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Dynamic Analysis of non Prestressed Rayleigh Beam Carrying an Added Mass and Traversed by Uniform Partially Distributed Moving Loads

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Abstract: The study deals with dynamic analysis of non-prestressed Rayleigh beam carrying an added mass and traversed by uniform partially distributed moving loads. The governing partial differential equations were analysed to determine the dynamical behaviour of the system under consideration. It is shown that the amplitude deflection decreases as the length of the load ($\in$) increases for a fixed value of the moving load (M_L) when a non-prestressed moving force problem W_F (x, t) is considered. Also we observed that for the moving mass problem, the amplitude deflection W_{ML} (x, t) decreases as the length of the load ($\in$) increases for various time t and a particular value of the moving load.

Key words: Dynamical, amplitude, deflection, observed

INTRODUCTION

This study investigates the problem of elastic structures (beams) subjected to moving loads. Such problems have received a greatly increased emphasis since the middle of the last century, when railway construction began. Useful insight into the dynamic behavior of bridges may be obtained by studying the response of beams under moving loads. The problem of oscillation of bridges under traveling loads has also interested many Engineers, Mathematician and Physicists and continues to motivate many investigations (Cifuentes, 1989; Cifuentes and Lalapet, 1992; Esmailzadeh and gorashi, 1992, 1995; Gbadeyan and Oni, 1992, 1995; Kalker, 1996; Lee, 1994; Lin, 1996).

Willis (1951) was the first to consider the problem of elastic beam under the action of moving loads. He made the assumption that the mass of the beam is smaller than that of the load. He obtained an approximate solution to the problem. Stokes (1849) approached the problem under similar assumption. The other extreme case was studied by Krylov (1995) in which he considered the problem for which the mass of the load is assumed to be smaller than that of the beam. The technique he used involved the expansion of the associated eigenvalues.

Timoshenko (1992) used energy method to obtain solution in series form for simply supported finite beams on elastic foundation subjected to time dependent point loads moving with uniform velocity across the beam.

Gbadeyan and Oni (1995) developed a theory concerning the dynamic response of finite Rayleigh beam (and rectangular plates) under an arbitrary number of moving concentrated masses. A method capable of solving this problem for all classical end conditions. (clamped, simply supported free and sliding conditions) was developed.

The moving load problem involving both the inertia effect as well as the force effects were not considered for several years. This type of dynamical problem was first considered by Saller (1991), later by Jeffcott whose iterative method become divergent in some cases.

In all the aforementioned discussion, load was idealized by a single mass point. It goes without saying that in reality point load does not exist. Recently work involving non-point moving loads that is due to Emailzadeh *et al*. (1992, 1995). It should be remarked at the juncture, that the moving load problems in all these previous studies have been for classical/ideal end conditions.

The main thrust of the study is to

- Develop a theory for the dynamic response of a finite Rayleigh beam which carries a lumped mass at one of its ends, to a distributed moving load.
- Present the analysis of the dynamic response of a finite Rayleigh beam which carries a lumped mass at the end x = L but arbitrarily supported at the end x = 0 to a uniform partially distributed moving load.

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- Present a very simple and practical analytical-numerical technique for determining the response of beams with non-classical boundary conditions.

MATHEMATICAL MODEL

We consider the case of a partially distributed load M which assumed to strike a finite Rayleigh beam of length L, initially at time t = 0 and advancing uniformly along the beam with a constant velocity, V. The beam is assumed to be simply supported at the left hand end of the beam (Fig. 1) while the beam has an attached mass at the other end x = L.

The governing equation of the model: The general equation governing the dynamic behavior of an elastic finite Rayleigh beam for the uniform partially distributed moving load is given by the following fourth order partial differential equation.

$$\frac{EI\partial^4 W}{\partial x^4}+\frac{m\partial^2 W}{\partial t^2}-\frac{mb^2\partial^4 W}{\partial x^2\partial t^2}=F(x,t) \qquad (1)$$

Where, E is the modulus of elasticity, I is the second moment of area of the beam's cross-section, m is the mass per unit length of the beam, W is the deflection of the beam, x is the spatial coordinate, t is the time, b is the radius of gyration and F (x, t) is the applied surface moving load.

The applied surface moving load F (x, t) defined as

$$F(x,t)=\frac{1}{\in}\left[-M_{Lg}-M_L\nabla W\right]\left[H(x-\xi+\in/2)-H(x-\xi-\in/2)\right] \qquad (2)$$

$$\nabla=\frac{\partial^2}{\partial t^2}+\frac{2v\partial^2}{\partial x\partial t}+\frac{v^2\partial^2}{\partial x^2} \qquad (3)$$

H being the Heaviside unit function and is defined as

$$H(x)=\begin{cases}0 & x<0\\ 1 & x>0\end{cases} \qquad (4)$$

Where

- M_L = The constant mass of the load which is assumed to be constant with the beam during the course of the motion.
- $\in$ = The length of the load
- g = Acceleration due to gravity
- ξ = $(Vt+\in/2)$ a particular distance along the length of the beam

