\ddot{\phi}_k(t)\int_0^L y_k(x)[H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi+\frac{\varepsilon}{2})] dx\frac{-2MV}{\varepsilon}\sum_{k=1}^{\infty}\phi_k\int_0^L y_{k'}(x)y_p(x)[H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi-\frac{\varepsilon}{2})] dx-\frac{M}{\varepsilon}V^2\sum_{k=1}^{\infty}\phi_k(t)\ 10(x)y''_k(x)y_p(x)[H(x-\xi+\frac{\varepsilon}{2}) -H(x-\xi+\frac{\varepsilon}{2})]dx\ =\sum_{k=1}^{\infty}\psi_k\int_0^L y_k(x)y_p(x)dx \quad (13)$$

evaluating the first definite integrals in Eq. 13 by carrying out integration by parts with respect to x and using the following properties of singularity function (Clough and Penzien, 1995).

$$\int_{x0}^{x2} yk(x)\delta\,(x\text{-}x_1)dx = y_k(x_1) \qquad (14)$$

Provided $x_0<x_1<x_2$

$$\frac{d}{dx} H\,(x\text{-}x1) = \delta\,(x\text{-}x1) \qquad (15)$$

Similar arguments to second, third to fifth definite integral in Eq. 13, hence evaluating the integrals using Taylor's series expansion and applying orthogonality properties of the characteristics function $y_k(x)$ to the right hand side of (13), we finally obtain

$$\psi_k(t) = -Mg[y_k(\xi) + \frac{\in^2}{24}\; y''(\xi)] - M\sum_{k=1}^{\infty}\ddot{\phi}_k(t) [y_k(\xi)y_p(\xi) + \frac{\in^2}{24}(y_k(\xi)y_p(\xi)''] - 2MV\sum_{k=1}^{\infty}\dot{\phi}_k(t) [y'_k(\xi)y_p(\xi) + \frac{\in^2}{24}(y'_k(\xi)y_p(\xi)''] - MV^2\sum_{k=1}^{\infty}\phi_k(t) [y''_k(\xi)y_p(\xi) + \frac{\in^2}{24}(y''_k(\xi)y_p(\xi)''] \qquad (16)$$

Note that in view of (10) and (11) Eq. 1 may be written as

$$EI\sum_{k=1}^{\infty}\phi_k(t)y^{iv}{}_k(x) + m\sum_{k=1}^{\infty}\ddot{\phi}_k(t)y_k(x) - N\sum_{k=1}^{\infty}\dot{\phi}_k(t)y''_k(x) = \sum_{k=1}^{\infty}\psi_k(t)y_k(x) \qquad (17)$$

The equation of free-vibration of the beam is satisfied by $y_k(x)$for any arbitrary k and is

$$Y^{iv}{}_k(x) = B^4{}_k\, y_k(x) = \frac{m\lambda^2}{EI} kYk(x) \qquad (18)$$

Substituting Eq. 18 into Eq. 17, we obtained

$$m\sum_{k=1}^{\infty}\lambda^2\phi_k(t)y_k(x) + m\sum_{k=1}^{\infty}\ddot{\phi}_k(t)y_k(x) - N\sum_{k=1}^{\infty}\dot{\phi}_k(t)y_k(x)\sum_{k=1}^{\infty}\psi_k(t)y_k(x) \qquad (19)$$

Substituting Eq. 16 into Eq. 19 we finally obtain

$$m\sum_{k=1}^{\infty}\ddot{\phi}_k(t)y_k(x) + m\sum_{k=1}^{\infty}\lambda^2{}_k\;\phi_k(t)y_k(x) - N \sum_{k=1}^{\infty}\phi_k(t)y''_k(x) = \sum_{k=1}^{\infty}y_k(x)[-Mg[y_k(\xi) + \frac{\varepsilon^2}{24}y''_k(\xi)] - M\sum_{k=1}^{\infty}\ddot{\phi}_k(t)[y_k(\xi)y_p(\xi) + \frac{\varepsilon^2}{24} (y_k(\xi)y_p(\xi)''] - 2MV\sum_{k=1}^{\infty}\ddot{\phi}_k(t)[y_k(\xi)y_p(\xi) + \frac{\varepsilon^2}{24} (y_k(\xi)y_p(\xi)''] - MV^2\sum_{k=1}^{\infty}\phi_k(t)[y''_k(\xi)y_p (\xi) + \frac{\varepsilon^2}{24}(Y''_k(\xi)yp(\xi)'']] \qquad (20)$$

Equation 20 is the desired set of generalized coupled differential equation which holds for any particular set of boundary conditions of the beam.

