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Extended the EPQ Inventory Model under Permissible Delay in Payments

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Abstract: The main purpose of this study is to investigate the case where the retailer's unit selling price and the purchasing price per unit are not necessarily equal within the EPQ framework. Under these conditions, we establish the retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory cycle time and optimal order quantity. Three easy-to-use theorems are developed to efficiently determine the optimal inventory policy for the retailer. Some previously published results of other researchers are deduced as special cases. Finally, numerical examples are given to illustrate these theorems obtained in this study.

Key words: EPQ, EOQ, permissible delay in payments, trade credit, inventory

INTRODUCTION

The traditional Economic Order Quantity (EOQ) model assumes that the retailer must be paid for the items as soon as the items are received. However, in practice the supplier will offer the retailer a delay period, that is the trade credit period, in paying for the amount of purchasing cost. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of trade credit period. Therefore, it is clear that the retailer will delay the payment up to the last moment of the permissible period allowed by the supplier. In the real world, the supplier often makes use of this policy to stimulate his/her customer's demand. Recently, several papers have appeared in the literature that treat inventory problems with varying conditions under the consideration of permissible delay in payments. Some of the prominent papers are discussed below.

Goyal[1] established a single-item inventory model under permissible delay in payments. Chung[2] developed an alternative approach to determine the economic order quantity under condition of permissible delay in payments. Aggarwal and Jaggi[3] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Chang *et al.*[4] extended this issue to the varying rate of deterioration. Liao *et al.*[5] and Sarker *et al.*[6] investigated this topic with inflation. Jamal *et al.*[7] and Chang and Dye[8] extended this issue with allowable shortage. Chang *et al.*[9] extended this issue with linear trend demand. Chen and Chuang[10] investigated light buyer's inventory policy under trade credit by the concept of discounted cash flow. Hwang and Shinn[11] modeled an inventory system for retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payment. Jamal *et al.*[12] and Sarker *et al.*[13] addressed the optimal payment time under permissible delay in payment with deterioration. Teng[14] assumed that the selling price not equal to the purchasing price to modify the Goyal's model[1]. Chung *et al.*[15] and Chung and Huang[16] discussed this issue under the selling price not equal to the purchasing price and different payment rule. Chang *et al.*[17] investigated the credit term that supplier provides a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. Shinn and Hwang[18] determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer's order size and also the demand rate is a function of the selling price. Chung and Huang[19] extended this problem within the EPQ framework and developed an efficient procedure to determine the retailer's optimal ordering policy. Huang[20] extended this issue under two levels of trade credit and developed an efficient solution procedure to determine the optimal lot-sizing policy of the retailer. Huang and Chung[21] extended

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Goyal's model[1] to cash discount policy for early payment. Arcelus *et al.*[22] modeled the retailer's profit-maximizing retail promotion strategy, when confronted with a vendor's trade promotion offer of credit and/or price discount on the purchase of regular or perishable merchandise. Abad and Jaggi[23] developed a joint approach to determine for the seller the optimal unit price and the length of the credit period when end demand is price sensitive. Salameh *et al.*[24] extended this issue to continuous review inventory model.

In 2003, Chung and Huang modeled the retailer's inventory system under payment delay within the Economic Production Quantity (EPQ) framework and developed an efficient procedure to determine the retailer's optimal ordering policy. However, they implicitly make the following assumptions:

- The unit selling price and the purchasing price per unit are assumed to be equal. However, as we know, the unit selling price for the retailer is usually significantly higher than the purchasing price per unit in order to obtain profit. Consequently, the viewpoint of Chung and Huang[19] is debatable sometimes.
- At the end of the credit period, the account is settled. The retailer starts paying for higher interest charges on the items in stock and returns money of the remaining balance immediately when the items are sold. What the above statement describes is just one arrangement of capitals of enterprises. Based on considerations of profits, costs and developments of enterprises, enterprises may invest their capitals to the best advantage.

According to the above arguments, this paper will adopt the following assumptions to modify Chung and Huang's[19] model.

- The selling price per unit and the unit purchasing price are not necessarily equal to match the most practical situations. This viewpoint also can be found in Teng[14], Chung *et al.*[15] and Chung and Huang[16].
- The retailer needs cash for business transactions. At the end of the credit period, the retailer pays off all units sold and keeps his/her profits for business transactions or other investment use. This viewpoint also can be found in Teng[14].

