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Trade off Between the Aperture Taper and Spillover Efficiencies of Prime Focus Reflector Antenna

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Abstract: In this study, we use a point source at the focus, so that energy would radiate uniformly in all directions both in magnitude and in phase. The problem is that the energy that is not radiated toward the reflector will be wasted. What we really want is a feed antenna that only radiates toward the reflector and has a phase pattern that appears to radiate from a single point If we can illuminate the whole reflector, then we should be using the whole aperture. However, when we look more closely at the parabolic surface, we find that the focus is farther from the edge of the reflector then from the centre, because a part of the feed radiation misses the reflector; this loss called spillover. Another part of the feed energy is reflected back into the feed and doesn't become part of the main beam; this loss is referred to as feed blockage. Ideally, all areas of the reflector should be illuminated with equal energy from the feed.

Key words: Parabolic dish antenna, aperture efficiency, spillover efficiency, illumination efficiency

INTRODUCTION

To determine the radiation characteristics (e.g. gain, aperture-taper and spillover efficiencies or loss, radiation pattern, etc.) of reflectors antennas, one may use the Bottler's formulas, provided one knows the aperture fields E_a, H_a on the effective aperture projected on the aperture plane. This study is referred to as the aperture-field method[1,2].

Alternatively, the current-distribution method determines the current J_S on the surface of the reflector induced by the incident field from the feed. The two methods yield slightly different, but qualitatively similar, results for the radiation patterns. The aperture fields E_a, H_a and the surface current J_S are determined by geometrical optics considerations based on the assumptions that

- the reflector lies in the radiation zone of the feed antenna and
- the incident field from the feed gets reflected as if the reflector surface is perfectly conducting and locally flat. These assumptions are justified because in practice the size of the reflector and its curvature are much larger than the wavelength λ .

In this study, we use the aperture-field method to determine the efficiencies and the edge illumination for parabolic reflectors antennas with feed at the focus.

PRINCIPLES OF PARABOLIC REFLECTOR ANTENNAS

A typical parabolic reflector, when the feed is positioned at the focus of the parabola, is shown in Fig. 1. A geometrical property of parabolas is that all rays originating from the focus get reflected in a direction parallel to the parabola's axis, that is, the z direction[3,4]. We use the polar and azimuthally angles ø and ÷ to characterize the direction $\hat{R}$ of an incident ray from the feed to the reflector surface. R and h be the lengths of the rays OP and PA. The sum R + h represent the total optical path length from the focus to the aperture plane. This length is constant, independent of ψ and is given by:

$$R + h = 2F \qquad (1)$$

where F is the focal length. The length 2F is the total optical length of the incident and reflected axial rays going from O to the vertex V and back to O.

Therefore, all the rays suffer the same phase delay traveling from the focus to the plane. The spherical wave radiated from the feed gets converted upon reflection into a plane wave. Conversely, for a receiving antenna, an incident plane wave gets converted into a spherical wave converging onto the focus.

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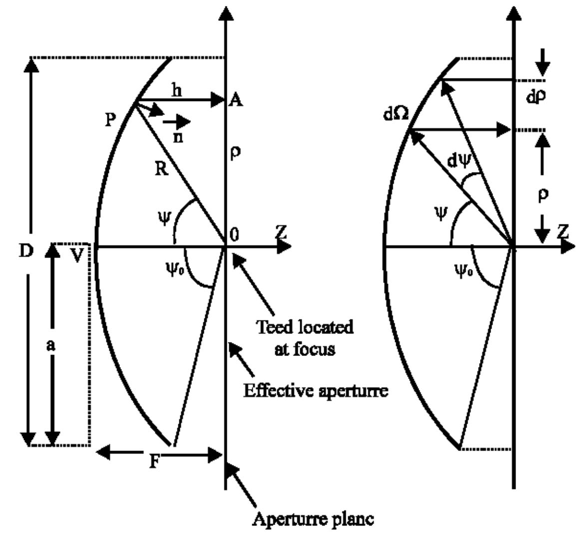

Fig. 1: Geometry of parabolic dish antenna

Since h = Rcos ψ, Equation 1 can be written in the following form, which is the polar representation of the parabolic surface:

$$R + R\cos\psi = 2F \Rightarrow R = \frac{2F}{1+\cos\psi} = \frac{F}{\cos^2(\psi/2)} \tag{2}$$

The radial displacement ñ of the reflected ray on the aperture plane is given by ρ = Rsin ψ. Replacing R from Eq. 2, we find

$$\rho = 2F\frac{\sin\psi}{1+\cos\psi} = 2F\tan\left(\frac{\psi}{2}\right) \tag{3}$$

Similarly, using Eq. 1 we have:

$$F - h = F\frac{1-\cos\psi}{1+\cos\psi} = F\tan^2\left(\frac{\psi}{2}\right) \tag{4}$$

It follows that h and ρ will be related by the equation for a parabola[2]:

$$4F(F-h) = \rho^2 \tag{5}$$

The radiated power from the horn within the solid angle d Ω = sin ψ d ψ d χ, must be equal upon reflection to the power propagating parallel to the z-axis and intercepting the aperture plane through the area dA = ρd ρd d χ, as shown in Fig. 1.

