

Optical Flow Computation in Colour Images Sequence

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Abstract: Motion computation is an important and challenging problem in the analysis of image sequences. Motion computation plays an important role in many applications such as target tracking and movement/change detection in surveillance systems; in compressing video images, if we have already compressed $I(t-1)$, we know much about $I(t)$. Typical approach for building a predicted image for $I(t)$, based on $I(t-1)$... etc. There are numerous other applications. From a sequence of images we can only estimate an approximation of the image motion field called optical flow. Motion estimation and computation in images sequence is a difficult and computationally expensive task. The computation of optical flow is an ill-posed problem, which expresses itself as the aperture problem. However, motion vectors can be estimated by using differential methods, where optic flow estimation is based on computing spatial and temporal image derivatives. A typical way to overcome the ill-posedness problems of differential optic flow methods consists of the use of smoothing techniques and smoothness assumption as a regularization methods, in which additional constraints functions are introduced. In this research we propose to improve optical flow estimation by including colour information as constraints functions in the optimization process. The proposed technique has shown encouraging results.

Key words: Optical flow, motion estimation, colour information

INTRODUCTION

The recent developments in computer vision, moving from static images analysis to video sequences, have focused the research on the understanding of motion analysis and representation. A fundamental problem in processing sequences is the computation of optical flow. This flow is a 2D vector field resulting from a perspective projection on the image plane of the 3D velocity field of a moving scene. Optical flow is a convenient and useful way for image motion representation and 3D interpretation. It often plays a key role in varieties of motion estimation techniques and has been used in many computer vision applications. Optical flow may be used to perform motion detection, autonomous navigation (knowledge of local motion of the environment relative to the observer system simplifies the calculation time-to-collision and focus of-expansion for example), scene segmentation (segmenting scene into moving and static objects), surveillance system (motion can be an important source for a surveillance system when objects of interest can be detected and tracked using the optical flow vector to define the future trajectories), motion compensation for encoding sequences and stereo disparity measurement

(Baron *et al.*, 1994; Beauchemin and Barro, 1995; Weickert and Schnow, 2001). Thus an optical flow algorithm is specified by three elements (Barron *et al.*, 1994):

- The spatiotemporal operators that are applied to the image sequence to extract features and improve the signal to noise ratio.
- How optical flow estimates are produced from a gradient search of the extracted feature space.
- The form of regularization applied to the flow field considering confidence measures if they exists.

Optical flow estimation and computation methods can be classified into three main categories: Differential approaches, block-matching approaches and frequential approaches (Baron *et al.*, 1994).

Despite more than two decades of research, the proposed methods for optical flow estimation are relatively inaccurate and non-robust. Many methods for the estimation of optical flow have been proposed (Horn and Shunck, 1981; Lucas and Kanade, 1981; Markanday and Flinchbaugh, 1990; Fleet and Jepson, 1994, 1995; Weber and Malik, 1995; Polina and Golland, 1995; Tsai *et al.*, 1999; Ming *et al.*, 2002; Zhang and Lu,

2000; Bruno and Pellerin, 2000; Barron and Klette, 2002; Arredondo *et al.*, 2004; Joachim Weickert *et al.*, 2003; Thomax *et al.*, 2004; Audre *et al.*, 2005; Volker Willert *et al.*, 2005).

We present in this study a differential approach using colour components as constraints functions for the optical flow computation. Differential methods belong to the widely used techniques for optic flow computation in images sequences. The rest of this study is organized as follows:

OPTICAL FLOW CONSTRAINT EQUATION

Optical flow is the apparent motion of brightness patterns in the images sequence. It corresponds to the motion field, but not always. For a rotating barber's pole example, the motion field and optical flow are different. The error is small at point with high spatial gradient under some simplifying assumptions. This is illustrated in Fig. 1.

In general, such cases are unusual, and for this lecture we will assume that optical flow corresponds to the motion field.