The main purpose of this study is to incorporate the assumptions (i) and (ii) to modify Chung and Huang's[19] model. That is, we incorporate both Chung and Huang[19] and Teng[14] to develop the retailer's inventory model. Then, we develop three easy-to-use theorems to efficiently determine the optimal inventory policy for the retailer. Finally, numerical examples are given to illustrate these theorems.

MODEL FORMULATION

For convenience, most notation and assumptions similar to Chung and Huang (2003a) will be used in this study.

Notation:

- A = Cost of placing one order
- c = Unit purchasing price per item
- D = Demand rate per year
- h = Unit stock holding cost per item per year excluding interest charges
- I_e = Interest earned per \$ per year
- I_k = Interest charges per \$ investment in inventory per year
- M = The trade credit period in years
- P = Replenishment rate per year, P>D
- s = Unit selling price per item
- T = The cycle time in years
- ρ = 1-D/P> 0
- TVC(T) = The total relevant cost per unit time when T>0
- T* = The optimal cycle time of TVC(T).

Assumptions:

- Demand rate, D, is known and constant.
- Replenishment rate, P, is known and constant.
- Shortages are not allowed.
- Time horizon is infinite.
- $s \geq c$; $I_k \geq I_e$.
- During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. When $T \geq M$, the account is settled at T = M, the retailer pays off all units sold and keeps his/her profits and starts paying for the higher interest charges on the items in stock. When $T \leq M$, the account is settled at T = M and we do not need to pay any interest charge.

Mathematical model: The total annual relevant cost consists of the following elements.

- Annual ordering cost = A/T.
- Annual stock holding cost (excluding interest charges) =

$$\frac{hT(P-D)\frac{DT}{P}}{2T} = \frac{DTh}{2}(1-\frac{D}{P}) = \frac{DTh\rho}{2}$$

- For inventory interest charges per year, we must consider three cases.

Case 1: $M < \frac{PM}{D} \le T$, as shown in Fig. 1.

Annual interest payable =

$$cI_k[\frac{DT^2\rho}{2} - \frac{(P-D)M^2}{2}]/T = cI_k\rho(\frac{DT^2}{2} - \frac{PM^2}{2})/T. \quad (1)$$

Case 2: $M \le T \le \frac{PM}{D}$, as shown in Fig. 2.

Annual interest payable = $$cI_k[\frac{D(T-M)^2}{2}]/T \quad (2)$$

Case 3: $T \le M$,

In this case, no interest charges are paid for the items.

- For interest earned per year, we must consider three cases.

Case 1: $M < \frac{PM}{D} \le T$

Annual interest earned = $$sI_e(\frac{DM^2}{2})/T. \quad (3)$$

Case 2: $M \le T \le \frac{PM}{D}$

Annual interest earned = $$sI_e(\frac{DM^2}{2})/T. \quad (4)$$

Case 3: $T \le M$, as shown in Fig. 3.

