Journal of Engineering and Applied Sciences 2 (1): 143-148, 2007

On Extended EOQ Model Under Cash Discount and Payment Delay Derived Without Derivatives

[1]Chung-Li Chou, [1]Yung-Fu Huang and [2]Hung-Fu Huang
[1]Department of Business Administration, Chaoyang University of Technology, Taichung, Taiwan, Republic of China
[2]Department of Electrical Engineering, National Cheng Kung University, Tainan, Republic of China

Abstract: In this study, we correct the model of Chang that discussed with the Economic Order Quantity (EOQ) under conditions of cash discount and payment delay. In addition, we try to use different method from Chang for obtaining optimal cycle time under cash discount and payment delay so that the annual total cost is minimized. This study provides an algebraic approach to determine the optimal cycle time. This approach provides one theorem to efficiently determine the optimal cycle time. Finally, numerical examples are solved to illustrate the results that may cause significant errors and penalties using wrong model when the cash discount rate is larger and the managerial insights from these numerical examples and sensitivity analysis are also obtained.

Key words: EOQ, payment delay, cash discount, trade credit, inventory control

INTRODUCTION

The traditional Economic Order Quantity (EOQ) model assumes that the retailer's capital is unconstrained and the retailer must be paid for the items as soon as the items are received. In practice, the supplier frequently offers the retailer a fixed delay period, that is the trade credit period, in settling the accounts. Before the end of trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of trade credit period. In real world, the supplier often makes use of this policy to promote his commodities.

Goyal[1] established a single-item inventory model under permissible delay in payments. Chung[2] developed an alternative approach to determine the economic order quantity under condition of permissible delay in payments. Aggarwal and Jaggi[3] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Chang *et al.*[4] extended this issue to varying rates of deterioration. Liao *et al.*[5,6] investigated this topic in the presence of inflation. Jamal *et al.*[7] and Chang and Dye[8] extended this issue with allowable shortage. Chang *et al.*[9] extended this issue with linear trend demand. Hwang and Shinn[10] modeled an inventory system for retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payment. Jamal *et al.*[11] and Sarker *et al.*[12] addressed the optimal payment time under permissible delay in payment with deterioration. Teng[13] assumed that the selling price is not equal to the purchasing price to modify Goyal's model[1]. Shinn and Hwang[14] determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer's order size and also the demand rate is a function of the selling price. Chung and Huang[15] examined this problem within the EPQ framework and developed an efficient procedure to determine the retailer's optimal ordering policy. Huang[16] extended this issue under two levels of trade credit and developed an efficient solution procedure to determine the optimal lot-sizing policy of the retailer. Huang and Chung *et al.*[17,1] to cash discount policy for early payment. Chang *et al.*[18] and Chung and Liao[19] investigated the problem of determining the economic order quantity for exponentially deteriorating items under permissible delay in payments depending on the ordering quantity. Huang[20] investigated that the unit selling price and the unit purchasing price are not necessarily equal within the EPQ framework under supplier's trade credit policy.

Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the

Corresponding Author: Yung-Fu Huang, No. 168, Jifong E. Rd., Wufong Township, Taichung County 41349, Taiwan, Republic of China

supplier. From the viewpoint of the supplier, the supplier hopes that the payment is paid from retailer as soon as possible. It can avoid the possibility of resulting in bad debt. So, in most business transactions, the supplier will not only offer the fixed period to settle the account but also may allow a cash discount to encourage the retailer to pay for his/her purchasing cost as soon as possible. The retailer can obtain the cash discount when the payment is paid within cash discount period offered by the supplier. Otherwise, the retailer will pay full payment within the trade credit period. In general, the cash discount period is shorter than the trade credit period. Many articles related to the inventory policy under cash discount and payment delay can be found in Chang[21], Ouyang *et al.*[17,21,22].