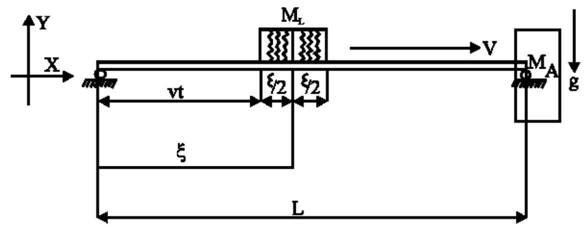

Fig. 1: The beam is assumed to be simply supported at the left hand end of the beam

Hence the governing equation of the model therefore becomes

$$\frac{EI\partial^4 W}{\partial X^4}+\frac{m\partial^2 W}{\partial t^2}-\frac{mb^2\partial^4 W}{\partial x^2\partial t^2}=\left[-M_{Lg}-M_L\frac{\partial^2 W}{\partial t^2}-2M_L\frac{V\partial^2 W}{\partial x\partial t}-M_L\frac{V^2\partial^2 W}{\partial x^2}\right]\left[H(X-\xi+\varepsilon/2)-H(X-\xi-\in/2)\right] \qquad (5)$$

The first term in the first square bracket on the r. h. s of Eq. 5 describes the constant gravitational force, while the second term accounts for the effect of acceleration in the direction of the transverse deflection W (x,t), the third term is for the complementary acceleration and the fourth term for the centripetal acceleration. The second square bracket describes the Hesviside unit function.

The bounddary condition: Equation 5 is subject to the following end supports;
At the end x = 0, one of the following holds.

$$\left.\begin{aligned}
W(x,t)&=\frac{\partial w(x,t)}{\partial x}=0 \text{ at } x=0 \text{ or } x=L\\
W(x,t)&=\frac{\partial^2 w(x,t)}{\partial x^2}=0 \text{ at } x=0 \text{ or } x=L\\
\frac{\partial^3 w(x,t)}{\partial x^3}&=\frac{\partial^2 w(x,t)}{\partial x^2}=0 \text{ at } x=0 \text{ or } x=L\\
\frac{\partial w(x,t)}{\partial x}&=\frac{\partial^3 w(x,t)}{\partial x^3}=0 \text{ at } x=0 \text{ or } x=L
\end{aligned}\right\} \qquad (6)$$

These conditions are sometimes called classical boundary conditions.
For the attached mass at the other end (x = L), we have

$$\begin{aligned}
EI\frac{\partial^2 w(L,t)}{\partial x^2}-\omega^2 JW(L,t)&=0\\
EI\frac{\partial^3 w(L,t)}{\partial x^3}-\omega^2 M_A(L,t)&=0
\end{aligned} \qquad (7)$$

(These conditions are called non-classical boundary conditions).
Where J is the mass moment of inertia at the end of the beam, ω^2 is the circular frequency and M_A is the attached mass at the end x = L.

The relevant initial conditions are

$$W(x,0)=\frac{\partial W(x,t)}{\partial t}=0 \tag{8}$$

OPERATION TRANSFORMATION OF THE GOVERNING EQUATION

We assume a solution of the form of the following series

$$w(X,T)=\sum_{i=1}^{\infty}\phi_i(t)X_i(x) \tag{9}$$

Where $\phi_i(x)$'s are the unknown function of time
$X_i(x)$'s are the known eigen function of free vibration of the beam.

Substituting Eq. 9 into Eq. 5 we have

$$\begin{aligned}&EI\sum_{i=1}^{\infty}\phi_i(t)X_i^{IV}(x)+m\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i(x)-mb^2\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i''(x)\\&=\frac{1}{\in}\left[-M_Lg-M_L\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i''(x)-2VM_L\sum_{i=1}^{\infty}\dot{\phi}_i(t)X_i'(x)-M_LV^2\sum_{i=1}^{\infty}\phi_i(t)X_i''(x)\right]\\&\left[H(X-\xi+\in)+H(x-\xi+\in)\right]\end{aligned} \tag{10}$$

We further assume that the load function can be expressed as

$$F(x,t)=\sum_{i=1}^{\infty}\psi_i(t)X_i(x) \tag{11}$$

Where ψ_i (t) are unknown function of time and X_i (x) are as said earlier.

Multiplying both sides of the right hand side of Eq. 10 by $X_j(x)$ and taking the definite integrals of both sides along the length L of the beam with respect to X, we have

$$\begin{aligned}&-\frac{MLg}{\in}\int_0^L X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2}\right]dx-\frac{M_L}{\in}\left[\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\int_0^L X_j(x)X_i(x)\left[H(x-\xi+\frac{\in}{2})-H)x-\xi-\frac{\in}{2})\right]\right.\\&dx\ \Big]\frac{-2M_LV}{\in}\left[\sum_{i=1}^{\infty}\dot{\phi}_i(t)\int_0^L X_j(x)X_i'(x)\left[H(x-\xi+\frac{\in}{2})-H)x-\xi-\frac{\in}{2})\right]dx\ \right]\frac{-M_LV^2}{\in}\left[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_i''(x)X_j(x)\right.\\&\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2})\right]dx\ \Big]\sum_{i=1}^{\infty}\psi_i(t)\int_0^L Xj(x)X_i(x)dx\end{aligned} \tag{12}$$

