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Application of Fuzzy Goal and Additive Fuzzy Goal Programming Approaches to Distributor Selection in Supply Chain

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Abstract: A supply chain, from an operations perspective, has three components: Sourcing or procurement, manufacturing and distribution and inventory disposal. The focus of this study is on decision making in the distributor selection and quota allocation, which is an important part of the supply chain of many firms. Such decision makings are may greatly affect a firm's ability to compete in the market place as they account for large portion of a product's production cost and may involve long-term contracts. In this study, the fuzzy goal and fuzzy additive goal programming approaches are applied for solving the distributor selection problem with multiple objectives, in which some of the parameters are fuzzy in nature. Because of all objectives and constraints are linear, the distributor selection problem has been formulated by using linear member ship function as a fuzzy goal and additive fuzzy goal programs. The formulated two programs are solved by using LINDO software. The proposed approaches have the capability to handle realistic situations in a fuzzy environment and provide a better decision tools for the distributor selection and quota allocation in a supply chain. The formulated fuzzy goal and additive fuzzy goal programs can be used to solve the real life problems.

Key words: Fuzzy goal, additive fuzzy, distributor, selection, supply chain

INTRODUCTION

In a supply chain, the flow of goods between a supplier and a customer passes through several echelons and each echelon may consist of many facilities. A typical simple supply chain is shown in Fig. 1. In this study, mainly concentrated on the distributor performance and the quota allocation to distributors and its selection. The distributors selection problem deals with issues related to the selection of right distributor and their quota allocations. In designing a supply chain, a decision maker must consider decisions regarding the selection of the right distributors and their quota allocation. The choice of the right distributor is a crucial decision with wide ranging implications in a supply chain. Hence, strategic partnership with better performing distributors should be integrated within the supply chain for improving the performance in many directions including reducing costs by eliminating wastages, continuously improving quality to achieve zero defects, improving flexibility to meet the needs of the end-customers, reducing lead time at different stages of the supply chain, etc. The distributor selection is a complex problem due to several reasons. By nature, the distributor selection in supply chain is a multi-criterion decision making problem. Individual distributor may perform differently on different criteria. A supply chain decision faces many constraints, some of these are related to distributors' internal policy and externally imposed system requirements. In such decision making situations, high degree of fuzziness and uncertainties are involved in the data set. Fuzzy set theory provides a framework for handling the uncertainties of this type (Kumar *et al*., 2003).

In this study, a fuzzy goal and additive fuzzy goal programming approaches are used to solve the multi-objective-optimization problems for quota allocation to the distributors in supply chain. Since crisp set assign a value of either 1 or 0. Whereas in fuzzy set is not assigned

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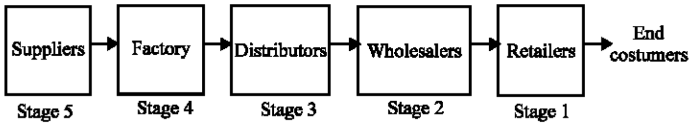

Fig 1: Supply chain

value of either 1 or 0, but the value of any set lies in between 1 and 0. For this a function can be generalized such that the value assigned to the element of the universe set fall within specified range and indicated member ship grade of this element in the set. Such a function is call a fuzzy membership function and set defined by it is a fuzzy set. Various types of membership functions can be used in fuzzy goal programming problem and its application such as a linear membership function, a tangent type of a membership function, an interval linear membership function, an exponential membership function, inverse tangent membership function, logistic type of membership function, concave piecewise linear membership function and piecewise linear membership function. As a tangent type of a membership function, an exponential membership function and hyperbolic membership function are non-linear function, a fuzzy mathematical programming defined with a non-linear membership function results in a non-linear programming. Usually a linear membership function is employed for linear programming in order to avoid non-linearity(Vasant, 2004). Therefore, in this study a linear membership function is employed to for objectives, which is having fuzziness.

The important four fuzzy goals consider in this study are maximization of the sales revenue, minimization of the transportation cost, minimization of the defective items rejected and minimization of the late deliveries with subjected constrain such as maximum capacity of the distributors, maximum budget allocated to distributors, maximum flexibility of the distributors and maximum sales value of the distributors. First the problem is formulated as multi-objective linear programming and then it is reformulated as fuggy goal and additive goal programs by using variable λ. Then formulated fuggy goal and additive fuggy goal programs are solved by using the LINDO software.