Annual interest earned = $$sI_e[\frac{DT^2}{2} + DT(M-T)]/T. \quad (5)$$

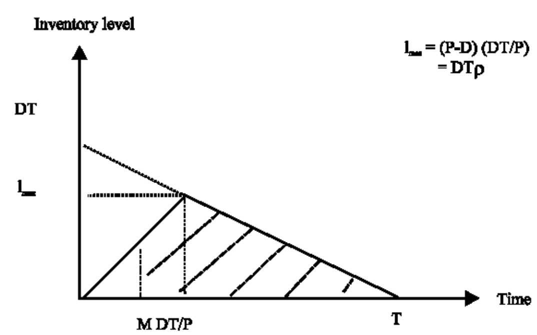

Fig. 1: The total accumulation of interest payable when $PM/D \le T$

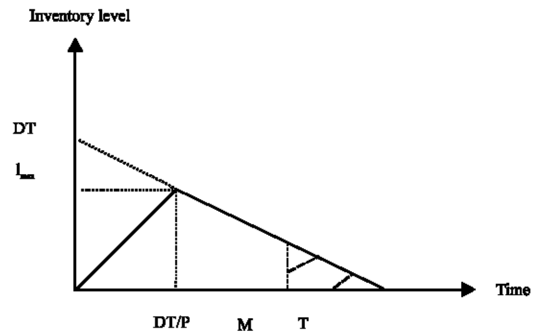

Fig. 2: The total accumulation of interest payable when $M \le T \le PM/D$

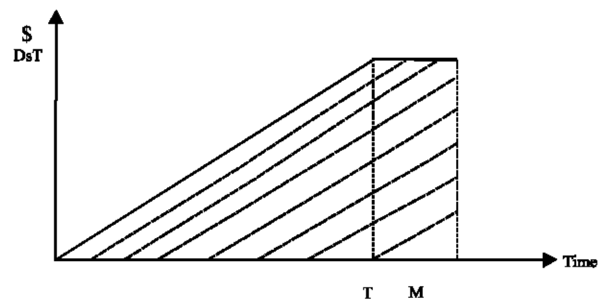

Fig. 3: The total accumulation of interest earned when $T \le M$

From the above arguments, the annual total relevant cost for the retailer can be expressed as TVC(T) = ordering cost + stock-holding cost + interest payable-interest earned.

We show that the annual total relevant cost, TVC(T), is given by

$$TVC(T) = \begin{cases} TVC_1(T) & \text{if} \quad T \ge \frac{PM}{D} & (6a) \\ TVC_2(T) & \text{if} \quad M \le T \le \frac{PM}{D} & (6b) \\ TVC_3(T) & \text{if} \quad 0 < T \le M & (6c) \end{cases}$$

where

$$TVC_1(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_k\rho(\frac{DT^2}{2} - \frac{PM^2}{2})/T - sI_e(\frac{DM^2}{2})/T \quad (7)$$

$$TVC_2(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_k[\frac{D(T-M)^2}{2}]/T - sI_e(\frac{DM^2}{2})/T \quad (8)$$

$$TVC_3(T) = \frac{A}{T} + \frac{DTh\rho}{2} - sI_e[\frac{DT^2}{2} + DT(M-T)]/T \quad (9)$$

Since $TVC_1(\frac{PM}{D}) = TVC_2(\frac{PM}{D})$ and $TVC_2(M) = TVC_3(M)$, TVC(T) is continuous and well-defined. All $TVC_1(T)$, $TVC_2(T)$, $TVC_3(T)$ and TVC(T) are defined on T>0. Equations (7), (8) and (9) yield

$$TVC_1'(T) = -\left[\frac{2A - M^2(cI_kP\rho + DsI_e)}{2T^2}\right] + D\rho(\frac{h + cI_k}{2})$$
$$= -\left[\frac{2A + DM^2(cI_k - sI_e) - PM^2cI_k}{2T^2}\right] + D\rho(\frac{h + cI_k}{2}), \quad (10)$$

$$TVC_1''(T) = \frac{2A + DM^2(cI_k - sI_e) - PM^2cI_k}{T^3} \quad (11)$$

$$TVC_2'(T) = -\left[\frac{2A + DM^2(cI_k - sI_e)}{2T^2}\right] + D(\frac{h\rho + cI_k}{2}) \quad (12)$$

$$TVC_2''(T) = \frac{2A + DM^2(cI_k - sI_e)}{T^3} \quad (13)$$

$$TVC_3'(T) = \frac{-A}{T^2} + D(\frac{h\rho + sI_e}{2}) \quad (14)$$

and

$$TVC_3''(T) = \frac{2A}{T^3} > 0 \quad (15)$$

Equation (15) implies that $TVC_3(T)$ is convex on T> 0. However, $TVC_1(T)$ is convex on T>0 if $\alpha>0$ and $TVC_2(T)$ is convex on T>0 if $\beta>0$, where $\alpha = 2A+DM^2(cI_k-sI_e)$ and $\beta = 2A+DM^2(cI_k-sI_e)$. We can find $\alpha<\beta$. Furthermore, we have $TVC'_1(PM/D) = TVC'_2(PM/D)$ and $TVC'_2(M) = =TVC'_3(M)$. Therefore, equations 6 (a, b, c) imply that TVC(T) is convex on T>0 if $\alpha>0$. Then, we have the following results.

Theorem 1 :

- If $\beta\le 0$, then TVC(T) is convex on (0, M] and concave on [M, ∞).
- If $\alpha\le 0$ and $\beta>0$, then TVC(T) is convex on (0, PM/D] and concave on [PM/D, ∞).
- If $\alpha>0$, then TVC(T) is convex on (0, ∞).