Assuming that U feed (ψ, χ) is the feed antenna's radiation intensity and noting that $|E_\alpha(\rho,\chi)|^2 / 2\eta$ is the power density of the aperture field and η is the characteristic impedance, the power condition reads:

$$\frac{1}{2\eta}\left|E_a(\rho,\chi)\right|^2 dA = U_{feed}(\psi,\chi)\, d\Omega$$
$$\Rightarrow \frac{1}{2\eta}\left|E_a(\rho,\chi)\right|^2 \rho\, d\rho = U_{feed}(\psi,\chi)\sin\psi\, d\psi \tag{6}$$

Where we divided both sides by χ *d* . Differentiating Eq. 3, we have:

$$d\rho = \frac{2F}{1+\cos\psi}d\psi = R\, d\psi \tag{7}$$

Which implies that $\rho d\rho = R^2 \sin\psi d\psi$. Solving Eq. 6 for $|E_\alpha(\rho,\chi)|$ we find:

$$\left|E^a(\rho,\chi)\right| = \frac{1}{R}\sqrt{2\eta U_{feed}(\psi,\chi)} \tag{8}$$

Where we think of $E_\alpha(\rho, \chi)$, as a function of ρ = 2F tan (ψ/2) and χ. Expressing R in terms of ρ, we have R = 2F-h = F + (F-h) = F + ρ^2/4F. Therefore, we may also write:

$$\left|E_a(\rho,\chi)\right| = \frac{4F}{\rho^2+4F^2}\sqrt{2\eta U_{feed}(\psi,\chi)} \tag{9}$$

Thus, the aperture fields get weaker towards the edge of the reflector. A measure of this tapering effect is the edge I llumination, that is, the ratio of the electric field at the edge (ρ = α) and at the center (ρ = 0). Replacing R = 2F/ (1+cosψ) = F/$\cos^2$ (ψ/2) in Eq. 8, we find the edge illumination:

$$\frac{\left|E_a(\alpha,\chi)\right|}{\left|E_a(0,\chi)\right|} = \frac{1+\cos\psi_0}{2}\sqrt{\frac{U_{feed}(\psi_0,\chi)}{U_{feed}(0,\chi)}} \tag{10}$$

Where ψ_0 is the subtended angle of the reflector. The directivity or gain of an aperture is given by:

$$G_a = \frac{4\pi U_{max}}{Pa} \tag{11}$$

where max U_{max} is the maximum of feed antenna's radiation intensity and P_α is the total power through the aperture given in terms of E_α as follows:

$$P_a = \frac{1}{2\eta}\int_A \left|E_a(\rho,\chi)\right|^2 sA$$
$$= \int_0^{\psi_0}\int_0^{2\pi} U_{feed}(\psi,\chi)\sin\psi d\psi d\chi \tag{12}$$

For a reflector antenna, the gain must be defined relative to the total power P_{feed} of the feed antenna, that is,

$$G_{ant} = \frac{4\pi U_{max}}{P_{feed}} = \frac{4\pi U_{max}}{P_a}\frac{P_\alpha}{P_{feed}} = G_\alpha e_{spl} \quad (13)$$

The factor $e_{spl} = P_a/P_{feed}$ is referred to as the spillover efficiency or loss and represents the fraction of the power P_{feed} that actually gets reflected by the reflector antenna[5].

The remaining power from the feed "spills over" the edge of the reflector and is lost. The aperture gain is given in terms of the geometrical area A of the aperture and the aperture-taper and phase-error efficiencies (e_{atl}, e_{pel}) by:

$$G_a = \frac{4\pi A}{\lambda^2} e_{atl}\, e_{pel} \quad (14)$$

It follows that the reflector antenna gain can be written as:

$$G_{ant} = G_a\, e_{spl}\, \frac{4\pi A}{\lambda^2} e_{atl}\, e_{pel}\, e_{spl} \quad (15)$$

The total aperture efficiency is $e_\alpha\ e_{\alpha tl}\ e_{pel}\ e_{spl}$. In practice, additional efficiency or loss factors must be introduced, such as those due to cross polarization or to partial aperture blockage by the feed.