MULTI-OBJECTIVE DISTRIBUTOR MODEL

The four distributors and four main objectives are considered for distribution model. The objectives are maximizing the sales revenue (Z_1), minimizing the transportation cost (Z_2), minimizing the defective items rejected (Z_3) and minimizing the late deliveries (Z_4). Besides the constraints considered for formulation of the problem are maximum selling capacity of the distributors, maximum budget allocation to distributors, flexibility of the distributors, sales value of the distributors. With above information the problem is formulated as multi objective linear program as shown below (Kumar *et al*., 2003).

Decision variable: x_i order quantity from the distributors i, where I = 1,2.….N

Parameters:

D. = Aggregate demand of the item over a fixed planning period.

N = Number of distributors competing for selection.

p_i = Price of a unit item at the distributors i.

t_i = Transportation cost of a unit item of the ordered quantity xi for the distributors i.

ld_i = Percentage of the late delivered units by the manufacturers to distributors i.

C_i = Maximum quantity can take with distributors I.

B_i = Budget allocated to each distributors.

d_i = Percentage of the rejected units delivered by the distributors i.

F_i = Supplier quota flexibility for supplier i.

F = Lower bound of flexibility in supply quota that a supplier should have.

R_i = Supplier rating value for supplier i.

PV = Lower bound to total purchasing value that a supplier should have

Min. $Z = (Z_1, Z_2, Z_3, Z_4)$

Subjected to

$\Sigma x_i \leq D$, D is varying form D min. to D max. (1)

$x_i \leq C_i$, C_i is varying form C_i min. to C_i max (2)

$x_i \leq B_i$ (3)

$\Sigma F_i x_i \geq F$, F is varying form F min. to F max (4)

$\Sigma R_i x_i \geq P$, P is varying form P min. to P max (5)

$x_i \geq 0$ and (6)

$Z_1 = \Sigma p_i x_i$

$Z_2 = \Sigma t_i x_i$

$Z_3 \; \Sigma d_i x_i$

$Z_4 = \Sigma ld_i x_i$

FUZZY MULTI-OBJECTIVE MODEL

Consider the fowling linear multi-objective model,

$$\text{Opt } Z = CX$$

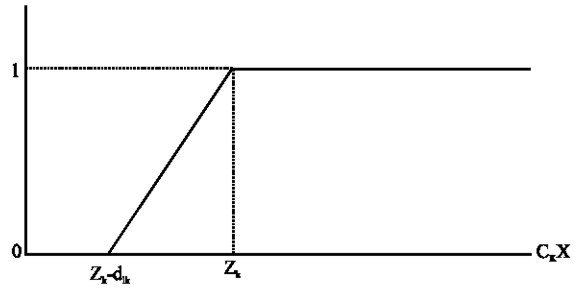

Fig. 2: Member ship function for maximization of fuzzy goal

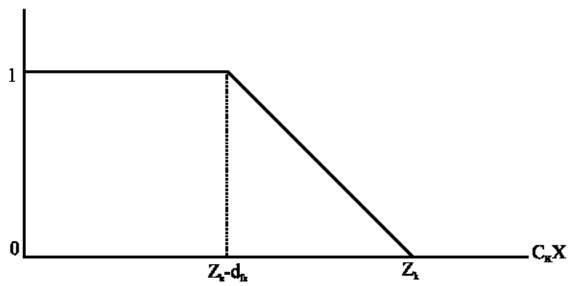

Fig. 3: Member ship function for minimization of fuzzy goal

$$\text{s.t. } AX \leq b$$

Where $Z = (z_1, z_2, \ldots\ldots z_k)$ is the vector of objectives, C is a K*N of constants, X is a an N*1 vector of the decision variables, A is an M*N matrix of constants and b is an M*1 vector of constants. This model can be applied to solve many real world problems. Fuzzy set theory can be useful in order to increase the model realism (Slim *et al*., 2004).To solve above problem a linear membership function can be used for each goal $\mu_{1k}(C_kX)$, Where

$$\mu_{1k}(C_kX) = \begin{cases} 1 & \text{if } CkX >= Z_k \\ 1 - \dfrac{(Z_k - C_kX)}{d_{lk}} & \text{if } Z_k - dl_k <= CkX <= Z_k \\ 0 & \text{if } CkX <= Z_k - dl_k \end{cases}$$

And another linear membership function $\mu_{2i}(a_iX)$, for the i^{th} constraint in the system constraints $AX \leq b$, where

$$\mu_{2i}(aiX)) = \begin{cases} 1 & \text{if } aiX <= bk \\ 1 - \dfrac{(a_iX - b_k)}{d_{2i}} & \text{if } bk <= a_iX <= b_i + d_{2i} \\ 0 & \text{if } a_iX >= b_i + d_{2i} \end{cases}$$