DETERMINATION OF THE OPTIMAL CYCLE TIME T*

Let $TVC'_i(T_i^*) = 0$ for all i = 1, 2, 3. We can obtain

$$T_1^* = \sqrt{\frac{2A + DM^2(cI_k - sI_e) - PM^2cI_k}{D\rho(h + cI_k)}} \text{ if } \alpha > 0, \quad (16)$$

$$T_2^* = \sqrt{\frac{2A + DM^2(cI_k - sI_e)}{D(h\rho + cI_k)}} \text{ if } \beta > 0 \quad (17)$$

and

$$T_3^* = \sqrt{\frac{2A}{D(h\rho + sI_e)}} \quad (18)$$

Equations 10, 12 and 14 yield that

$$TVC_1'(\frac{PM}{D}) = TVC_2'(\frac{PM}{D}) = \frac{-2A + \frac{M^2}{D}[P(P-D)h + cI_k(P^2 - D^2) + sI_eD^2]}{2(\frac{PM}{D})^2} \quad (19)$$

and

$$TVC_2'(M) = TVC_3'(M) = \frac{-2A + DM^2(h\rho + sI_e)}{2M^2} \quad (20)$$

Furthermore, we let

$$\Delta_1 = -2A + \frac{M^2}{D}[P(P-D)h + cI_k(P^2 - D^2) + sI_eD^2] \quad (21)$$

and

$$\Delta_2 = -2A + DM^2(h\rho + sI_e) \quad (22)$$

Then, we have $\Delta_1>\Delta_2$.

Suppose that $\beta\le 0$: When $\beta\le 0$, then $\alpha<0$. We can find $TVC_1(T)$ is increasing on [PM/D, ∞) from equation (10) and $TVC_2(T)$ is increasing on [M, PM/D] from Eq. 12. In addition, we can obtain $\Delta_1>\Delta_2>0$ from Eq. 21 and 22. By the convexity of $TVC_3(T)$, we see

$$TVC_3'(T)\begin{cases} <0 & \text{if } T < T_3^* & (23a) \\ =0 & \text{if } T = T_3^* & (23b) \\ >0 & \text{if } T > T_3^*. & (23c) \end{cases}$$

Then, we have the following result to determine the optimal cycle time T*.

Theorem 2: Suppose that $\beta\le 0$, then $TVC(T^*) = TVC(T_3^*)$ and $T^* = T_3^*$.

Proof. If $\beta \le 0$, then we can find $\alpha<0$, $\Delta_1>\Delta_2>0$ and

- $TVC_1(T)$ is increasing on [PM/D, ∞) from equation (10).
- $TVC_2(T)$ is increasing on [M, PM/D] from equation (12).

Since $\Delta_2>0$, then $TVC'_2(M) = TVC'_3(M)>0$. Equations 23 (a, b, c) imply that

- $TVC_3(T)$ is decreasing on $(0, T_3^*]$ and increasing on $[T_3^*, M]$.

Combining (i), (ii), (iii) and equations 6(a, b, c), we have that TVC(T) is decreasing on $(0, T_3^*]$ and increasing on $[T_3^*, \infty)$. Consequently, $T^* = T_3^*$.

Suppose that $\alpha \le 0$ and $\beta > 0$: When $\alpha \le 0$ and $\beta > 0$, we can find $TVC_1(T)$ is increasing on $[PM/D, \infty)$ from equation (10) and $\Delta_1>0$ from equation (21). By the convexity of $TVC_i(T)$ (i = 2, 3), we see

$$TVC_i'(T)\begin{cases} <0 & \text{if } T < T_i^* \quad (24a)\\ =0 & \text{if } T = T_i^* \quad (24b)\\ >0 & \text{if } T > T_i^*. \quad (24c)\end{cases}$$

Then, we have the following result to determine the optimal cycle time T*.

Theorem 3: Suppose that $\alpha \le 0$ and $\beta > 0$. Then,

(A) If $\Delta_2 \ge 0$, then $TVC(T^*) = TVC(T_3^*)$ and $T^* = T_3^*$.
(B) If $\Delta_2 < 0$, then $TVC(T^*) = TVC(T_2^*)$ and $T^* = T_2^*$.