Dynamic response of simply supported beam: For the present configuration, we made use of the Eigen functions.

$$yk(x) = \sin\frac{a_k}{L}\,x + B_k\,\sinh\frac{a_k}{L}\,x \quad k = 1,2,3,....n \qquad (21)$$

Where $B_k \frac{\sin a_k}{\sinh a_k}$ = and a_k is determine from the trancedental equation
$\sinh a_k$

$$\text{Cos } a_k\ \sinh a_k\ \text{-}\sin a_k\ \cosh a_k\ \frac{2\lambda^2{}_k\sinh a_k}{EIak_k{}^3} = 0 \qquad (22)$$

We obtain the set of exact governing differential equation for the vibration of the beam by employing Eq. 21 and evaluating the values of the integral in Eq. 13 and finally obtain

$$m\sum_{k=1}^{\infty}\ddot{\phi}_k(t)y_k(x) + m\sum_{k=1}^{\infty}\lambda^2{}_k\,\phi_k(t)y_k(x) - N \sum_{k=1}^{\infty}\phi_k(t)y''_k(x) = \sum_{k=1}^{\infty}y_k(x)\left\{\frac{-Mg}{\varepsilon a_p}[\frac{\sin a_k(\xi)}{L}\frac{\sin a_p\varepsilon + B^2{}_k}{2L} \frac{\sinh a_k\xi}{L}\frac{\sinh a_p{}^{\varepsilon}}{2L}] - M\sum_{k=1}^{\infty}\ddot{\phi}_k(t)\cos(\frac{a_k - a_p}{L})\xi \sin(\frac{a_k - a_p}{2L})\varepsilon + \frac{L}{a_k + a_p}[\cos(\frac{a_k + a_p}{L})\xi\sin (\frac{a_k + a_p}{2L})\varepsilon + \frac{MB_k}{\varepsilon(a_k^2 + a_p^2)}[a_k\ \cos\left(\frac{1 - a_k}{L}\right)\xi\sin\left(\frac{1 - a_k}{2L}\right)(-\varepsilon) + \right.$$

$$\begin{bmatrix} \cos\left(\frac{1+a_k}{L}\right)\xi\sin\left(\frac{1+a_K}{2L}\right)(-\varepsilon)+a_p\cos\left(\frac{a_k-a_p}{L}\right) \\ \xi\sin\left(\frac{a_k-a_p}{2L}\right)\varepsilon+\cos\left(\frac{a_k+a_p}{L}\right)\xi\sin\left(\frac{a_k+a_p}{2L}\right) \end{bmatrix}$$

$$+MB_k^2L\left[\cosh\left(\frac{a_k+a_p}{L}\right)\xi\sinh\left(\frac{a_k+a_p}{2L}\right)\varepsilon\right]$$

$$-\frac{MB_k^2L}{\varepsilon(a_k-a_p)}\left[\cosh\left(\frac{a_k-a_p}{L}\right)\xi\sinh\left(\frac{a_k-a_p}{2L}\right)\varepsilon\right]$$

$$\frac{-2MVa_k}{L}\sum_{k=1}^{\infty}\dot{\phi}_k(t)\begin{Bmatrix} \frac{L}{a_k+a_p}\left[\sin\xi\left(\frac{a_k+a_p}{L}\right)\cos\left(\frac{a_k+a_p}{2L}\right)\varepsilon\right]+ \\ \frac{L}{a_k-a_p}\left[\sin\xi\left(\frac{a_k-a_p}{L}\right)\cos\left(\frac{a_k-a_p}{2L}\right)\varepsilon\right] \end{Bmatrix}$$

$$\frac{+B_kL}{\varepsilon(a_k^2-a_p^2)}\left[a_k\begin{pmatrix} \cos\left(\frac{1+a_k}{L}\right)\xi\sin\left(\frac{1+a_k}{2L}\right) \\ +\cos\left(\frac{1-a_k}{L}\right)\xi\sin\left(\frac{1-a_k}{2L}\right)\varepsilon \end{pmatrix}+\right]$$