These membership functions are illustrated in Fig. 2-4 respectively. Where d_{1k} ($k = 1,2,\ldots.K$) and d_{2i} ($I = 1,2,\ldots.M$)

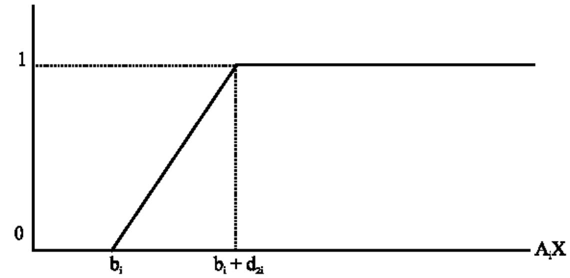

Fig. 4: Member ship function of fuzzy constraints

are chosen constants of admissible violations and a_i is the i^{th} row of matrix A. Thus to build a fuzzy multi-objective programming model, the decision maker may establish aspiration levels for values of the objective function to be minimized and maximized respectively, as well as each of the constraint may be modeled as fuzzy set by specific member ship functions. Hence the conventional distinction between objectives and constraints no longer applies in the FLP models. $\mu_{1k}(C_kX)$ and $\mu_{2i}(a_iX)$ denote the degree of the membership of goals and constraints respectively. The degree of the membership of goals and constraints express the satisfaction of the decision maker with the solution. So, values of the membership functions must be maximized (Selim *et al*., 2004).

In one of the fuzzy set theorems, the membership function of the intersection of any two(or more) sets is the minimum membership function of these sets. After eliciting the linear membership functions and by applying this theorem, objective function of the multi-objective linear programming model incorporating the fuzzy goals and fuzzy constraints can be formulated as follows (Selim *et al*., 2004).

$$\max_x \min(\mu_{11}(C_1X), \ldots\ldots, \mu_{1k}(C_kX), \mu_{21}(a_1X), \ldots\ldots, \mu_{2i}(a_iX))$$

By introducing the auxiliary variable λ, this problem can be equivalently transformed as (Selim *et al*., 2004).

$$\max \lambda \qquad \mu_{1k}(C_kX) \geq \lambda \quad k = 1,2,\ldots.K$$
$$\mu_{2i}(a_iX) \geq \lambda \qquad I = 1,2,\ldots.M$$

According to above descriptions fuzzy linear program can be rewritten as following

$$\max \lambda$$
$$\lambda \leq 1\text{-}(Z_k\text{-}C_kX)/\, d_{1k} \qquad k=1,2,\ldots.K$$
$$\lambda \leq 1\text{-}(a_iX\text{-}b_i)\, /\, d_{2k} \qquad i=1,2,\ldots.M$$
$$0 \leq \lambda \leq 1 \text{ and } x \geq 0$$

FUZZY GOAL PROGRAMMING (FGP)

Fuzzy Goal Programming (FGP) is one of the most powerful multi-objective decision making approach. If there no priorities and also no relative importance assigned to objectives, formulation of the FGP model is same as in general Fuzzy Linear Programming (FLP) model. The main difference between FGP and FLP is that FLP uses the definite intervals determined from solution of the LP models and so the solution does not change from decision maker to decision maker, whereas in FGP, aspiration levels are specified by decision maker and reflect relative flexibility (Selim *et al.*, 2004).

ADDITIVE FUZZY GOAL PROGRAMMING (FAGP)

Since some goals may be more important than others, so different priority levels are used in FAGP formulations frequently. In order to reflect the relative importance of the goal, the weighted average of membership function values can be used. Weights in the weighted fuzzy additive model reveal the relative importance of the fuzzy goals (Selim *et al.*, 2004).

AN ILLUSTRATION OF QUOTA ALLOCATION TO DISTRIBUTORS

The effectiveness of the multi-objective linear approach for the quota allocation to the distributors is presented in this study. The distributors source data of the illustrative example is prepared based on data available in Kumar *et al.* (2003), where the data is presented for the supplier selection problem. The data relates to a realistic situation of a manufacturing sector dealing with any kind parts. The adopted situation can easily be extended to any other industry (Kumar *et al.*, 2003). The four objectives, viz. maximizing the net sales revenues, minimizing the transportation cost, minimizing the net defective items rejected and minimizing the net late deliveries have been considered subject to few practical constraints regarding aggregate demand of the items, maximum distributors' capacity limitations, distributors' budget allocations, maximum flexibility of supply chain maximum sales value rating of supply chain.

