Journal of Engineering and Applied Sciences 2 (11): 1661-1668, 2007
ISSN: 1816-949X

A Finite Element Analysis of Corroded Plates

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Abstract: In the several structural applications of plates, thickness in service may not always be considered constant as, in addition to other sources, a variable thickness profile can result from metal surface losses due to corrosion. A further analytical step to the finite element analysis of plate has been proposed in this study, by modeling this arbitrarily varying thickness profile as a 3rd degree polynomial. The plate continuum was discretized into rows and columns and thickness losses measured randomly along the plate length for the number of idealized rows and the least square method used to fit a unique curve through the data points for each row. Equivalent element thicknesses were obtained as simple averages of its corresponding boundary nodal thickness, determined from the polynomial curve. A finite element program incorporating this novel concept was developed and validated by a comparative analysis of classical results for a uniformly loaded encastre plate of constant cross section.

Key words: Finite elements analysis, marine plates corrosion, corroded plates discretization

INTRODUCTION

One significant feature of a structural plate is the dependence of its bending properties on its transverse dimension, the thickness, which is normally very small compared to the other two sides (Coates *et al*., 1987). In majority of its classification and use, however, plate thickness is usually considered constant.

Plates of uniform thickness find use in several structural applications especially supporting surface loads between spans. As wall panels, they transmit load along their length or in the transverse direction as in slabs, decks, etc. Plates confine pressure as in pipes, boilers, reactors, heat exchangers and several other containment structures. The automobile and aeronautic industries also depend on the use of plates in the form of panels, stiffeners, aircraft wings, fuselages, etc.

The response of plates under various forms of loading and support condition is of great value in structural design and numerous investigators have studied problems of flexure, buckling and postbuckling analysis since Von Karman first developed the compatibility and equilibrium equation. These attempts and several motivations at studying the behavior of loaded plates have led to developing solutions to the problem of plate flexure. Thus, approximations for the deflection behavior of isotropic plates with geometries and boundary conditions have been severally obtained. Some of the most important results are as described by Timoshenko and Woinowsky-Krieger (1959). Some exact solutions have similarly been obtained using Theory of Elasticity (Timoshenko and Goodier, 1951).

The growing interests for a better understanding of plate behaviour are enormous. There is the need to develop enhanced methods of calculation and analyses resulting in smaller moment distributions on plates and improved factor of safety, so that their durability as well as integrity in service can be better predicted. To this end, plate thickness cannot always be considered constant in all of its application, especially in the course of its use in service. Consequently, plate thickness can vary due to action of loads, as a result of mechanical wear and tear and the combined effect of environmental activities in operation. In addition, a deliberate attempt at a better surface finish in the course of plate fabrication and handling can result in variation of plate thickness.

The bending analysis of plate of non-uniform thickness dates back to the work of Olsson (1934). Several other studies of stress and deformation of plates with discontinuous changes of thickness have been undertaken employing classical and analytical methods (Fok and Rhodes, 1977; Rushton, 1969; Cul and Dowell, 1983; Raju, 1966; Ttretyak, 1963). Thickness variation profile has most recently been considered in the solution of plate stability, where the variation due to corrosion was analysed (Lakhote *et al*., 2002) and modelled by the two-dimensional Fourier infinite series approach (Roorda *et al*., 1996) applicable to uniaxially loaded thin rectangular simply supported plate of plane geometrical configuration.

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In summary, thickness variation and its associated structural problems is real in the overall life cycle of any plate structure and requiring further investigation, as this will affect stress distribution and structural integrity of plates under exploration. The resulting stress redistribution often leads to overstress in critical areas and threat of failure and even collapse. Thus, assessment of available level of safety in existing structures in service becomes a critical subject (Johnary, 2005).

This study presents, therefore, the description of the computer approaches adapted from the finite element method in analyzing the structural responses of a plate of an arbitrarily varying thickness profile.

THEORETICAL BACKGROUND

The variation of thickness can be linear, varying according to a relation (Tretyak, 1963), or can be of an unpredictable nature as can be observed on corroded plate surfaces (Lakhote *et al.*, 2002).