Of all the loss factors, the ATL and SPL are the primary ones that significantly affect the gain. Their tradeoff is captured by the illumination efficiency or loss, defined to be the product of ATL and SPL, $e_{il} = e_{\alpha tl}\, e_{spl}$.

The ATL and SPL may be expressed in terms of the radiation intensity $U_{feed}(\psi, \chi)$.

Using $\rho d\rho = R^2 \sin\psi d\psi = \rho R\, d\psi = 2\, FR_{tan}(\psi/2)\, d\psi$ and Eq. 8, we have:

$$\begin{aligned} |E_a(\rho,\chi)|dA &= \sqrt{2\eta U_{feed}(\psi,\chi)}\frac{1}{R}2FR_{tan}\frac{\psi}{2}d\psi d\chi \\ &= 2F\sqrt{2\eta U_{feed}(\psi,\chi)}\,\tan\frac{\psi}{2}d\psi\chi \\ |E_a(\rho,\chi)|^2 dA &= 2\eta U_{feed}(\psi,\chi)\frac{1}{R^2}R^2\sin\psi\, d\psi\, d\chi \\ &= 2\eta U_{feed}\sin\psi\, d\psi\, d\chi \end{aligned} \quad (16)$$

The aperture area is $A = \pi a^2 = \pi\,(2F)^2 \tan^2(\psi_0/2)$. The aperture taper efficiency or loss, e_{atl} and the spillover efficiency or loss, e_{spl}, as follows:

$$e_{atl} = \frac{\left|\int_A |E_a| dA\right|^2}{A\int_A |E_a|^2 dA} = \frac{(2F)^2\left|\int_A \sqrt{2\eta U_{feed}}\tan\frac{\psi}{2}d\psi d\chi\right|}{\pi(2F)^2\tan^2\left(\frac{\psi_0}{2}\right)\int_A 2\eta U_{feed}\sin\psi\, d\psi\, d\chi} \text{ or,}$$

$$e_{atl} = \frac{1}{\pi}\cot^2\left(\frac{\psi_0}{2}\right)\frac{\left|\int_0^{\psi_0}\int_0^{2\pi}\sqrt{U_{feed}(\psi,\chi)}\tan\frac{\psi}{2}\,d\psi\, d\chi\right|^2}{\int_0^{\psi_0}\int_0^{2\pi}U_{feed}\sin\psi\, d\psi\, d\chi} \quad (17)$$

$$e_{spl} = \frac{P_a}{P_{feed}} = \frac{\int_0^{\psi_0}\int_0^{2\pi}U_{feed}(\psi,\chi)\sin\psi\, d\psi\, d\chi}{\int_0^{\pi}\int_0^{2\pi}U_{feed}(\psi,\chi)\sin\psi\, d\psi\chi} \quad (18)$$

where we replaced P_{feed} by the integral of U_{feed} over all solid angles. It follows that the illumination efficiency $e_{ill} = e_{atl}\, e_{spl}$ will be:

$$e_{atl} = \frac{1}{\pi}\cot^2\left(\frac{\psi_0}{2}\right)\frac{\left|\int_0^{\psi_0}\int_0^{2\pi}\sqrt{U_{feed}(\psi,\chi)}\tan\frac{\psi}{2}\,d\psi\, d\chi\right|^2}{\int_0^{\psi_0}\int_0^{2\pi}U_{feed}(\psi,\chi)\sin\psi\, d\psi\, d\chi} \quad (19)$$

An example of a feed pattern that approximates practical patterns is the following azimuthally symmetric radiation intensity[2]:

$$U_{feed}(\psi,\chi)\begin{cases} U_0\cos^4\psi, & \text{if } 0 \le \psi \le \frac{\pi}{2} \\ 0, & \text{else} \end{cases} \quad (20)$$

Where $U_0 = U_{feed}(\psi_0, 0)$ is the feed antenna's radiation intensity at $\psi = \psi_0$ and $\chi = 0$. For this example, the SPL, ATL and ILL can be computed in closed form:

$$\begin{aligned} e_{atl} &= 40\cot^2\left(\frac{\psi_0}{2}\right)\frac{\left[\sin^4\left(\frac{\psi_0}{2}\right) + \text{In}\left(\cos\left(\frac{\psi_0}{2}\right)\right)\right]^2}{1-\cos^5\psi_0} \\ e_{atl} &= 40\cot^2\left(\frac{\psi_0}{2}\right)\left(\frac{\psi_0}{2}\right)\left[\sin^4\left(\frac{\psi_0}{2}\right) + \text{in}\left(\cos\left(\frac{\psi_0}{2}\right)\right)\right]^2 \end{aligned} \quad (21)$$