Distributors source data of the illustrative example shown in Table 1 represent the data source for the sale price quoted (p_i in rupees per unit); transportation cost (t_i in rupees per unit); the percentage defective items rejections d_i; the percentage of late deliveries ld_i; maximum distributors' capacity limitations C_i units; the budget allocations for the suppliers B_i; Distributors' quota flexibility F_i and distributors sales value rating R_i on a scale of 0-1. The maximum value of flexibility in distributors' quota and maximum total purchase value of distributed items are policy decisions and depend on the demand. The maximum value of flexibility in suppliers' quota is given as $F = F_0 D$ and the maximum total purchase value of supplied items is given as PV =RD: If overall maximum flexibility is 0.95 on the scale of 0-1, the overall maximum distributors' Rating (R) is 0.90 on the scale of 0-1 and the mean Demand (D)of normal distribution is 500, then the maximum value of flexibility in distributors' quota (F) and the maximum total purchase value of distributed items (PV) are 475 and 450, respectively. Then the formulated multi-objective linear problem for quota allocation to distributors can be written as from Table 1.

$$\text{Maximize } Z_1 = 100x_1+300x_2+250x_3+350x_4$$
$$\text{Minimize } Z_2 = 15x_1+10x_2+5x_3+20x_4$$
$$\text{Minimize } Z_3 = 0.03x_1 +0.04 x_2 + 0.02 x_3 + 0.07 x_4$$
$$\text{Minimize } Z_4 = 0.02x_1+0.03x_2+0.09x_3+0.05x_4$$

Subjected to

$$x_1 + x_2 + x_3 + x_4 = 500$$
$$x_1 <= 200$$
$$x_2 <= 235$$
$$x_3 <= 225$$
$$x_4 <= 220$$
$$100x_1 <= 20000$$
$$300x_2 <= 70500$$
$$250x_3 <= 56250$$
$$350x_4 <= 77000$$
$$0.996x_1 + 0.999 x_2 + 0.998 x_3 + 0.997 x_4 >= 475$$
$$0.86x_1 +0.92 x_2 + 0.98 x_3 + 0.88 x_4 >= 450$$
$$x_1 , x_2 ,x_3,x_4 \geq 0$$

FORMULATION OF FUZZY GOAL AND ADDITIVE FUZZY GOAL PROGRAMS

Fuzzy goal and additive goal programs for the problem of quota allocation to distributors is formulated by using linear membership function (Fig. 5).

Table 1: Distributors source data of the illustrative example

Distributors No.	p_i Rs.	t_i Rs	d_i (%)	ld_i (%)	C_i Max. Units	B_i Rs	Fi (%)	R_i (%)
1	100	15	3	2	200	20000	99.6	86
2	300	10	4	3	235	70500	99.9	92
3	250	5	2	9	225	56250	99.8	98
4	350	20	7	5	220	77000	99.7	88

Table 2: LPP results of the individual objectives for maximum bound of constraints

S. No	Objective	Min. bound	LPP results	Max bound	Fuzzy range
1	Sales revenue	8750	158750.0	308750.0	300000
2	Transportation cost	200	1895.000=1900	3600	3400
3	Defectives rejected	2	13.5=14	26	24
4	Late deliveries	2	16.14000=17	32	30

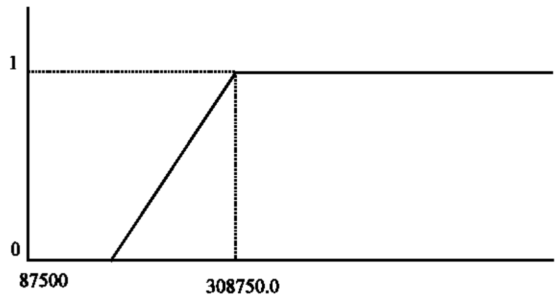

Fig. 5: Linear member ship function for sales revenue (Maximization)

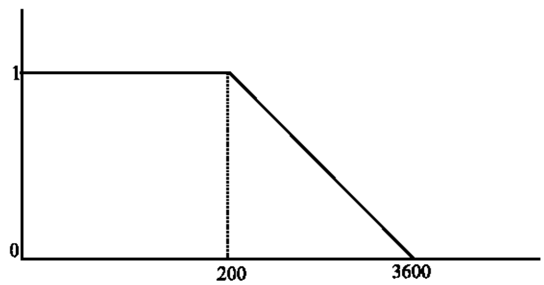

Fig. 6: Linear member ship function for transportation cost (Minimization)

The Linear membership function for other objectives like minimization of defective items rejected and minimization of late deliveries is same as Fig. 6.