$$a_p\begin{pmatrix} \cos\left(\frac{1-a_k}{L}\right)\xi\sin\left(\frac{1-a_k}{2L}\right) \\ \varepsilon+\cos\left(\frac{1+a_k}{L}\right)\xi\sin\left(\frac{1+a_k}{2L}\right)\varepsilon \end{pmatrix}+$$

$$\frac{B_KL}{(a_K+a_p)}\left[\sinh\xi\left(\frac{a_k+a_p}{L}\right)\cosh\left(\frac{a_k+a_p}{2L}\right)\varepsilon\right]$$

$$-\frac{B_kL}{(a_k-a_p)}\left[\sinh\xi\left(\frac{a_k-a_p}{L}\right)\cosh\left(\frac{a_k-a_p}{2L}\right)\varepsilon\right]$$

$$-\frac{MV^2}{L^2}a_k^2\sum_{k=1}^{\infty}\phi_k(t)\{\frac{L}{(a_k+a_p)}$$

$$\left[\cos\frac{\xi}{L}(a_k+a_p)\sin(a_k+a_p)\frac{\varepsilon}{2L}\right]+\frac{L}{(a_k-a_p)}$$

$$\left[\cos\xi\left(\frac{a_k-a_p}{L}\right)\sin\left(\frac{a_k-a_p}{2L}\right)\varepsilon\right]$$

$$-\frac{B_kL}{\varepsilon(a_k^2+a_p^2)}\begin{bmatrix} a_p\cos\left(\frac{1+a_k}{L}\right)\xi\sin\left(\frac{1+a_k}{2L}\right) \\ \varepsilon+a_k\cos\left(\frac{1+a_k}{L}\right)\xi\sin\left(\frac{1+a_k}{2L}\right)\varepsilon \end{bmatrix}$$

$$+a_p\cos\left(\frac{1-a_k}{L}\right)\xi\sin\left(\frac{1-a_k}{2L}\right)\varepsilon+\cos\left(\frac{1-a_k}{L}\right)\xi\sin\left(\frac{1-a_k}{2L}\right)\varepsilon$$

$$+\frac{B_k^2L}{a_k+a_p}\left[\cosh\xi\left(\frac{a_k+a_p}{L}\right)\xi\sin\left(\frac{a_k+a_p}{2L}\right)\varepsilon\right]$$
$$+\frac{B_k^2L}{(a_k-a_p}\left[\cosh\xi\left(\frac{a_k-a_p}{L}\right)\sinh\left(\frac{a_k-a_p}{2L}\right)\varepsilon\right] \quad (23)$$

Remark: We remark here that two cases were a solved for in Eq. 23.

Case (I): The moving force initially stressed Euler Bernoulli beam Problem. By moving force Problem, we mean the case in which only the first force effects are taken into consideration; by so doing, only the first term on the right hand side of Eq. 23 is retain, neglecting all the other terms.

Case (II): The moving mass initially stressed Euler Bernoulli beam problem. By moving mass initially stressed Euler Bernoulli; we mean the situation in which both the inertia effect as well as force effect are taken into consideration. For the system under consideration, the entire Eq. 23 is the moving mass problem

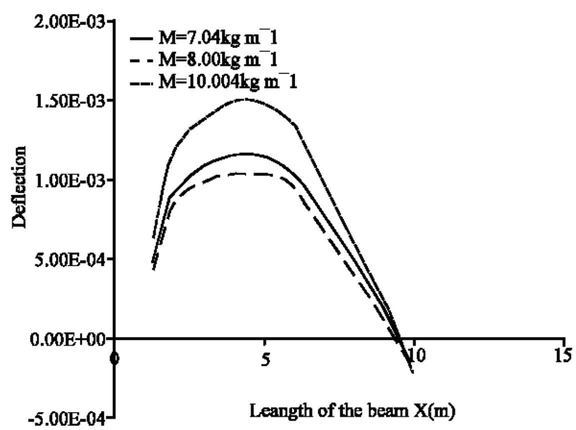

Fig. 2: The variation of the lateral deflection of the simply supported initially stressed Euler- Bernoulli Beam carring a mass at its end x=L and traversed b moving force, for t=0.5s, E=0.1m and different values of M

Table 1: The variation of the lateral deflection Y_F (x, t) of the simply supported initially stressed Euler Beam for t = 0.5s, ε = 0.1m at different M