Evaluating the improper integrals in the l h s of the Eq. 12 term by term by integration by parts.
We finally obtained

$$
\begin{aligned}
&-M_L g\left[X_i(\zeta)+\frac{\in^2}{24}X''_i(\zeta)\right]-M_L(t)\sum_{i=1}^{\infty}\ddot{\phi}_i\left\{X_i(\zeta)X_j(\zeta)+\frac{\in^2}{24}\begin{bmatrix}X''_i(\zeta)X_j(\zeta)+2X'_i(\zeta)X'_j(\zeta)\\+X_i(\zeta)X''_j(\zeta)\end{bmatrix}\right\}\\
&-2M_L V\sum_{i=1}^{\infty}\dot{\phi}_i(t)\left\{X_i'(\zeta)X_j(\zeta)+\frac{\in^2}{24}\begin{bmatrix}X'''_i(\zeta)X_j(\zeta)+2X''_i(\zeta)X'_j(\zeta)\\+X'(\zeta)X_j''(\zeta)\end{bmatrix}\right\}\\
&-M_L V^2\sum_{i=1}^{\infty}\phi_i(t)\left\{X_i(\zeta)X''_j(\zeta)+\frac{\in^2}{24}\begin{bmatrix}X^{iv}{}_i\zeta X_j(\zeta)+2X'''_i(\zeta)X'_j(\zeta)\\+X''_i(\in)X''_j(\zeta)\end{bmatrix}\right\}=\psi_{i(t)}
\end{aligned}
\tag{13}
$$

A detailed analysis of the present problem, with the derivation of the Eq. 13 is given in the appendix.
Note (1) In the r. h. s of Eq. 12, we have made use of the orthonormal principle
(2)On noting that Eq. 11 is the applied force F (x, t), Eq. 10 now becomes

$$
\begin{aligned}
&EI\sum_{i=1}^{\infty}\phi_i(t)X_i^{iv}(x)+m\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i(x)-mb^2\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i''(x)=\sum_{i=1}^{\infty}X(x)\\
&\left\{\left[Xi(\zeta)+\frac{\in^2}{24}X''i(\zeta)\right]-M_L\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\left[X_i(\zeta)X_j(\zeta)+\frac{\in^2}{24}\left[X''_i(\zeta)X_j(\zeta)+2X'_i(\zeta)X'_j(\zeta)+X_i(\zeta X''_j(\zeta)\right]\right]\right.\\
&-2M_L V\sum_{i=1}^{\infty}\dot{\phi}_i(t)\left[X'_i(\zeta)X_j(\zeta)+\frac{\in^2}{24}\left[X'''_i(\zeta)X_j(\zeta)+2X''_i(\zeta)X'_j(\zeta)+X'_i(\zeta)X''_j(\zeta)\right]\right]\\
&\left.-M_L V^2\sum_{i=1}^{\infty}\phi_i(t)\left[X''_i(\zeta)X_j(\zeta)+\frac{\in^2}{24}\left[X^{iv}{}_i(\zeta)X_j(\zeta)+2X'''_i(\zeta)X'_j(\zeta)+X''_i(\zeta)X''_j(\zeta)\right]\right]\right\}=\psi_i(t)
\end{aligned}
\tag{14}
$$

The equation of free-vibration of beam is given as

$$X^{iv}{}_i(x)-\beta^4{}_iX_i(x)=0 \tag{15}$$

Where

$$\beta^4{}_i=\frac{m\omega^2{}_i}{EI} \tag{16}$$

$$X^{iv}{}_i(x)=\beta^4{}_i=\frac{m\omega^2{}_i}{EI}X_i(x) \tag{17}$$

By putting Eq. 17 into Eq. 14, we have

$$M\sum_{i=1}^{\infty}\omega_i^2\,\phi_i(t)X_i(x)+m\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i(x)-mb^2\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i''(x)=\sum_{i=1}^{\infty}X_i(x)$$
$$\left\{\left[Xi(\zeta)+\frac{\in}{24}X''(\zeta)\right]-M_L\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\Big[X_i(\zeta)X_j(\zeta)+\frac{\in^2}{24}\big[X''_i(\zeta)X_j(\zeta)+2X'_i(\zeta)X'_j(\zeta)+X_i(\zeta X''_j(\zeta)\ \big]\Big]\right.$$
$$-2M_LV\sum_{i=1}^{\infty}\dot{\phi}_i(t)\Big[X'_i(\zeta)X_j(\zeta)+\frac{\in^2}{24}\big[X'''_i(\zeta)X_j(\zeta)+2X''_i(\zeta)X'_j(\zeta)+X'_i(\zeta)X''_j(\zeta)\ \big]\Big]$$
$$-M_LV^2\sum_{i=1}^{\infty}\phi_i(t)\Big[X''_i(\zeta)X_j(\zeta)+\frac{\in^2}{24}\big[X^{iv}_i(\zeta)X_j(\zeta)+2X'''_i(\zeta)X'_j(\zeta)+X''_i(\zeta)X''_j(\zeta)\ \big]\Big] \quad (18)$$

Equation 18 is a set of coupled ordinary linear second order differential equations.
Remark: By considering Eq. 18, two interesting special cases of the problem may be analysed as follows:

- As $\in$ tends to zero, then the model would revert to the problem of a single point mass traveling on a suspension bridge which has been fully treated and analysed by some scholars (Esmailzadeh, 1992; Gbadeyan and Oni, 1992; Jeffcott).
- If the inertia effect of the load is ignored then Eq. 18 becomes uncoupled. This can then be verified by replacing M_Lg by P and M_L (not involving g) by zero in equation 18. Then it can be concluded that we are having moving force solution in a special case of the more general form of the moving mass one.