Proof. If $\alpha \le 0$ and $\beta > 0$, we can find $\Delta_1 > 0$ and $TVC_1(T)$ is increasing on $[PM/D, \infty)$ from equation (10). Then,
(A) If $\Delta_2 \ge 0$, then $TVC'_2(M) = TVC'_3(M) \ge 0$. Equations 24 (a, b, c) imply that

(i) $TVC_2(T)$ is increasing on [M, PM/D].
(ii) $TVC_3(T)$ is decreasing on $(0, T_3^*]$ and increasing on $[T_3^*, M]$.

Combining (i), (ii) and equations 6 (a, b, c), we have that TVC (T) is decreasing on $(0, T_3^*]$ and increasing on $[T_3^*,\infty)$. Consequently, $T^* = T_3^*$.

(B) If $\Delta_2 < 0$, then $TVC'_2(M) = TVC'_3(M) < 0$. Equations 24 (a, b, c) imply that

(i) $TVC_2(T)$ is decreasing on $[M, T_2^*]$ and increasing on $[T_2^*, PM/D]$.
(ii) $TVC_3(T)$ is decreasing on (0, M].

Combining (i), (ii) and equations 6 (a, b, c), we have that TVC(T) is decreasing on $(0, T_2^*]$ and increasing on $[T_2^*,\infty)$. Consequently, $T^* = T_2^*$.

Suppose that $\alpha > 0$: When $\alpha > 0$, all T_i^* (i = 1, 2, 3) are well-defined. By the convexity of $TVC_i(T)$ (i = 1, 2, 3), we see

$$TVC_i'(T)\begin{cases} <0 & \text{if } T < T_i^* \quad (25a)\\ =0 & \text{if } T = T_i^* \quad (25b)\\ >0 & \text{if } T > T_i^*. \quad (25c)\end{cases}$$

Then, we have the following results to determine the optimal cycle time T*.

Theorem 4: Suppose that $\alpha > 0$. Then

(A) If $\Delta_1 > 0$ and $\Delta_2 \ge 0$, then $TVC(T^*) = TVC(T_3^*)$ and $T^* = T_3^*$.
(B) If $\Delta_1 > 0$ and $\Delta_2 < 0$, then $TVC(T^*) = TVC(T_2^*)$ and $T^* = T_2^*$.
(C) If $\Delta_1 \le 0$ and $\Delta_2 < 0$, then $TVC(T^*) = TVC(T_1^*)$ and $T^* = T_1^*$.

Proof.

(A) If $\Delta_1 > 0$ and $\Delta_2 \ge 0$, then $TVC_1'(\frac{PM}{D}) = TVC_2'(\frac{PM}{D})$ >0 and $TVC_2'(M) = TVC_3'(M) \ge 0$.

Equations 25 (a, b, c) imply that

- $TVC_1(T)$ is increasing on $[PM/D, \infty)$.
- $TVC_2(T)$ is increasing on [M, PM/D].
- $TVC_3(T)$ is decreasing on $(0, T_3^*]$ and increasing on $[T_3^*, M]$.

Combining (i), (ii), (iii) and equations 6 (a, b, c), we have that TVC(T) is decreasing on $(0, T_3^*]$ and increasing on $[T_3^*, \infty)$. Consequently, $T^* = T_3^*$.

(B) If $\Delta_1 > 0$ and $\Delta_2 < 0$, then $TVC_1'(\frac{PM}{D}) = TVC_2'(\frac{PM}{D})$ > 0 and $TVC_2'(M) = TVC_3'(M) < 0$. Equations 25 (a, b, c) imply that

- $TVC_1(T)$ is increasing on $[PM/D, \infty)$.
- $TVC_2(T)$ is decreasing on $[M, T_2^*]$ and increasing on $[T_2^*, PM/D]$.
- $TVC_3(T)$ is decreasing on (0, M].

Combining (i), (ii), (iii) and equations 6 (a, b, c), we have that TVC(T) is decreasing on $(0, T_2^*]$ and increasing on $[T_2^*, \infty)$. Consequently, $T^* = T_2^*$.

(C) If $\Delta_1 \le 0$ and $\Delta_2 < 0$, then $TVC_1'(\frac{PM}{D}) = TVC_2'(\frac{PM}{D})$ ≤ 0 and $TVC_2'(M) = TVC_3'(M) < 0$.