The edge illumination is from Eq. 10:

$$\frac{|E_a(\psi_0)|}{|E_a(0)|} = \frac{1+\cos\psi_0}{2}\cos^2\psi_0 \quad (22)$$

RESULTS AND DISCUSSION

Figure 2 shows a plot of Eq. 21 and 22 versus ψ_0. The ATL is a decreasing and the SPL an increasing function of ψ_0 but when $\psi_0 = 90^0$ the SPL

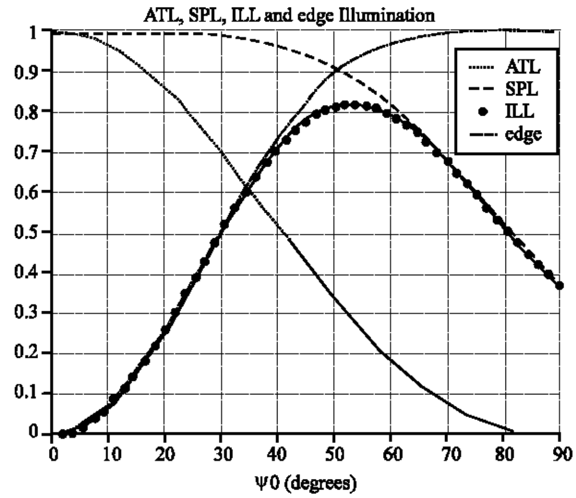

Fig. 2: Trade off between ATL and SPL

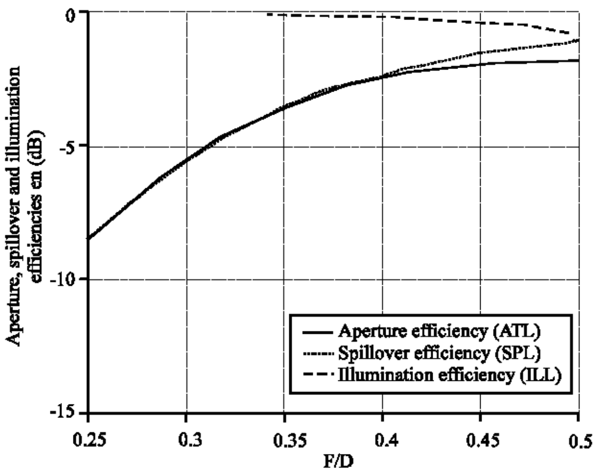

Fig. 3: Influence the F/D ratios of the aperture, spill over and illumination efficiencies in the practical range 0.25-0.5

becomes maximum and equal to one or 0dB and ATL equal ILL and equal 0.3731 or-8.5635 dB, on the other hand the F/D decreases to the minimal value 0.25 this is shown in Fig. 3.

The product $e_{ill} = e_{ant}\, e_{spl}$ reaches the maximum value of 0.8196 at $\psi_0 = 31.53^0$. The corresponding edge illumination is 0.285 or-10.9031 dB. The F/D ratio is $(\psi_0/2)/4 = 0.498$.

Expressing the physical area in terms of the diameter D, we can summarize the gain of a parabolic antenna:

$$G_{ant} = \frac{4\pi A}{\lambda^2} e_a = \left(\frac{\pi D}{\lambda}\right)^2 e_a \quad (23)$$

3 The 3-dB beam width of a reflector antenna with diameter D can be estimated by rule of thumb[6]:

$$\Delta\theta_{3dB} = 70^0 \frac{\lambda}{d} \quad (24)$$

The beam width depends also on the edge illumination. Typically, as the edge attenuation increases, the beam width widens and the side lobes decrease. By studying various reflector sizes, types and feeds, Komen[7] arrived at the following improved approximation for the 3-dB width, which takes into account the edge illumination:

$$\Delta\theta_{3dB} = \left(1.05^0 A_{edge} + 55.95^0\right)\frac{\lambda}{d} \quad (25)$$

where A_{edge} is the edge attenuation in dB, that is, $A_{edge} = -20\log_{10}[|E_\alpha(\psi_0 /) E_\alpha(0)|]$ For example, for A_{edge} = 11 dB, the angle factor becomes 67.5^0.

CONCLUSION

In this study, the calculation of the efficiencies of parabolic reflector antenna gives rise to the rule of thumb that the best tradeoff between ATL and SPL is achieved when the edge illumination is about -11 dB. According to the results, obtained one can say that the aperture efficiency is a decreasing and the spillover efficiency an increasing function of ψ_0 but the F/D is decreasing.

Taking into account other losses (cross polarization efficiency and blockage efficiency), the aperture efficiency of practical parabolic reflectors is typically of the order of 0.55-0.65. Therefore, the value 0.82 for the illumination efficiency is an overestimate. Then the illumination and spillover losses are the principal causes of gain degradation.

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