Simply supported rayleigh beam with an attached mass: The dynamic response of the system under consideration (a beam carrying a mass at the end x = L and traversed by partially distributed moving load) having a simply supported boundary conditions is considered. (In particular the beam under consideration is simply supported at x = 0 while carrying a mass at the end x = L).

The end conditions are as prescribed in Eq. 7 and corresponding kernel can be easily shown as

$$X_i(x)=\sin\frac{q_i}{L}x+\beta_i\sinh\frac{q_i}{L}x \quad (19)$$

Where

$$\beta_i=\frac{EI\,q_i\sin q_i\,x+\omega^2\,JL\cos q_i}{EI\,q_i\sin q_i\,x-\omega^2\,JL\cos q_i\,x} \quad (20)$$

and q_i is the roots of the associated transcendental frequency equation given as

$$\cos q_i\sinh q_i-\sin q_i\cos q_i-\frac{2\omega^2M_L\sin q_i\sinh q_i}{EIq_i^{\,2}}-\frac{2\omega^2\,JL\cos q_i\cosh q_i}{EIq_i^{\,2}}$$
$$+\frac{\omega^2M_LJ}{(EI)^2q_i^{\,2}}\left[\sin q_i\cosh q_i-\cos q_i\sinh q_i\right]=0 \quad (21)$$

The transcendental frequency Eq. 21 is solved, using Newton Raphson's method

The governing differential equation for vibration of the beam, for the particular case under consideration could be obtained thus by deriving exact governing equations by employing Eq. 19 and evaluating the exact values of the integral in Eq. 12. After along lengthy simplification, we finally have

$$m\sum_{i=1}^{\infty}\omega_i^2\,\phi_i(t)X_i(x)+m\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i(x)-mb^2\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i''(x)=\sum_{i=1}^{\infty}\ddot{\phi}_i\sin\frac{q_i}{L}\Big(\zeta+\frac{\in}{2}\Big)+B_i\sinh\frac{q_i}{L}\Big(\zeta+\frac{\in}{2}\Big)$$

$$\Big\{\Big[-\frac{M_Lg}{\in}\Big[2\sin\frac{q_i}{L}\xi\sin\frac{q_i}{2L}(\in+B_i)\sinh\frac{q_i}{L}\xi\sinh\frac{q_i\in}{2L}\Big]-\sum_{i=1}^{\infty}\phi_i(t)\Big\{\frac{LM_L}{q_i-q_j}\Big[\sin\frac{\in}{2L}(q_i-q_j)\Big]$$

$$-\frac{LM_L}{\in(q_i-q_j)}\Big[\sin\frac{\in}{2L}(q_i+q_j)\cos\frac{\xi}{L}(q_i+q_j)\Big]+\frac{LB_iM_L}{\in(q_i^2-q_j^2)}\Big[q_i\sin\frac{(-\in)}{2L}(1-q_i)\Big\}+\sin\frac{(-\in)}{2L}(1-q_i)$$

$$\cos\frac{\xi}{L}(1+q_i)+q_j(\sin\frac{(-\in)}{2L}(q_i-q_j)\xi\cos\frac{\xi}{L}(q_i-q_j)+\sin\frac{\in}{2L}(q_i-q_j)\xi\cos\frac{\xi}{L}(q_i+q_j))\Big]$$

$$+\frac{M_LB_i^2M_L}{\in(q_i-q_j)}\Big[\cosh\frac{\xi}{L}(q_i+q_j)\sin\frac{\in}{2L}(q_i+q_j)\Big]-\frac{M_LB_i^2M_L}{\in(q_i-q_j)}\Big[\cosh\frac{\xi}{L}(q_i-q_j)\sinh\frac{\in}{2L}(q_i+q_j)\Big]$$

$$-2M_LV\sum_{i=1}^{\infty}q_i\dot{\phi}\Big\{\frac{1}{(q_i+q_j)}\Big\}\Big[\sin\frac{\xi}{L}(q_i+q_j)\sin\frac{\in}{2L}(q_i+q_j)\Big]+\frac{1}{\in(q_i-q_j)}\Big[\cos\frac{\xi}{L}(q_i-q_j)\sin\frac{\in}{2L}(q_i+q_j)\Big]$$