A plate is a continuum which can be modelled as a plane stress problem described by the bi-harmonic partial differential equation as

$$\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial^4 y} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} = \frac{q}{D} \quad (1)$$

For a set of boundary conditions and any given load intensity q, the solution of (1) yields the distribution of stress, which are actually bending/twisting moments.

In general terms for an orthotropic plate, the stress-strain relations from plate bending theory is given as

$$\sigma(x,y) = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_x & D_1 & 0 \\ D_1 & D_y & 0 \\ 0 & 0 & D_{xy} \end{bmatrix} \begin{Bmatrix} -\partial^2 w / \partial x^2 \\ -\partial^2 w / \partial y^2 \\ 2\partial^2 w / \partial x \partial y \end{Bmatrix} \quad (2)$$

For the consideration of geometrical non-linearity based on the concept of a varying thickness profile due largely to surface losses from corrosion, plate flexural rigidity becomes largely dependent on the value of the thickness, t, at any point on the plate continuum. This kind of stochastic variation presents discontinuities which are difficult to describe analytically. However, an approximate solution in which the governing differential equation is replaced by a set of algebraic simultaneous equations, which represent the value of the unknown variables (displacements, stress, etc) at the discrete points or nodes and particularly suited for automatic computation involving a number of repetitive steps, can be used. One such numerical method is the method of Finite Elements, formulated to take advantage of the capabilities of a computer program. The Finite Element method which is an extension of the analysis of ordinary frames to two and three-dimensional structures such as plates, shells etc, was pioneered in the aircraft industry for the accurate analysis of complex airframes has been extensively developed and published (Zienkiewicz, 1971; Rockey *et al.*, 1973; Hrennikoff, 1941; Clough *et al.*, 1956; Ray, 1974; Holland, 1974).

In the Finite Element analysis of problems of a discrete nature, the structure is considered as an assemblage of all individual structural elements following a well defined procedure of establishing local equilibrium at each 'node' or connecting point of the structure.

The key to the method is the general matrix equation relating elemental load to their corresponding nodal displacements given as

$$\{F^e\} = [K^e]\{\partial^e\} \quad (3)$$

A partitioned method of matrix summation of the contributions of all element stiffness matrices results in

$$\{F\} = [K]\{\partial\} \quad (4)$$

The overall stiffness matrix [K] relates the applied nodal forces to the unknown displacement, upon which the displacement vector for the entire structure can be determined for any given load, after consideration of the given boundary conditions.

COMPUTATIONAL METHODS

In the present study, we adopt all the conventional idealisation of classical small-deflection, thin-plate theory and the triangular element, with three corresponding nodal forces and displacement quantities as defined (Fig. 1).

The estimation of element stiffness matrix generally involves the definition of a suitable coordinate system and superimposition of displacement functions as a rigid body and as a simply supported element such that (Zienkiewicz, 1971)

$$w = w^{rb} + w^{ss} \quad (5)$$

Where:

$$w^{rb} = w_1 L_1 + w_2 L_2 + w_3 L_3 \quad (6)$$

and

$$w^{ss} = N_{x1}\theta_{x1}^{ss} + N_{y1}\theta_{y1}^{ss} + N_{x2}\theta_{x2}^{ss} + N_{y2}\theta_{y2}^{ss} + N_{x3}\theta_{x3}^{ss} + N_{y3}\theta_{y3}^{ss} \quad (7)$$

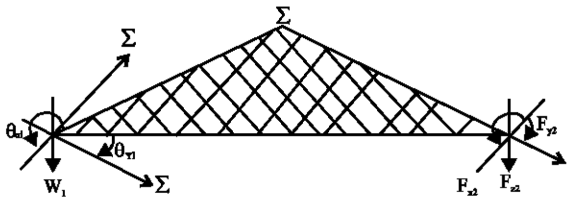


Fig. 1: Nodal degrees of freedom

Each N_x and N_y term representing the individual shape functions, chosen such that boundary conditions at the nodes of the simply supported elements are satisfied. The shape functions are defined as

$$\begin{aligned} N_{x1} &= L_1^2\left(b_3L_2 - b_2L_3\right) + \frac{b_3 - b_2}{2}L_1L_2L_3 \\ N_{y1} &= L_1^2\left(c_3L_2 - c_2L_3\right) + \frac{c_3 - c_2}{2}L_1L_2L_3 \end{aligned} \quad (8)$$