Equations 25(a, b, c) imply that

- $TVC_1(T)$ is decreasing on $[PM/D, T_1^*]$ and increasing on $[T_1^*, \infty)$.
- $TVC_2(T)$ is decreasing on $[M, PM/D]$.
- $TVC_3(T)$ is decreasing on $(0, M]$.

Combining (i), (ii), (iii) and equations 6 (a, b, c), we have that TVC(T) is decreasing on $(0, T_1^*]$ and increasing on $[T_1^*, \infty)$. Consequently, $T^* = T_1^*$.

Incorporating the above arguments, we have completed the proof of Theorem 4.

SPECIAL CASES

In this section, some previously published models are deduced as special cases.

(I) Chung and Huang's (2003a) model
When s = c, let

$$TVC_4(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_k\rho\left(\frac{DT^2}{2} - \frac{PM^2}{2}\right)/T - cI_e\left(\frac{DM^2}{2}\right)/T,$$

$$TVC_5(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_k\left[\frac{D(T-M)^2}{2}\right]/T - cI_e\left(\frac{DM^2}{2}\right)/T,$$

$$TVC_6(T) = \frac{A}{T} + \frac{DTh\rho}{2} - cI_e\left[\frac{DT^2}{2} + DT(M-T)\right]/T,$$

$$T_4^* = \sqrt{\frac{2A + DM^2c(I_k - I_e) - PM^2cI_k}{D\rho(h + cI_k)}},$$

$$T_5^* = \sqrt{\frac{2A + DM^2c(I_k - I_e)}{D(h\rho + cI_k)}}$$

and

$$T_6^* = \sqrt{\frac{2A}{D(h\rho + cI_e)}}.$$

Then $TVC'_i(T_i^*) = 0$ for i = 4, 5, 6. Equations 6 (a, b, c) will be reduced as follows:

$$TVC(T) = \begin{cases} TVC_4(T) & \text{if} \quad T \ge \frac{PM}{D} & (26a) \\ TVC_5(T) & \text{if} \quad M \le T \le \frac{PM}{D} & (26b) \\ TVC_6(T) & \text{if} \quad 0 < T \le M & (26c) \end{cases}$$

Equations 26 (a, b, c) will be consistent with equations 6 (a, b, c) in Chung and Huang[19], respectively. Hence, Chung and Huang[19] will be a special case of this study.

(II) Teng's (2002) model

When $P \to \infty$, let

$$TVC_7(T) = \frac{A}{T} + \frac{DTh}{2} + cI_k\left[\frac{D(T-M)^2}{2}\right]/T - sI_e\left(\frac{DM^2}{2}\right)/T,$$

$$TVC_8(T) = \frac{A}{T} + \frac{DTh}{2} - sI_e\left[\frac{DT^2}{2} + DT(M-T)\right]/T,$$

$$T_7^* = \sqrt{\frac{2A + DM^2(cI_k - sI_e)}{D(h + cI_k)}}$$

and

$$T_8^* = \sqrt{\frac{2A}{D(h + sI_e)}}.$$

Then $TVC'_i(T_i^*) = 0$ for I = 7, 8. Equations 6 (a, b, c) will be reduced as follows:

$$TVC(T) = \begin{cases} TVC_7(T) & \text{if} \quad M \le T & (27a) \\ TVC_8(T) & \text{if} \quad 0 < T \le M & (27b) \end{cases}$$

Equations 27 (a, b) will be consistent with equations (1) and (2) in Teng[14], respectively. Hence, Teng[14] will be a special case of this study.

(III) Goyal's (1985) model

When $P \to \infty$ and s = c, let

$$TVC_9(T) = \frac{A}{T} + \frac{DTh}{2} + cI_k\left[\frac{D(T-M)^2}{2}\right]/T - cI_e\left(\frac{DM^2}{2}\right)/T,$$

$$TVC_{10}(T) = \frac{A}{T} + \frac{DTh}{2} - cI_e\left[\frac{DT^2}{2} + DT(M-T)\right]/T,$$

$$T_9^* = \sqrt{\frac{2A + DM^2c(I_k - I_e)}{D(h + cI_k)}}$$

and

$$T_{10}^* = \sqrt{\frac{2A}{D(h + cI_e)}}$$

Then $TVC'_i(T_i^*) = 0$ for I = 9, 10. Equations 6(a, b, c) will be reduced as follows:

$$TVC(T) = \begin{cases} TVC_9(T) & \text{if} \quad M \le T & (28a) \\ TVC_{10}(T) & \text{if} \quad 0 < T \le M & (28b) \end{cases}$$

Equations 28(a, b) will be consistent with equations (1) and (4) in Goyal[1], respectively. Hence, Goyal[1] will be a special case of this study.