Optical flow techniques are based on the idea that for most points in the image, neighbouring points have approximately the same brightness. In other words, the world is made up of continuous objects over which brightness varies smoothly. So optical flow can be computed from a sequence by making assumptions about the variations of the scene brightness. One such assumption (Horn and Schunck, 1981) known as the brightness constancy assumption, is represented by the following Equation:

$$I(x, y, t) = I(x + \delta x, y + \delta y, t + \delta t) \quad (1)$$

Where $I(x, y, t)$ represents the luminance function at pixel (x, y) at time t and $(\delta x, \delta y)$ is the displacement occurring at pixel (x, y) during δt .

We perform a Taylor development limited to the first order and we get:

$$I(x, y, t) = I(x, y, t) + \frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t \quad (2)$$

Cancelling $I(x, y, t)$ on both sides and dividing by δt ($\delta t \rightarrow 0$) we obtain:

$$I_x u + I_y v + I_t = 0 \quad (3)$$

Where:

I_x , I_y and I_t are first partial derivatives of I , respectively with respect to x , y and t and u and v are the optical flow

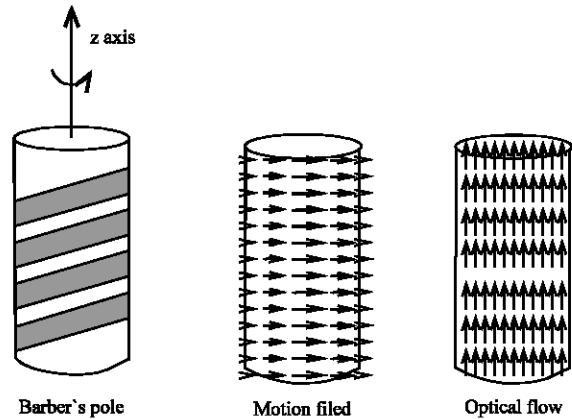


Fig. 1: The motion field and optical flow of a barber's pole

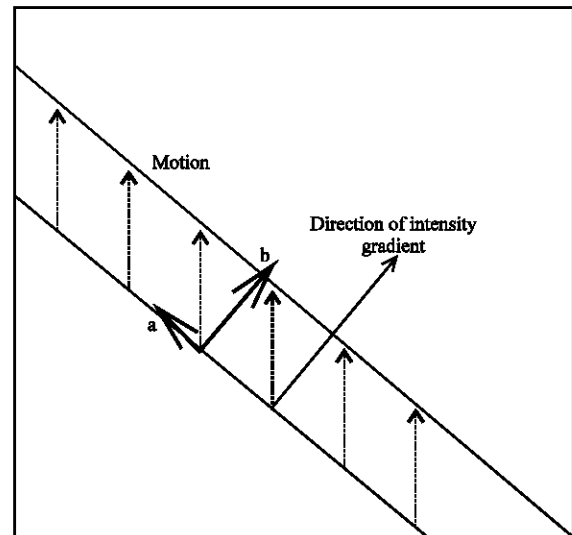


Fig. 2: The aperture problem

components, respectively in the x and y directions. Eq. 3 is called optical flow constraint equation. It provides only the normal velocity component. So we are only able to measure the component of optical flow that is in the direction of the intensity gradient. We are unable to measure the component tangential to the intensity gradient (aperture problem). This problem is illustrated in Fig. 2.

The system is undetermined because we only have one equation for two unknowns. To overcome this problem, it is necessary to add additional constraints. Another problem is that are assuming that δt is very small. Since most video cameras generate images every thirtieth of a second or so, this limit is usually not approached in practice. This means that the dismissal of the higher order terms of the Taylor expansion is only a reasonable

assumption for scenes with slow motion. The sampling error in the spatial domain also leads to errors in the computation of the observables I_x, I_y .

USE OF COLOUR INFORMATION AS ADDITIONAL CONSTRAINT

The brightness assumption implies that the (R, G, B) components of each image remain unchanged during the motion undergone within a small temporal neighbourhood (Weber and Malik, 1995). Therefore, R, G and B images can be used in a similar way as the luminance function: they have to satisfy the optical flow constraint equation. Markandey and Flinchbaugh (1990) have proposed a multispectral approach for optical flow computation. Their two-sensors proposal is based on solving a system of two linear equations having both optical flow components as unknowns. The equations are deduced from the standard optical flow constraint (3). In their experiments, they use colour TV camera data and a combination of infrared and visible images. Finally, they use two channels to resolve the ill-posed problem (Barron *et al.*, 1994).