Length of the Beam X(m)	W_{ML} (x,t) for M_L=7.04kg m^{-1}	W_{ML} (x,t) for M_L=8.0kg m^{-1}	W_{ML} (x,t) for M_L=10kg m^{-1}
1.4644	4.77E-04	5.42E-04	6.77E-04
2.2788	9.37E-04	1.00E-03	1.27E-03
4.2481	1.06E-03	1.21E-03	1.54E-03
5.8575	1.06E-03	1.14E-03	1.45E-03
7.1215	7.37E-04	8.44E-04	1.07E-03
8.5353	2.87E-04	3.29E-04	4.17E-04
9.9501	-2.66E-04	-3.05E-04	-3.87E-04

RESULTS AND DISCUSSION

To solve the two cases discussed above, in Eq. 23. We made use of approximate Central difference formula for the derivatives in Eq. 23. The resulting equation were solved by MATLAB package for the following data:

M = 7.04 kg m^{-1}, 8.0 kg M^{-1} and 10 kg m^{-1}; m = 70, E = 2.07×10^{11}N M^{-2}, I = 1.04×10^{-6}m^4, V = 12k m h^{-1}, g = 9.8 m s^{-2}, L = 10 m, ε = 0.1 m and 1.0 m, t = 0.5s, 1.0s and 1.5s, h = 0.01, N = 0.5.

Hence we have the following graphs: Figure 2 shows the variation of the latter deflection $y_f(x,t)$ of the simply supported initially stressed Euler Bernoulli Beam carrying a lumped mass at its end x = L and traversed by a moving force, for t = 0.5s, ε = 0.1 m and different values of M shown in 1. (In this graph, the deflection y_f (x, t) indicating the moving force is plotted against various values of (x). It is noted here that the response amplitudes increases as M increases shown in Table 1. Figure 3 depicts the response curve of the system for a moving force when t increases

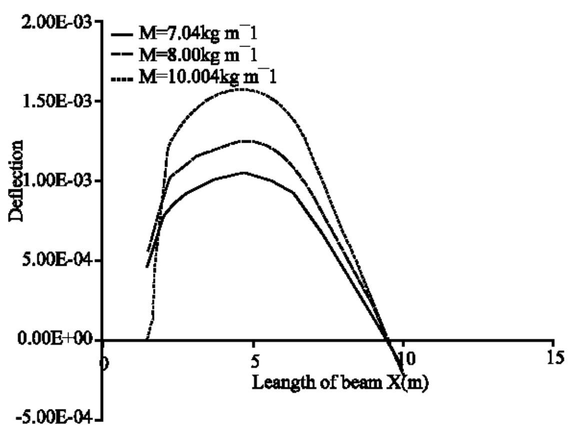

Fig. 3: Te responce curve of the system for amoving force of initially srressed simply supported Euler-Bernoulli Beam carrying a lumped mass at its end x=L for t=1.0s, E=0.1m and Different values of M

Table 2: The response of curve of the system for a moving force when t = 1.0s, ε = 0.1m at different values of M

Length of the Beam X(m)	W_{ML} (x,t) for M_L=7.04kg m^{-1}	W_{ML} (x,t) for M_L=8.0kg m^{-1}	W_{ML} (x,t) for M_L=0kg m^{-1}
1.4644	4.63E-04	5.51E-04	6.88E-08
2.2788	8.51E-04	1.02E-03	1.28E-03
4.2481	1.03E-03	1.23E-03	1.56E-03
5.8575	9.71E-04	1.16E-03	1.48E-03
7.1215	7.17E-04	8.49E-04	1.09E-03
8.5353	2.79E-04	3.52E-04	4.24E-04
9.9501	-2.59E-04	-3.11E-04	-3.93E-04

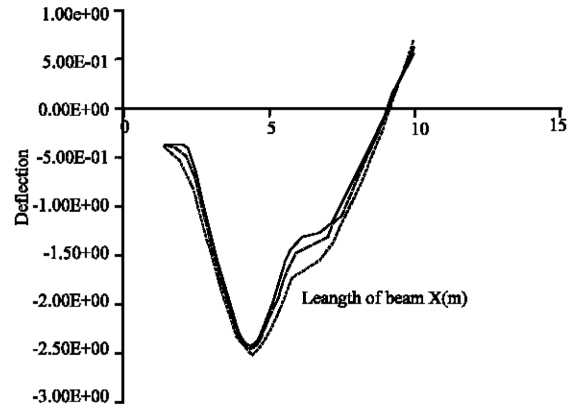

Fig. 4: The variation of the lateral deflection YM (x, t) of the simply supported initially stressed Euler-Bernoulli Beam carrying a lumped mass at its end x = L and traversed b moving mass, for t = 0.5s, E = 0.1 m and different values of M