Other individual shape functions are similarly obtained by changing the subscript in a cyclic order and the nodal and area coordinates, a_1, b_1, c_1 etc and L_1, L_2, L_3 have been, respectively defined (Rockey *et al*., 1973).

In plane elasticity problems, strains are usually associated with curvatures and are related to simply supported nodal displacements as

$$\left\{\varepsilon^{ss}(x,y)\right\} = \begin{Bmatrix} -\dfrac{\partial^2 w^{ss}}{\partial x^2} \\ -\dfrac{\partial^2 w^{ss}}{\partial y^2} \\ 2\dfrac{\partial^2 w^{ss}}{\partial x \partial y} \end{Bmatrix} \quad (9)$$

and in general as

$$\left\{\varepsilon^{ss}(x,y)\right\} = \left[B^{ss}\right]\left\{\delta^{ess}\right\} \quad (10)$$

$[B^{ss}]$ is a partitioned matrix presented as

$$\left[B^{ss}\right] = \left[\left[B_1^{ss}\right]\left[B_2^{ss}\right]\left[B_3^{ss}\right]\right] \quad (11)$$

Where:

$$\left[B_1^{ss}\right] = \begin{bmatrix} -\dfrac{\partial^2 N_{x1}}{\partial x^2} & -\dfrac{\partial^2 N_{y1}}{\partial x^2} \\ -\dfrac{\partial^2 N_{x1}}{\partial y^2} & -\dfrac{\partial^2 N_{y1}}{\partial y^2} \\ 2\dfrac{\partial^2 N_{x1}}{\partial x \partial y} & 2\dfrac{\partial^2 N_{y1}}{\partial x \partial y} \end{bmatrix} \quad (12)$$

$[B_2^{ss}]$ and $[B_3^{ss}]$ involve terms in Nx_2, Ny_2 and Nx_3, Ny_3, respectively and the $[B^{ss}]$ will thus be established by carrying out the required differentiation of the shape function as indicated in (12) above.

Stress, strain and nodal displacements are related by a 6×6 -stiffness matrix for the simply supported element as

$$\left[K^{ess}\right] = \iint \left[B^{ss}\right]^T [D] \left[B^{ss}\right] dxdy \quad (13)$$

By appropriate substitutions, the slopes at any point on the simply supported element can be expressed in terms of the total slopes and lateral displacements at that point as

$$\begin{aligned} \theta_{x1}^{ss} &= \theta_{x1} + \frac{1}{2\Delta}\left(w_1c_1 + w_2c_2 + w_3c_3\right) \\ \theta_{y1}^{ss} &= \theta_{y1} + \frac{1}{2\Delta}\left(w_1b_1 + w_2b_2 + w_3b_3\right) \end{aligned} \quad (14)$$

A complete set of slope equation for all nodes is related to a transformation matrix as

$$\left\{\delta^{ess}\right\} = \begin{Bmatrix} \theta_{x1}^{ss} \\ \theta_{y1}^{ss} \\ \theta_{x2}^{ss} \\ \theta_{y2}^{ss} \\ \theta_{x3}^{ss} \\ \theta_{y3}^{ss} \end{Bmatrix} = \frac{1}{2\Delta}\left[\begin{array}{ccc|ccc|ccc} 2\Delta & 0 & c_1 & 0 & 0 & c_2 & 0 & 0 & c_3 \\ 0 & 2\Delta & -b_1 & 0 & 0 & -b_2 & 0 & 0 & -b_3 \\ 0 & 0 & c_1 & 2\Delta & 0 & c_2 & 0 & 0 & c_3 \\ 0 & 0 & -b_1 & 0 & 2\Delta & -b_2 & 0 & 0 & -b_3 \\ 0 & 0 & c_1 & 0 & 0 & c_2 & 2\Delta & 0 & c_3 \\ 0 & 0 & -b_1 & 0 & 0 & -b_2 & 0 & 2\Delta & -b_3 \end{array}\right] \begin{Bmatrix} \theta_{x1} \\ \theta_{y1} \\ w_1 \\ \hline \theta_{x2} \\ \theta_{y2} \\ w_2 \\ \hline \theta_{x3} \\ \theta_{y3} \\ w_3 \end{Bmatrix} \quad (15)$$