LPP results of the individual objectives are tabulated in the Table 2. In case of goal programming, selection of fuzzy range is most important. The accuracy of the final results depends upon fuzzy range. Selected fuzzy ranges for each objective are shown in Table 2. By using above results and linear membership function, the formulated fuzzy goal and additive fuzzy goal programs are given below.

Fuzzy goal programming:

Max λ

Subjected to

$\lambda <= (Z_1\text{-}8750)/ 300000$

$\lambda <= (3600\text{-}Z_2)/ 3400$

$\lambda <= (26\text{-}Z_2)/ 24$

$\lambda <= (32\text{-}Z_2)/ 30$

$x_1 + x_2 + x_3 + x_4 = 500$

$x_1 <=200$

$x_2 <= 235$

$x_3 <= 225$

$x_4 <=220$

$100x_1 <= 20000$

$300x_2 <= 70500$

$250x_3 <= 56250$

$350x_4 <= 77000$

$0.996x_1 + 0.999 x_2 + 0.998 x_3 + 0.997 x_4 >= 475$

$0.86x_1 +0.92 x_2 + 0.98 x_3 + 0.88 x_4 >= 450$

$x_1, x_2, x_3 , x_4 >=0$

$0 < = \lambda <=1$

Additive fuzzy goal programming:

Max $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$

Subjected to

$\lambda_1 <= (Z_1\text{-}8750)/ 300000$

$\lambda_2 <= (3600\text{-}Z_2)/ 3400$

$\lambda_3 <= (26\text{-}Z_2)/ 24$

$\lambda_4 <= (32\text{-}Z_2)/ 30$

$x_1 + x_2 + x_3 + x_4 = 500$

$x_1 <=200$

$x_2 <= 235$

$x_3 <= 225$

$x_4 <=220$

$100x_1 <= 20000$

$300x_2 <= 70500$

$250x_3 <= 56250$

$350x_4 <= 77000$

$0.996x_1 + 0.999 x_2 + 0.998 x_3 + 0.997 x_4 >= 475$

$0.86x_1 +0.92 x_2 + 0.98 x_3 + 0.88 x_4 >= 450$

$x_1, x_2, x_3 , x_4 >=0$

$0 < = \lambda_1 <=1, 0 < = \lambda_2 <=1, 0 < = \lambda_3 <=1, 0 < = \lambda_4 <=1$

RESULTS AND DISCUSSION

Formulated above fuzzy goal and additive fuzzy goal programs are solved by using LINDO software, which is available in the Internet. The corresponding results are tabulate in Table 3 and 4. In Table 3 the quantity allocated to four distributors is tabulated. In the Table 4, optimized values of all four objectives are tabulated. The effect of minimized defective items rejected and minimization of late deliveries is nominal. But effect of maximization sales revenue and minimization of transportation cost is significant. So that in additive goal

Table 3: Allocation of items for four distributors by Fuzzy goal programming

	X_1	X_2	X_3	X_4
Fuzzy goal programming	000	212	142	146
Additive of fuzzy goal programming	000	70	210	220

Table 4: Optimized values of the four objectives

Objectives	---------------------Fuggy goal programming---------------------		---------------Additive fuggy goal programming------------	
Sales revenue	λ =0.183529	150200	λ_1= 0.4725	150500
Transportation cost	λ=0.183529	2976	λ_2=0.4794	1970
Defective items rejected	λ =0.183529	21.54	λ_3=0.1500	22.4
Late deliveries	λ=0.183529	26.44	λ_4=0.0000	32

programming first weight age is given to the maximization of sales revenue, second weight is given to transportation cost, third weight age is given to minimization of defective items rejected and final weight age is given to minimization of the late deliveries. Due to weight ages given to objectives, the sales revenue is maximized to more value than fuzzy goal programming and also transportation cost is minimized to least value than fuzzy goal programming. Difference of maximized sales revenue and minimized transportation cost is more with additive goal programming than fuzzy goal programming. Hence additive goal program is more effective tool for solving real life problems, where the weight ages are required for the objectives.

CONCLUSION

- Quota allocation to the distributors is an important goal in supply chain management. For this fuzzy goal and additive fuzzy goal programs are successfully formulated for multi objectives and is solved by LINDO software.
- Neglecting the effect of optimized results of defective items rejected and late deliveries, the difference amount of sales revenue and transportation cost is more with additive goal programming than fuzzy goal programming. So that additive goal programming is effective tool to use for any real life problems, where the weight ages are required for the objectives.
- Any commercially available soft wares such as LINDO software can also be used to solve the proposed fuzzy goal and additive fuzzy goal programs for quota allocation to distributors and distributors selection in supply chain.

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