Golland and Bruckstein (1995) follow the same algebraic method. They compare a straightforward 3-channels approach using RGB data with two 2-channel methods, the first based on normalized RGB values and the second based on a special hue-saturation definition. The standard optical flow constraint may be applied to each one of the RGB quantities, providing an over determined system of linear equations (Barron *et al.*, 1994):

$$\begin{cases} R_x u + R_y v + R_t = 0 \\ G_x u + G_y v + G_t = 0 \\ B_x u + B_y v + B_t = 0 \end{cases} \quad (4)$$

Then the pseudo-inverse computation gives the following solution for the system:

$$V = (A^T A)^{-1} A^T b \quad (5)$$

Where:

$$A = \begin{bmatrix} R_x & R_y \\ G_x & G_y \\ B_x & B_y \end{bmatrix}, \quad b = \begin{bmatrix} -R_t \\ -G_t \\ -B_t \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} u \\ v \end{bmatrix} \quad (6)$$

This assumes that the matrix $(A^T A)$ has rank 2, i.e., it is non-singular. By definition this matrix is singular if its rank is equal to 1, i.e., its columns or lines are linearly

dependent, which means that the first order spatial derivatives of the colour components (R, G, B) are dependent. Since the sensitivity functions $D_r(\lambda)$, $D_g(\lambda)$ and $D_b(\lambda)$ of the light detectors are linearly independent, the first derivatives of the R, G, B functions will also be independent for images sequence with colour changing in two different directions. But if the colour is a uniform distribution, the (R, G, B) functions are linearly dependent or if the colours of the considered region change in one direction only, the gradient vectors of (R, G, B) are parallel so that the spatial derivatives are dependent and the matrix $(A^T A)$ is singular. In addition to the estimates of the image flow components at a certain pixel of the image, we would like to get some measure of confidence in the result at this pixel, which would tell us to what extent we could trust our estimates. It is common to use the so-called condition number of the coefficient matrix of a system $(A^T A)$ as a measure of confidence of this system (Polina Golland, 1995).

To improve this problem, the idea is the use of two independent functions for colour characterization so that their gradient directions are not parallel.

If the quantities used here are denoted f and ff . The colour conservation assumption implies:

$$\begin{cases} f_x u + f_y v + f_t = 0 \\ ff_x u + ff_y v + ff_t = 0 \end{cases} \quad (7)$$

Here the solution is given by simple matrix inversion:

$$V = A^{-1} \cdot b \quad (8)$$

The ideal case is obtained when the gradient directions of the two chosen functions are normal. One possible solution is the use of two different colour systems: the normalized RGB system, denoted rgb system and the HSV system (Barron and Klette, 2002).

The rgb system is computed in the following way:

$$\begin{cases} r = \frac{R}{R + G + B} \\ g = \frac{G}{R + G + B} \\ b = \frac{B}{R + G + B} \end{cases} \quad \text{where: } r + g + b = 1 \quad (9)$$

It is clear that any pair of (r, g, b) forms a system of two independent functions. If we are taking the r and g components, the optical flow computation system to be solved is given by Eq. 8, where:

$$A = \begin{bmatrix} r_x & r_y \\ g_x & g_y \end{bmatrix}, \quad b = \begin{bmatrix} -r_t \\ -g_t \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} u \\ v \end{bmatrix} \quad (10)$$

Now we consider the HSV system. This representation uses three other values to define colour (Hue, Saturation and Value). While value is an intensity measure and corresponds to non-chromatic light characteristics, hue and saturation are chromaticity parameters, encoding the colour information. Saturation is a measure of pure colour in a certain spectrum (ratio between pure colour white light) and hue encodes the colour of wavelength information. Similar to HSV, the YUV model decomposes the colour as a brightness Y and a colour coordinate system (U, V). The different between the two is the description of the colour plan. H and S describe a vector in polar form, representing the angular and magnitude components, respectively. Y, U and V, however, form an orthogonal euclidean space (Robert and Brian, 2003).