By virtual work principle, the element nodal displacements and forces in the simply supported case is related by a 6×6 –stiffness matrix $[K^{ess}]$ as

$$\left\{F^{ess}\right\} = \left[K^{ess}\right]\left\{\delta^{ess}\right\} \quad (16)$$

No curvatures are set up during rigid body movements and thus elemental force is related to the corresponding displacement by appropriate substitution as

$$\{F^e\} = [T]^T [K^{ess}][T]\{\delta^e\} \quad (17)$$

The required nine degree of freedom stiffness matrix for a triangular element is obtained as

$$\{K^e\} = [T]^T [K^{ess}][T] \quad (18)$$

Consequently, the stress-displacement matrixes are internal moments obtained as

$$[H] = [D][B^{ss}][T]\{\delta^e\}$$

MODEL APPLICATION TECHNIQUE

The objective of the analysis is the determination of the unknown nodal displacement from where element stresses can be determined using the stress matrix for an encastre plate of a varying thickness profile. A finite element solution thus involves calculating the stiffness matrix for every element in the idealized structure and then assembling the overall structural stiffness matrix, [K] for the complete structure.

The finite element algorithm was developed using the Visual Basic version 6.0 programming language employing triangular elements throughout (Ibekwe, 2006). The analysis was carried out using input data, which fully described the idealized structure and its loading and boundary conditions built up in various subroutines.

Node numbering: The first step in the simulation process was to choose a suitable coordinate and node numbering system for the idealization. The convention of numbering the nodes of each element in a counter-clockwise manner was adopted. The generation of element nodal connectivity (mesh) proceeded automatically by use of suitable declarations (program instructions). Furthermore, each node was uniquely defined with respect to a reference coordinate (x, y) at (0, 0).

In order to keep the 'band width' of the overall stiffness matrix as small as possible (i. e. to achieve a narrow, dense, diagonal band), the nodes were numbered such that the maximum difference between node numbers was kept as small as possible. Node numbers start from zero at the reference coordinate to n along the y-axis and continue from n + 1 at the base of the next discretized column or vice versa.

Input data: The source code provided input data specifying the geometry of the idealized structure (length, width and thickness), its material properties and loading and support conditions. Controls such as total number of elements, n, number of rows, n_r or columns n_c, yielding automatically the total number of nodes corresponding to n elements, were imputed to enable the main routine ascertain how much storage will be required for each individual analysis. An interactive interface containing the input information for the analysis is presented in Appendix 1.

In order to successfully develop an algorithm for this research from the established Finite Element derivation procedure (Zienkiewicz, 1971) the continuum was split into rows and columns, with the grid like intersections being the nodal points. As such, the nodes are considered as being arranged either longitudinally along the plate length or transversely along its width in line with the pattern established for node numbering.

Of special interest is the arbitrary natural mode of geometric variation as a result of thickness loses due to corrosion. The thickness of corroded sections, t_o, were approximately taken as factors of the original plate thickness, t measured along corresponding rows on the discretized plate continuum. As opposed to the general case of constant cross section, where thickness is factored from the overall structural stiffness matrix, [K], the thickness factor here is built into the individual element stiffness matrix $[K^e]$ before the assemblage is carried out.

The novel concept considered was to discretize the plate continuum, idealized as rows and columns and physically taking thickness gauging in a random manner along the row length, which as is often encountered in engineering measurements, are susceptible to errors. The stochastic nature of corroded thickness variation was then approximated to a polynomial of degree three and the method of least squares (Uhunmwangho, 1997) used to fit a unique curve through the given data points, such that nodal thicknesses could be determined along the length, on a row by row or along the width on a column by column basis.