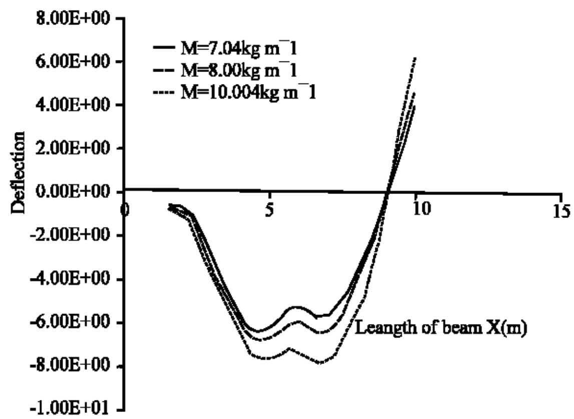

Fig. 5: The responce curve of the system for a moving mass of initilly stressed simply supported Euler-Bernoulli Beam carrying a lumped mass at its end x=L. For t=1.0s, E=0.1m and different values of M

Table 3: The variation of the lateral deflection Y_M (x, t) of the simply supported moving mass of initial stressed Bernoulli Beam for t = 0.5s, ε = 0.1m

Length of the Beam X(m)	W_{ML} (x,t) for M_L = 7.04kg m^{-1}	W_{ML} (x,t) for M_L = 8.0kg m^{-1}	W_{ML} (x,t) for M_L=10kg m^{-1}
1.4644	-3.89E-01	-3.77E-01	-3.87E-01
2.2788	-4.17E-01	-5.20E-01	-6.88E-01
4.2481	-2.41E+00	-2.42E+00	-2.47E+00
5.8575	-1.42E+00	-1.54E+00	-1.75E+00
7.1215	-1.18E+00	-1.25E+00	-1.42E+00
8.5353	-3.64E-01	-4.17E-01	-4.99E-01
9.9501	5.82E-01	6.24E-01	6.92E-01

Table 4: The response of curve of the system for a moving mass when $t = 1.0s$, $\varepsilon = 0.1m$ and different values of M

Length of the Beam X(m)	W_{ML} (x,t) for M_L=7.04kg m^{-1}	W_{ML} (x,t) for M_L=8.0kg m^{-1}	W_{ML} (x,t) for M_L=10kg^{-1}m
1.4644	-6.82E-01	-6.76E-01	-6.76E-01
2.2788	-1.09E+00	-1.26E+00	-1.58E+00
4.2481	-6.16E+00	-6.52E+00	-7.18E+00
5.8575	-5.30E+00	-5.97E+00	-7.23E+00
7.1215	-5.52E+00	-6.23E+00	-7.65E+00
8.5353	-2.05E+00	-2.47E+00	-3.33E+00
9.9501	3.95E+00	4.64E+00	6.08E+00

to 1.0s and $\varepsilon = 1.0m$ with different values of M shown in Table 2. we observe that as t increases, the response amplitudes also increases with increase in M.

Furthermore, Fig. 4 shows the variation of the deflection y_m (x, t) i.e. the moving mass problem of initially stressed simply supported Euler Bernoulli Beam carrying a lumped mass at its end x = L and traversed by a moving mass. Shown in Table 3. For t = 0.5s, $\varepsilon = 0.1m$ for various values of M, it was observed that as t increases we also have the amplitude deflection increasing as M increases.

In addition, Fig. 5, shows the variation of the deflection y_m (x, y) of the initially stressed simply supported Euler-Bernoulli beam carrying a lumped mass at x = L and traversed by a moving mass for $\epsilon = 0.1$ m, t = 1.0s for various values of m. Shown in Table 4 It was also observed that the amplitude deflection increases as M increases.

CONCLUSION

We have investigated the response of initially stressed Euler Bernoulli Beam with an attached mass to uniform partially distributed load. We have modeled the problem mathematically in such a way that the mass of the moving load is more compared to with the mass of the beam.

The interesting conclusions of the problem are as follows:

- The response amplitude increases as mass of the load M increases under a moving force problem
- The response amplitude was found to increase with an increase in mass of the load M for various values of time and ε.

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