$$+\frac{LB_i}{\in(q_i^2-q_j^2)}\Big[q_i(\sin\frac{\in}{2L}(1+q_i)+\sin\frac{\xi}{L}(1+q_i)\sin\frac{\in}{2L}(1+q_i)+q_i(\sin\frac{\xi}{2L}(1-q_i)\Big]$$

$$+q_i(\sin\frac{\in}{2L}(1-q_i)\cos\frac{\xi}{L}(1-q_i)+\sin\frac{\in}{2L}(1+q_i)\cos\frac{\xi}{L}(1+q_i)\Big]+\frac{B_i^2}{\in(q_i+q_j)}\Big[\sinh\frac{\xi}{L}(q_i+q_j)\sinh\frac{\in}{2L}(q_i+q_j)\Big]$$

$$+\frac{B_i^2}{\in(q_i+q_j)}\Big[\sinh\frac{\xi}{L}(q_i+q_j)\sinh\frac{\in}{2L}(q_i+q_j)\Big)\Big]\Big]\Big\}+M_LV^2\sum_{i=1}^{\infty}\phi_i(t)\Big\{q_i^2\Big(\frac{1}{(q_i+q_j)}\Big[\sin\frac{\in}{2L}(q_i-q_j)\cos\frac{\xi}{L}(q_i+q_j)+\frac{1}{(q_i+q_j)}\Big[\sin\frac{\in}{2L}(q_i+q_j)\cos\frac{\xi}{L}(q_i+q_j)\Big]$$

$$-\frac{B_iL}{(q_i^2+q_j^2)}\Big[q_i\sin\frac{\in}{2L}(1-q_i)\cos\frac{\xi}{L}(1-q_i)+\sin\frac{\in}{2L}(1+q_i)\cos\frac{\xi}{L}(1+q_i)+\Big]$$

$$q_i\Big(\sin\frac{\in}{2L}(1-q_i)\cos\frac{\xi}{L}(1-q_i)+\sin\frac{\in}{2L}(1+q_i)\cos\frac{\xi}{L}(1+q_i)\Big)\Big]+\frac{B_i^2}{(q_i+q_j)}\Big[\sinh\frac{\xi}{L}(q_i+q_j)\sinh\frac{\in}{2L}(q_i+q_j)\Big]$$

$$-\frac{B_i^2}{(q_i+q_j)}[\cosh\frac{\xi}{L}(q_i-q_j)\sinh\frac{\in}{2L}(q_i-q_j)\Big]\Big\}\Big\} \qquad (22)$$

Equation 22 is the desired exact differential equation describing the behaviour of Rayleigh beam carrying an added mass at one of its ends by a distributed moving load.

The highly coupled equation is solved numerically.

Note. For the case of $q_i = q_j$, we replace the expression involving

$$\frac{1}{q_i-q_j} \text{ by } \frac{q_i\in}{2L}$$

To solve Eq. 22, recourse can be made to a numerical method, but two cases are to be tackled

Case 1: The moving force Rayleigh beam problem: A moving force is one in which the inertia effects of the load are neglected and only the force effects are retained. This is done in Eq. 22 by neglecting all the three terms on the right hand side of the later except the first term in the first curly bracket [i.e., by neglecting all the terms apart from the first term on the r. h. s of Eq. 22.

Case II the moving mass Rayleigh beam problem: is one in which both the inertia effects and the force effects are retained i.e., the whole Eq. 22 is the moving mass problem.

To obtained results given in this paper, approximate central difference formulas have been utilized for the derivatives in Eq. 22 for both cases [cases I and II]. Thus for N modal shapes, Eq. 22 are transformed to a set of N linear algebraic equations, which are to be solved for each interval of time. Regarding the degree of approximations involved, in order to ensure the stability and convergence of the solution, sufficiently small time steps have been utilized.

RESULTS AND DISCUSSION

For the purpose of discussing the results, some numerical calculations are carried out for the two cases of our consideration (i.e., the moving force problem and the moving mass problems, equations.

In order to provide numerical solution, the work of Esmailzadeh and Gorashi (1995) is followed there by choosing the following values. The length of the beam in each problem was taken to be 10 m, the velocity (V) of the moving force or mass is such that V = 3.3 m/s. The parameters 1, E g,b,h and m are assumed to take up the following values 1.04×10^{-6}m, 2.07×10^{11} N m^{-2},. 9.8 m/s, 0.05, 0.01 and 1.5s, while E = 0.1 m and 1.0 m were used. Hence we have the following tables.

Table 1: Variation of the deflection W_F (x, t) of the non-prestressed simply supported Rayleigh beam carrying a lumped mass at its end x = L and traversed by a moving force. For t = 0.5s, $\in$ = 0.1m and various values of M_L.