To fit an nth degree polynomial through a given set of data points (x_i, y_i), given that

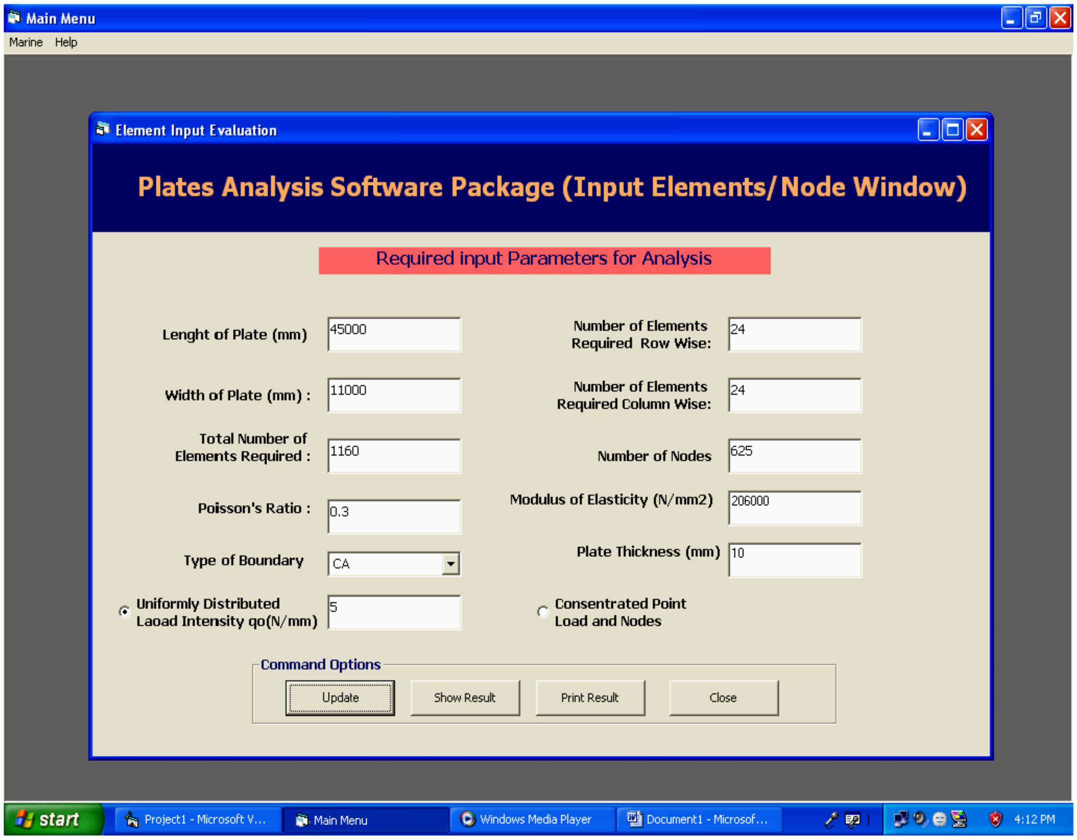
$$y_0 = a_0 x_i^0 + a_1 x_i + a_2 x_i^2 + a_3 x_i^3 + ... + a_n x_i^n \quad (20)$$

Such that

$$f(x_1) = t_1, f(x_2) = t_2, ..., f(x_{n-1}) = t_{n-1}, f(x_n) = t_n$$

along the longitudinal or transverse axes.