For HSV space, V is an intensity measure and corresponds to non chromatic light characteristics, H and S are chromaticity parameters, encoding the colour information. S is a measure of pure colour in a certain spectrum (ratio between pure colour and white light) and H encodes the colour of wavelength information. The HSV system is computed in the following way:

$$\begin{aligned} V &= \text{Max}(R, G, B) \\ S &= \frac{\text{Max}(R, G, B) - \text{Min}(R, G, B)}{\text{Max}(R, G, B)} \\ H &= \begin{cases} \frac{G - B}{\text{Max}(R, G, B) - \text{Min}(R, G, B)} & \text{If } R = \text{Max}(R, G, B) \\ 2 + \frac{B - R}{\text{Max}(R, G, B) - \text{Min}(R, G, B)} & \text{If } G = \text{Max}(R < G < B) \\ 4 + \frac{R - G}{\text{Max}(R < G < B) - \text{Min}(R < G < B)} & \text{If } B = \text{Max}(R, G, B) \end{cases} \end{aligned} \quad (11)$$

The solution is given by Eq. 8, where:

$$A = \begin{bmatrix} h_x & h_y \\ s_x & s_y \end{bmatrix}, \quad b = \begin{bmatrix} -h_t \\ -s_t \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} u \\ v \end{bmatrix} \quad (12)$$

MATERIALS AND METHODS

It was shown that a colour sequence could be straightforwardly considered as a set of three different sequences produced by three types of light sensors with different sensitivity functions in response to the same input sequence (Markany and Flinchbaugh, 1990; Polina

Golland, 1995). So we propose to use the same formulation as those proposed by Horn and Schunck for the luminance function and to apply it to the three colour components.

In the first stage we have to minimize a function error containing the three colour components for the considered colour space, each component satisfying the optical flow constraint equation without any smoothness term, for the RGB space we have:

$$\text{Min}_{u,v} \begin{cases} F = (R_x \cdot u + R_y \cdot v + R_t)^2 + (G_x \cdot u + G_y \cdot v + G_t)^2 \\ \quad + (B_x \cdot u + B_y \cdot v + B_t)^2 \\ = \epsilon_R^2 + \epsilon_G^2 + \epsilon_B^2 \end{cases} \quad (13)$$

The problem will be posed as finding (u, v) optical flow components minimising F. The solution was given by using Eq. 8, Where:

$$\begin{aligned} A &= \begin{bmatrix} R_x^2 + G_x^2 + B_x^2 & R_x R_y + G_x G_y + B_x B_y \\ R_x R_y + G_x G_y + B_x B_y & R_y^2 + G_y^2 + B_y^2 \end{bmatrix}; \\ b &= - \begin{bmatrix} R_x R_t + G_x G_t + B_x B_t \\ R_y R_t + G_y G_t + B_y B_t \end{bmatrix} \end{aligned} \quad (14)$$

The matrix A must be non-singular. The smallest eigenvalue of $A^T A$ or the condition number of $A^T A$ can be used to measure numerical stability, i.e., if the smallest eigenvalue is below a threshold or the condition number is above a threshold, then we set the optical flow vector to be undefined at this image location. So, in the second stage we add a local (on a small region around each pixel) smoothness term on the magnitude of optical flow vector with a weight α . The motion of any object between two following times (t_0 and $t_0 + \partial t$ where $\partial t \rightarrow 0$) is supposed to be very small and it can be used as a small displacement in any direction. So Eq. 9 with the smoothness term will be:

$$\text{Min}_{u,v} \begin{cases} F = (R_x \cdot u + R_y \cdot v + R_t)^2 + (G_x \cdot u + G_y \cdot v + G_t)^2 \\ \quad + (B_x \cdot u + B_y \cdot v + B_t)^2 + \frac{1}{2} \alpha^2 \|V\|^2 \\ = \epsilon_R^2 + \epsilon_G^2 + \epsilon_B^2 + \epsilon_s^2 \end{cases} \quad (15)$$