Table 1: The optimal cycle time and optimal order quantity using Theorem 2, Theorem 3 and Theorem 4

A	D	P	c	s	I_k	I_e	h	M	α	β	Δ_1	Δ_2	Theorem	Optimal cycle time, T*	Optimal order quantity, DT*
100	2000	3000	60	160	0.15	0.12	5	0.1	<0	<0	>0	>0	2	T_3 = 0.069227*	138
100	4000	5000	50	100	0.15	0.12	5	0.1	<0	>0	>0	>0	3-(A)	T_3 = 0.062017*	248
100	2000	3000	50	60	0.15	0.12	5	0.1	<0	>0	>0	<0	3-(B)	T_2 = 0.106002*	212
150	2000	3500	30	90	0.15	0.12	10	0.1	>0	>0	>0	>0	4-(A)	T_3 = 0.099716*	199
200	3000	4000	50	80	0.15	0.12	5	0.1	>0	>0	>0	<0	4-(B)	T_2 = 0.113305*	340
150	2500	3000	35	40	0.15	0.12	5	0.1	>0	>0	<0	<0	4-(C)	T_1 = 0.189737*	474

Table 2: The optimal cycle time with various values of *P* and *s*

Let A = $200/order, D = 3000units/year, c = $50/unit, h = $10/unit/year, Ik = $0.14/$/year, Ie = $0.12/$/year and M = 0.1year

	s = $60/unit					s = $80/unit					s = $100/unit				
P	α	β	Δ_1	Δ_2	T*	α	β	Δ_1	Δ_2	T*	α	β	Δ_1	Δ_2	T*
3500	>0	>0	<0	<0	T_1 = 0.14301*	>0	>0	>0	<0	T_2 = 0.11285*	>0	>0	>0	>0	T_3 = 0.09964*
4000	>0	>0	>0	<0	T_2 = 0.11758*	>0	>0	>0	<0	T_2 = 0.10629*	<0	>0	>0	>0	T_3 = 0.09589*
5000	>0	>0	>0	<0	T_2 = 0.10927*	<0	>0	>0	>0	T_3 = 0.09901*	<0	>0	>0	>0	T_3 = 0.09129*

NUMERICAL EXAMPLES

To illustrate all results obtained in this paper, let us apply the proposed method to efficiently solve the following numerical examples. The optimal cycle time and optimal order quantity are summarized in Table 1 and Table 2, respectively.

To study the effects of replenishment rate per year, P and unit selling price per item, *s*, on the optimal cycle time and optimal order quantity for the retailer derived by the proposed method, we solve the example on Table 2 with various values of P and s. The following inferences can be made based on Table 2. When *P* is increasing, the optimal cycle time and optimal order quantity for the retailer are decreasing. So, the retailer will shorten the ordering time interval since the replenishment speed is faster. When s is increasing, the optimal cycle time and optimal order quantity for the retailer are decreasing. This result implies that the retailer will order less quantity to take the benefits of the payment delay more frequently.

CONCLUSION

This study is to amend Chung and Huang's EPQ model (2003a) to the case where the retailer's unit selling price and the purchasing price per unit are not necessarily equal, reflecting the real-life situations. Then, we provide a very efficient solution procedure to determine the optimal cycle time T*. Theorems 2, 3 and 4 help the retailer accurately and quickly determining the optimal inventory policy under minimizing the annual total relevant cost. In addition, we deduce some previously published results of other researchers as special cases. From the final numerical example, we find a result that is interesting. This result implies that the retailer will order less quantity to take the benefits of the payment delay more frequently when the unit selling price per item is larger than the unit purchasing price per item more and more. A future study will further incorporate the proposed model into more realistic assumptions, such as probabilistic demand, deteriorating items and allowable shortages.

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