The sum of the squares of the error or deviation e_i, is given by



Appendix 1: Finite element interactive interface

$$s(a_0, a_1, ...a_n) = \sum_{i=1}^{i=n}(e_i)^2 = \sum(y_i - a_0 - a_1x_i - a_2x_i^2 - .... - a_nx_i^n)^2 \tag{21}$$

The function is a minimum when

$$\begin{aligned}
\frac{\partial s}{\partial a_0} &= 0 = \sum_{i=1}^{i=n} 2(y_i - a_0x_i^0 - a_1x_i - a_2x_i^2 - a_3x_i^3 - .... - a_nx_i^n)x_i^0 \\
\frac{\partial s}{\partial a_1} &= 0 = \sum_{i=1}^{i=n} 2(y_i - a_0x_i^0 - a_1x_i - a_2x_i^2 - a_3x_i^3 - .... - a_nx_i^n)x_i \\
&\vdots \\
\frac{\partial s}{\partial a_n} &= 0 = \sum_{i=1}^{i=n} 2(y_i - a_0x_i^0 - a_1x_i - a_2x_i^2 - a_3x_i^3 - .... - a_nx_i^n)x_i^n
\end{aligned} \tag{22}$$

Algebraically rearranging the above will give in matrix form

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 & \sum x_i^3 & . & . & \sum x_i^n \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & . & . & \sum x_i^{n+1} \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & . & . & . & \sum x_i^{n+2} \\ \sum x_i^3 & \sum x_i^4 & \sum x_i^5 & . & . & . & \sum x_i^{n+3} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ \sum x_i^n & \sum x_i^{n+1} & \sum x_i^{n+2} & \sum x_i^{n+3} & . & . & \sum x_i^{2n} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ . \\ . \\ a_n \end{bmatrix} = \begin{bmatrix} \sum t_i \\ \sum t_i x_i \\ \sum t_i x_i^2 \\ \sum t_i x_i^3 \\ . \\ . \\ \sum t_i x_i^n \end{bmatrix} \quad (23)$$

The above matrix equation is general from which a unique curve for each row or column is fitted and nodal thicknesses along the longitudinal or transverse axis, respectively determined. As the finite element theory is defined for constant cross section, an equivalent thickness for each triangular element was approximated as a simple average of its boundary nodal thicknesses as

$$t_e = \frac{1}{3}\sum_{i=1}^{3} t_{node\,i} \quad (24)$$

Similarly, to determine the consistent load vector, which is the sum of the contributions of the various loading considered (Rao, 1989; Pepper and Henrich, 1992), the load was first evaluated for a simply supported case $\{F^{ess}\}$ and then for the element $\{K^e\}$ by employing the transformation matrix [T].

Element stiffness: Handling of the strain matrix whose second order partial derivative involves terms in x and y variables were the main difficulty encountered at this point. Numerical differentiation could not be employed as this is usually applied at specific points to give definite values of the derivatives at those points. Algebraic expressions (Ibekwe, 2006) in terms of the nodal coordinates for these derivatives were generated from Eq. 11 in the form a + bx +cy and directly employed accordingly for each term of the $[B^{ss}]$ matrix to obtain the product $[B^{ss}]^T[D][B^{ss}]$ in the same algebraic form as above.

A simple algorithm for normal analytical integration was employed to handle the double matrix integration of the above product. The integration was carried out for the different variables x and y in turns within the element boundaries to obtain the $[K^{ess}]$ matrix. The required 9×9 element stiffness $[K^e]$ was then obtained by the matrix multiplication of the transformation matrix [T] together with its transpose.

Assembly of the overall structural stiffness: The matrix equations that describe the individual Finite Elements are essential, but not in themselves sufficient to solve the problem. Thus, the overall Structural Stiffness Matrix, which is made up of the sums of all the contributions of all individual Element Stiffness Matrices $[K^e]$, was determined using a partitioning method of matrix addition. Due attention was paid and special declarations made to ensure that the forces and displacements calculated for all individual elements were inserted into the appropriate locations to which the actual overall stiffness term relate.

The overall stiffness matrix, which is symmetrical of size n×n gives an indication of the total number of simultaneous algebraic equations to be solved to obtain the required solution. In this case of 3-degrees of freedom per node, the number of equations will be 3 times the number of node, resulting from the total number of element ch.