Deriving F over u and v and solving the result system. The same solution is found when adding the smoothness term in the function F to minimize. Deriving This solution is obtained by Eq. 8, where:

$$A = \begin{bmatrix} R_x^2 + G_x^2 + B_x^2 + \alpha^2 & R_x R_y + G_x G_y + B_x B_y \\ R_x R_y + G_x G_y + B_x B_y & R_y^2 + G_y^2 + B_y^2 + \alpha^2 \end{bmatrix};$$

$$b = - \begin{bmatrix} R_x R_t + G_x G_t + B_x B_t \\ R_y R_t + G_y G_t + B_y B_t \end{bmatrix} \quad (16)$$

We do not use iterative method to compute the optical flow components here and the proposed method is only based on the function optimisation and matrix inversion.

RESULTS AND DISCUSSION

This study examines the quantitative performance and the implementation of the studied and proposed method.

Error measurement: In order to quantify the accuracy of the estimated range flow, the following errors measures are used. Let the correct range flow be denoted as V_c and the estimated flow as V_e . The first error measure describes the relative error in the velocity magnitude (Barron *et al.*, 1994; Barron and Klette, 2002):

$$Er = \frac{\|V_c - V_e\|}{\|V_c\|} \cdot 100\% \quad (17)$$

Er measures only the difference between the estimated and the correct velocity magnitude. So we use the directional error as a second error measure:

$$Ed = \arccos \left(\frac{V_c \cdot V_e}{\|V_c\| \cdot \|V_e\|} \right) [^\circ] \quad (18)$$

This quantity gives the angle in 3D between the correct velocity vector and the estimated vector and thus describes how accurately the correct direction has been recovered. For the real images sequences we can only show the computed flow fields and discuss qualitative properties. We address this table, to prove the efficiency of Horn and Shunck method for white and black sequences and for a precise confidence measure (Barron *et al.*, 1994; Robert and Brian, 2003; Joachim, 2003; Thomax *et al.*, 2004; Andre *et al.*, 2005; Volker *et al.*, 2005).

Implementations and results: In the implementation of all studied methods, the images of R, G and B, r and g and H and S are obtained from the brightness function of images sequence (R, G, B).

Table 1: Time taken by proposed methods for computation by s CPU time

Proposed method	64*64	128*128	240X320	Panning sequence
Using rgb	2.1250	7.0790	103.7040	56.4220
Using HSV	2.0470	8.2660	114.7970	73.0310
Using (Min RGB)	2.9840	10.2190	144.9530	78.5780
Using (Min RGB+ smoothing)	3.0620	10.6250	146.7190	83.7810

Fig. 3: Original Image, of colour ball sequence with 240X320 size

The first order derivatives of the sequence functions are computed by using the $(1/12) \begin{pmatrix} -1 & 8 & 0 \\ 0 & -8 & 1 \end{pmatrix}$ kernel. We used a 5×5 neighbourhood, where each line was a copy of the estimation kernel mentioned above. For the computation of temporal derivatives, a $3 \times 3 \times 2$ spatiotemporal neighbourhood was used.

In our case, we first computed the time taken by any proposed method addressed in Table 1, using Matlab implementation on Toshiba PC Intel® pentium®, Mprocessor 1.70 GHz and 0.99 Go of RAM. We used the ball sequence Fig. 3 with different sizes (Toby Breckon, 2006) and Barron and Klette synthetic panning sequence Fig. 4. The first synthetic sequence Fig. 5, contains ball moving in the horizontal direction with 4 pixels/frame and in the vertical direction with 3 pixels/frames, with variable sizes, derived from the colour ball sequence (Toby Brecken, 2006) Fig. 3. We have constructed this sequence to use it in the testing phase of the proposed methods. The second one, is generated by Barron and Klette Fig. 4 where the correct flow is known. For panning, they simply translated an image of the Tamaki campus computer science building to the left by 3 pixels to make each new frame. The real image sequence was at Point Englan-d on the Tamaki river in Auckland (Baron and Klette, 2002; Bolker *et al.*, 2005).