Length of the Beam X(m)	W_F (x,t) for $M_L = 7.04$ kg m^{-1}	W_F (x, t) for $M_L = 8.0$ kg m^{-1}	W_F (x, t) for $M_L = 10$ kg m^{-1}
1.469	1.13E-03	9.74E-04	7.82E-04
2.888	-5.11E-02	-4.46E-02	-3.57E-02
4.307	7.53E-02	6.67E-02	5.33E-02
5.726	4.7172	3.2833	3.2833
7.145	-30.5796	-26.3228	-21.1228
8.564	-215.752	-186.117	-149.5205

Table 2: Variation of the deflection W_F (x, t) of the non-prestressed simply supported Rayleigh beam carrying a lumped mass at its end x = L and traversed by a moving force. For t = 0.5s, $\in$ = 1.0 m and various values of M_L

Length of the Beam X(m)	W_F (x,t) for $M_L = 7.04$ kg m^{-1}	W_F (x, t) for $M_L = 8.0$ kg m^{-1}	W_F (x, t) for $M_L = 10$ kg m^{-1}
1.853	1.96E-04	1.72E-04	1.37E-04
3.206	-8.90E-03	-7.83E-03	-6.26E-03
4.559	1.32E-02	1.16E-02	9.32E-03
5.912	8.19E-01	7.21E-01	5.76E-01
7.265	-5.27E+00	-4.64E+00	-3.71E+00
8.618	-3.74E+01	-3.29E+01	-2.63E+01

Table 3: Variation of the deflection W_F (x, t) of the non-prestressed simply supported Rayleigh beam carrying a lumped mass at its end x = L and traversed by moving force. For $\in$ = 0.1 m, M_L = 7.04 kg m^{-1} at various values of t

Length of the Beam X(m)	W_F (x,t) for t = 0.5 sec	W_F (x, t) for t = 0.1 sec	W_F (x, t) for t = 1.5 sec
1.469	1.13E-03	3.95E-03	8.74E-03
2.888	-5.11E-02	-1.79E-01	-3.96E-01
4.307	7.53E-02	2.64E-01	5.84E-01
5.726	4.72E+00	1.65E+01	3.66E+00
7.145	-3.06E+01	-1.06E+02	-2.35E+02
8.564	-2.16E+02	-7.55E+02	-1.67E+03
9.983	3.91E+02	1.37E+04	3.03E+04

Table 1 shows the values of the transverse deflection $W_f(x, t)$ of the non prestressed simply supported Raleigh beam carrying a lumped mass at the end x = L at the t = 0.5s, while E = 0.1, The analysis was carried out for the various values of X. Similar results were presented in (Table 2) but for E = 1.0 m. Uniform partially distributed moving force was considered. It was observed from each of these two tables, that the amplitude deflection decreases as M_L increases. However the amplitude deflection decreases as E increases for fixed values of M. In particular, amplitude increases as M_L increases.

Furthermore, (Table 3) shows the variation of the deflection, W_f (x, t) of the system involving moving force problem, against various values of x and for different values of time, t. it was found that the amplitude deflection increases as t increases.

Table 4 deals with variation of the lateral deflection W_{ML} (x, t). The moving mass problem, of a non-prestressed Rayleigh beam traversed by a uniform partially distributed mass against various values of coordinate x, three different values of the moving load M_L (i.e., 7.04, 8.0 and 10 kg m^{-1} were considered for time t = 0.5s and $\in$ = 0.1 m.

Table 4: Variation of the deflection W_{ML} (x, t) of the moving mass non-prestressed simply supported Rayleigh beam traversed by a uniform partially distributed mass against various values of x, for different values of M_L (i.e., M_L = 7.04, 8.0 and 10 kg m^{-1}), t = 0.5s and $\in$ = 0.1 m

Length of the Beam X(m)	W_{ML} (x,t) for $M_L = 7.04$ kg m^{-1}	W_{ML} (x,t) for $M_L = 8.0$ kg m^{-1}	W_{ML} (x,t) for $M_L = 10$ kg m^{-1}
1.469	-2.96E-06	-2.91E-05	-2.80E-05
2.888	-9.19E-05	-8.13E-05	-7.97E-05
4.307	-3.47E-05	-3.53E-05	-3.66E-05
5.726	8.45E-05	8.38E-05	8.14E-05
7.145	2.40E-05	2.51E-05	8.14E-05
8.564	-2.77E-04	-2.76E-04	-2.72E-04
9.983	-4.91E-04	-4.91E-04	-1.91E-04

Table 5: Variation of the deflection W_{ML} (x, t) of the moving mass non-prestressed simply supported Rayleigh beam traversed by a uniform partially distributed mass against various values of x, for different values of M_L (i.e., M_L = 7.04, 8.0 and 10 kg m^{-1}), t = 1s and $\in$ = 0.1m

Length of the Beam X(m)	W_{ML} (x,t) for $M_L = 7.04$ kg m^{-1}	W_{ML} (x,t) for $M_L = 8.0$ kg m^{-1}	W_{ML} (x,t) for $M_L = 10$ kg m^{-1}
1.469	-5.71E-05	-5.74E-05	-5.54E-05
2.888	-1.52E-04	-1.52E-04	-1.50E-04
4.307	-5.83E-05	-5.89E-05	-6.02E-05
5.726	1.60E-04	1.58E-04	1.56E-04
7.145	3.78E-04	3.86E-05	4.02E-05
8.564	-5.11E-04	-5.09E-04	-5.05E-04
9.983	-8.81E-04	-8.81E-04	-8.81E-04