Application of boundary conditions: Once the overall structural stiffness matrix had been established, the main task became that of determining the corresponding nodal displacements for every defined node making up the continuum. A simplifying principle of eliminating the effect of the influence of the prescribed zero displacements by deleting their corresponding rows and columns, was employed. The reduced matrix was then inverted. Finally, the product of the inverse stiffness matrix and the consistent load vector for the entire plate was solved using the Gauss elimination iterative method (Ross, 1982) to obtain the solution for nodal displacements.

Stress-displacement solution: The required 3×1 element stresses M_x, M_y and M_{xy} was obtained by premultiplying each of the 9×1 element nodal displacement vector by the transformation matrix according to Eq. 18.

Plotting: In line with the pattern established for node numbering, results of nodal parameters obtained were imported into an excel spreadsheet and their graphical representation generated using the Microsoft Excel chart wizard.

RESULTS AND DISCUSSION

A validation of the finite element software by a comparative analysis of results with those obtained experimentally or classically from other analytical formulations was carried out.

Maximum deflection for a 5m length, 0.01m thickness encastre plate of aspect ratio (b/a) 1.1, uniformly loaded by a 5KN/m load from the Navier solution is given as

$$w_{max} = \frac{0.00150 q_o a4}{D}$$

And obtained as 2.457m. As a measure of validity, the optimal solution (using 400 elements) from the finite element software[19] for the same aspect ratio (5.0 m×5.0 mx 0.01m plate) under the same load and boundary conditions gave a maximum of 2.532 m representing an error margin of 1.031% from the classical result.

The analysis as is usual for plate flexure also shows results of angular deflection ((θ_x, θ_y) and reactions (M_x, M_y, M_{xy}), representing both bending and twisting moments. Further analysis extends to the simulation of a plate whose thickness profile is varying in an unpredictive manner. This could possibly have been as a result of an intentional variation of thickness to meet specific design considerations, or from natural wear and tear including cavitations of submerged structures and/or by an unintentional attack through reactions with the environment leading to metal surface deterioration with time.

CONCLUSION

Following the analysis and results obtained, it can thus be stated that the assumption of uniform structural responses common in homogeneous plates is theoretically no longer valid due to metal surface losses which have been modelled to behave as a polynomial of degree three.

As a general consequence of this, progressive loss of thickness is to be monitored by regular thickness gauging and corroded members refitted at the earliest possible time to reduce the risk of failure.

The assumptions made in the development of this program for plate of a varying thickness profile, considered the plate to be only of a rectangular configuration and built-in on all sides. This regrettably is one major limitation of the software.

Furthermore, it cannot be claimed that the developed visual basic program is as sophisticated as already existing industrial finite element software such as STAAD, NASTRAND etc. However, an incorporation of the concept of variable thickness proposed in this study will be particularly useful in large scale finite element programming, resulting in improved versatility of these tools in structural analysis of plates.

Notation:

a,b	=	Plate dimension
x, y	=	Nod al coordinates
a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3,	=	Relation between nodal coordinates
L_1, L_2, L_3	=	Area coordinates
A_1, A_2, A_3,	=	Area of elemental sections
Δ	=	Total element area
M_x, M_y, M_{xy}	=	Moment per unit length
q	=	Load distribution
E	=	Elastic tensile modulus
ν	=	Poisson's ratio
D, [D]	=	Plate flexural rigidity and rigidity matrix, respectively
t_0, t, t_e	=	Original, measured and equivalent element plate thicknesses, respectively
w	=	Deflection
w^{rb}, w^{ss}	=	Rigid body and simply supported components of deflection, respectively
θ_x, θ_y	=	Rotations about x and y axes
ε	=	Curvature (strain)
N_x, N_y,	=	Shape functions
$\{\delta^e\}$,$\{\delta\}$	=	Element and structural displacement vectors, respectively
$\{F^e\}$, $\{F\}$	=	Element and structural force vectors, respectively
$[K^{ess}]$	=	Stiffness matrix for simply supported component
$[K^e]$,$[K]$	=	Element and overall structural stiffness matrices, respectively
[T]	=	Transformation matrix
$[B^{ss}]$	=	Matrix relating element strain to element nodal displacements
[H]	=	Stress-displacement matrix

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