Figure 6-9 illustrate the results of the first proposed method using, respectively the normalized space colour rgb and the HSV space colour. Figure 10-16 illustrate the results of the proposed method using RGB colour space with or without smoothness performance term. and the Fig. 17 illustrate our proper validation of Horn-Schunck method using the Y component of space colour with specific parameters.

Table 2: Comparison between the results using synthetic colour ball sequence with 64×64 size

Method	AME: Er±Std (Er) %	AAE: Ed±Std (Ed) %
Using rgb space RGB	5.50±2.44	3.15±1.39
Using HSV space RGB	22.2±25.45	11.6±12.14
Min. RGB space RGB	10.4±11.41	5.83±6.13
Min. RGB space RGB with smoothing term	6.16±4.11	3.52±2.33

Table 3: Comparison between the results from the literature using yosemite sequence

Method	AME (Er) %	AAE (Ed) %	Density (%)
HS (original)	32.43	30.28	100
HS (original) $ \nabla I > 5.0$	25.41	28.14	59.6
HS (modified)	11.26	16.41	100
HS (modified) $ \nabla I > 5.0$	5.48	10.41	32.9

Table 4: Comparison between the results Fig. 7, 9, 12-15 and 17 using synthetic panning sequence

Method	AME: Er±Std (Er) %	AAE: Ed±Std (Ed) %
Horn-Schunck RGB	17.44±17.77	2.64±4.08
Goland-Bruckstein RGB	11.38±17.36	5.04±11.80
Baron-Klette RGB	16.14±17.57	0.16
Using rgb space RGB	3.04±0.72	1.74±0.40
Using HSV space RGB	9.66±19.14	5.04±8.63
Min. RGB space RGB	6.06±6.96	3.43±3.79
Min. RGB space RGB with smoothing term	3.52 ±2.04	2.01±1.16

Fig. 17: Horn-schunck flow for the Y component ($Y = 0.299R + 0.578G + 0.114B$) with $\alpha = 3$ and 100 iterations

In the second stage, we used the first synthetic colour sequence (Ball sequence with 64×64 size Fig. 5) to compare quantitatively the obtained results Fig. 6 to 8, using the Average Magnitude Error (AME) and the Average Angular Error (AAE) for each studied method results, reported in Table 2.

In the last stage, we used the synthetic panning sequence Fig. 4 to compare quantitatively the obtained results Fig. 10 to 17, using (AME) and (AAE) mentioned above Table 3. In Table 4, we added from the fourth line our results to the presented in (Baron and Klette, 2002; Valiczer *et al.*, 2005).

CONCLUSION

Colour optical flow computed via the three colour components seems better than Gray value optical flow. The normalized rgb colour space method gives good results Fig. 6 and 7 followed by the RGB space with smoothing term method Fig. 8, 16 and 17, after that we found the RGB space without smoothing term method Fig. 10, 14 and finally the HSV space method Fig. 6 and 14. In our case we used a 100% density of dense optical flow computation.

The proposed and studied methods using any colour space require the presence of significant gradients of the colour functions. If the gradient magnitude of these functions is too small (≈ 0), these methods would fail to give reliable results. This implies that all methods based on gradient computation, are not reliable when the scene contains objects with uniform colour. Gradient based methods are simple and highly speed in implementation. So we can use them for detecting foreground objects in the sequence by using the flow optic magnitude constraint.

We can also use the same iterative solution to Horn and Schunck for each colour component. So, iterative solution will take enough time and can't resolve the problem! We can also extend the smoothness function with other forms (as the combination of the local and global constraints) and we can use a bidirectional multigrid methods for variational optical flow computation to resolve the real-time computation problem and the solving of the linear system of equations that result from a discretisation of the Euler-Lagrange equations. We plan to investigate all these to find a robust and sufficiently method for optical flow computation for any given sequences in some specific applications.

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