Table 6: Variation of the displacement W_{ML} (x, t) of the moving mass non-prestressed simply supported Rayleigh beam for different values of time t against $\in$ (i.e., $\in$ = 0.1 and 1 m) and M_L = 7.04 kg m^{-1}

Time (t)s	W_{ML} (x,t) for $\in$ = 0.1 m	W_{ML} (x,t) for $\in$ = 1.0 m
0.43	-2.96E-06	8.55E-05
0.86	-8.19E-05	-8.27E-05
1.29	-3.47E-05	-1.61E-04
1.72	8.45E-05	7.06E-05
2.15	2.40E-05	4.51E-06
2.58	-2.77E-04	2.26E-04
3.01	-4.90E-04	2.52E-04

Table 5 contains similar values of W_{ML} (x, t) but for t = 1.0s. Furthermore values of W_{ML} (x, t) similar to those in (Table 4) are presented in (Table 5). Table 6 shows the variation of the displacement W_{ML} (x, t) for different values of $\in$ ($\in$ = 0.1 m and 1 m) against t. (Table 4-6) shows that the amplitude deflection decreases as M_L increases for fixed values of t and $\in$.

CONCLUSION

We have examined dynamic analysis of non-prestressed Rayleigh beam carrying an added mass and traversed by uniform partially distributed moving mass. We have modeled the problem mathematically in such a way that the mass of the moving load is small compared with the mass of the beam. The material of the beam is linearly elastic and homogenuous at any cross section. The beam is finite, initially straight and uniform cross section area.

It is further assumed that the beam carries the whole load at the left hand support initially. The load moves at constant velocity from left to right and keeps contact with the beam at all times.

Some of the interesting, conclusions of the problem are as summarized as follows:

- The amplitude deflection decreases as the length of the load ($\in$) increases for a fixed value of the moving load (M_L) when a non-prestressed moving force problem is considered.
- The amplitude deflection of non-prestressed moving force problem decreases as the value of the moving load M_L increases for a particular time t.
- The amplitude, deflection of the non-prestressed moving force problem increases as time t increases for various values of X.
- The amplitude deflection $W_{ML}(x, t)$ decreases as $\in$ increases for various time, t and a particular value of the moving load when a non-prestressed moving mass problem is considered.

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APPENDIX

In order to derive Eq. 13 the function F (x, t) is assumed to be expressible as

$$F(x,t) = \sum_{i=1}^{\infty} \psi_i(t) X_i(x) \tag{A1}$$

Where the $\Psi_i(t)$'s are unknown functions of time. By substituting for W (x, t) from Eq. 9, multiplying both sides of the r.h.s. of Eq. 10 by $X_j(x)$ and taking the definite integral of both sides along the length of the beam with respect to X we obtain Eq. 12.

$$\begin{aligned}
&-\frac{M_L}{\in}\int_0^L X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2})\right] \\
&dx-\frac{M_L}{\in}\left[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_j(x)X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2})\right]dx\right] \\
&\frac{-2M_L V}{\in}\left[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_j(x)X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2})\right]dx\right] \\
&\frac{-M_L V^2}{\in}\left[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_i''(x)X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2})\right]dx\right] \\
&=\sum_{i=1}^{\infty}\psi_i(t)\int_0^L X_j(x)X_i(x)dx
\end{aligned} \tag{A2}$$

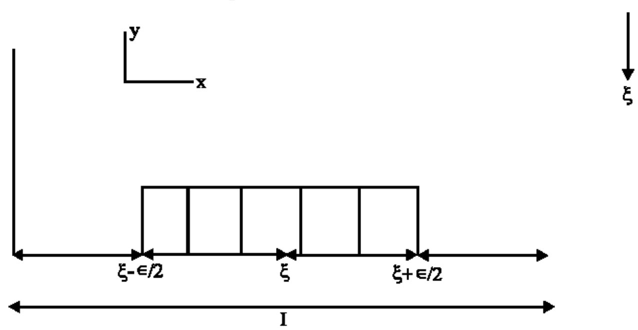
The above integrations can be convienently carried out term by term by defining them as follows:

$$\left.\begin{aligned}
K &= -\frac{M_L g}{\in}\int_0^L X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2}\right]dx \\
L &= \frac{M_L}{\in}\left[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_j(x)X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2}\right]dx\right] \\
M &= \frac{-2M_L V}{\in}\left[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_j(x)X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2}\right]dx\right] \\
N &= \frac{-M_L V^2}{\in}\left[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_i''(x)X_j(x)\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2}\right]dx\right] \\
O &= \sum_{i=1}^{\infty}\psi_i(t)\int_0^L X_j(x)X_i(x)dx
\end{aligned}\right\} \quad \text{(A3)}$$

Consider the first term that is K, using integration by parts,

$$K = -\frac{M_L g}{\in}\left\{\left[\left[H(x-\xi+\frac{\in}{2})-H(x-\xi-\frac{\in}{2}\right]\int_0^L X_j(x)dx\Big|_0^L\right] - \int_0^L\left[\int_0^L X_j(x)dx\left[H'(x-\xi+\frac{\in}{2})-H'(x-\xi-\frac{\in}{2}\right]\right]\right\} \quad \text{(A4)}$$

Using the property of Heaviside function and the figure below:



$$K = \frac{-M_L g}{\in}\int_0^L\int_0^L X_j(x)\delta(x-\xi+\frac{\in}{2})dx - \int_0^L\int_0^L X_j(x)\delta(x-\xi-\frac{\in}{2})dx \quad \text{(A5)}$$

Now setting $D(x) = \int_0^L X_j(x)dx$ equation K becomes;

$$K = \frac{-M_L g}{\in}\left[\int_0^L D(x)\delta(x-\xi+\frac{\in}{2})dx - \int_0^L D(x)\delta(x-\xi-\frac{\in}{2})dx\right] \quad \text{(A6)}$$

Thus by the property of Dirac delta function, we have

$$K = \frac{-M_L g}{\in}\left[D(\xi + \frac{\in}{2}) - D(\xi + \frac{\in}{2})\right] \quad \text{(A7)}$$

$$K = \frac{-M_L g}{\in}\left[\int_0^L X_j(\xi + \frac{\in}{2})d\xi - \int_0^L X_j(\xi - \frac{\in}{2})d\xi\right] \quad \text{(A8)}$$

Using Taylor's series expansion, we obtain

$$K = \frac{-M_L g}{\in}\left[\int_0^L \left[X_j(\xi) - \left(\frac{\in}{2}\right)X'\ (\xi) + \frac{\left(\frac{\in}{2}\right)^2 X''(\xi)}{2!} + \frac{\left(\frac{\in}{2}\right)^3 X'''(\xi)}{3!} + \frac{\left(\frac{\in}{2}\right)^4 X^{iv}(\xi)}{4!} + \ldots - X_j(\xi) + \left(\frac{\in}{2}\right) X'(\xi) - \right.\right.$$
$$\left.\frac{\left(\frac{\in}{2}\right)^2 X''(\xi)}{2!} + \frac{\left(\frac{\in}{2}\right)^3 X'''(\xi)}{3!} - \frac{\left(\frac{\in}{2}\right)^4 X^{iv}(\xi)}{4!}\ldots\ \right]dx \quad \text{(A9)}$$

Hence

$$K = -M_L g\left[Xi(\zeta) + \frac{\in^2}{24}X''_i(\zeta)\right] + 0(\varepsilon^3) \quad \text{(A10)}$$

Following similar argument, the second improper integral

$$L = -M_L \sum_{i=1}^{\infty} \ddot{\phi}_i(t)\int_0^L \left[X_i(\xi + \frac{\in}{2})X_j(\xi + \frac{\in}{2}) - X_i(\xi - \frac{\in}{2})X_j(\xi - \frac{\in}{2})\right]dx \quad \text{(A11)}$$

Which reduces to

$$L = -M_L\ (t)\sum_{i=1}^{\infty} \ddot{\phi}_i\left\{X_i(\zeta)X_j(\zeta) + \frac{\in^2}{24}\left[X''_i(\zeta)X_j(\zeta) + 2X'_i(\zeta)X'_j(\zeta) + X_i(\zeta)X''_j(\zeta)\right]\right. \quad \text{(A12)}$$

Also the third and the fourth improper integrals M and N becomes

$$M = -2M_L V\sum_{i=1}^{\infty} \dot{\phi}_i(t)\int_0^L \left[X_i'(\xi + \frac{\in}{2})X_j(\xi + \frac{\in}{2}) - X_i'(\xi - \frac{\in}{2})X_j(\xi - \frac{\in}{2})\right]dx \quad \text{(A13)}$$

$$N = -M_L V^2\sum_{i=1}^{\infty} \phi_i(t)\int_0^L \left[X_i''(\xi + \frac{\in}{2})X_j(\xi + \frac{\in}{2}) - X_i''(\xi - \frac{\in}{2})X_j(\xi - \frac{\in}{2})\right]dx \quad \text{(A14)}$$

Respectively which finally reduces to

$$M = -2M_L V\sum_{i=1}^{\infty}\dot{\phi}_i(t)\Big[X_i'(\zeta)X_j(\zeta) + \frac{\in^2}{24}\Big[X'''_i(\zeta)X_j(\zeta) + 2X''_i(\zeta)X_j(\zeta) + X'(\zeta)X_j''(\zeta)\Big]\Big] + 0(\varepsilon^3) \quad (A15)$$

And

$$N = -M_L V^2\sum_{i=1}^{\infty}\phi_i(t)\Big[X_i(\zeta)X''_j(\zeta) + \frac{\in^2}{24}\Big[X^{iv}{}_i\zeta X_j(\zeta) + 2X'''_i(\zeta)X'_j(\zeta) + X''_i(\in)X''_j(\zeta)\Big]\Big] + 0(\varepsilon^3) \quad (A16)$$

Respectively.
By using orthogonality relation

$$0 = \sum_{i=1}^{\infty}\psi_i(t)\int_0^L X_j(x)X_i(x)dx \text{ decomes } \psi_i(t). \quad (A17)$$

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