The purpose of this study is to correct the model of Chang[21] discussed with the EOQ under conditions of cash discount and payment delay. In addition, we try to use different method from Chang[21] for obtaining the optimal cycle time so that the annual total cost is minimized. This study provides an algebraic approach to determine the optimal cycle time. In previous most published papers that have been derived using differential calculus to find the optimal solution and to prove optimality condition with second-order derivatives. In recent papers, Grubbström and Erdem[23] and Cárdenas-Barrón[24] showed that the formulae for the EOQ and EPQ with backlogging can be derived without differential calculus. Yang and Wee[25] developed algebraically the optimal replenishment policy of the integrated vendor-buyer inventory system without using differential calculus. Wu and Ouyang[26] modify Yang and Wee[25] to allow shortages using algebraic method. In this study, we provide one theorem to efficiently determine the optimal cycle time. Finally, numerical examples are solved to illustrate the results that may cause significant errors and penalties using wrong model when the cash discount rate is larger and the managerial insights from these numerical examples and sensitivity analysis are also obtained.

MODEL FORMULATION

The same notation and assumptions as in Chang[21] are used.

Notation

- D = Annual demand
- S = Cost of placing one order
- c = Unit purchasing price per item
- p = Unit selling price per item
- h = Unit stock holding cost per item per year excluding interest charges
- I_d = Annual interest rate that can be earned
- I_c = Annual interest charges for inventory item
- r = Cash discount rate, $0 < r < 1$
- M_1 = The period of cash discount in years
- M_2 = The period of trade credit in years, $M_1 < M_2$
- T = The cycle time in years
- $Z(T)$ = The annual total variable cost in Chang[21]
- $TVC(T)$ = The annual total cost in this study
- T^* = The optimal cycle time of $TVC(T)$

Assumptions

- Demand rate is known and constant.
- Shortages are not allowed.
- Replenishment is instantaneous.
- Time horizon is infinite.
- $s > c$; $I_c \geq I_d$.
- During the time the account is not settled, generated sale revenue is deposited in an interest-bearing account. At the end of this period, the retailer pays units sold, keeps profits and starts paying the higher interest charges on the items in stock.

Algebraic modeling: However, we want to correct the term of the interest payable per unit time in Case 1in Chang[21]. Since the supplier offers cash discount if payment is paid within M_1. Therefore, in this case, the retailer will pay the annual purchasing cost, $c(1\text{-}r)D$, to the supplier. Then, the annual cost of interest charges for the items kept in stock is based on the annual purchasing cost, $c(1\text{-}r)D$. So, we change the interest payable per unit time in Case 1 in Chang[21].

$$\frac{cDI_c(T-M_1)^2}{2T} \text{ to } \frac{c(1-r)DI_c(T-M_1)^2}{2T}.$$

Therefore, the annual total cost functions is as follows.

Annual total cost = ordering cost + stock-holding cost + purchasing cost + interest payable-interest earned.

Then, we rewrite four annual total variable cost functions, $Z_i(T)$ for i = 1, 2, 3 and 4, in Chang[21] to $TVC_i(T)$ for i = 1, 2, 3 and 4 as follows.

$$TVC_1(T) = \frac{S}{T} + \frac{hDT}{2} + c(1-r)D + \frac{c(1-r)I_cD(T-M_1)^2}{2T} - \frac{pI_dDM_1^{\,2}}{2T} \text{ if } T \geq M_1, \tag{1}$$

$$TVC_2(T) = \frac{S}{T} + \frac{hDT}{2} + c(1-r)D - pI_dD(M_1 - \frac{T}{2}) \text{ if } T < M_1, \tag{2}$$

$$TVC_3(T) = \frac{S}{T} + \frac{hDT}{2} + cD + \frac{cI_cD(T-M_2)^2}{2T} - \frac{pI_dDM_2^2}{2T} \text{ if } T \geq M_2 \quad (3)$$

and

$$TVC_4(T) = \frac{S}{T} + \frac{hDT}{2} + cD - pI_dD(M_2 - \frac{T}{2}) \text{ if } T < M_2. \quad (4)$$

At $T = M_1$, we find $TVC_1(M_1) = TVC_2(M_1)$. And at $T = M_2$, we find $TVC_3(M_2) = TVC_4(M_2)$.

Then, we can rewrite

$$TVC_1(T) = \frac{S}{T} + \frac{hDT}{2} + c(1-r)D + \frac{c(1-r)I_cD(T-M_1)^2}{2T} - \frac{pI_dDM_1^2}{2T}$$

$$= \frac{D[h+c(1-r)I_c]}{2T}\left\{T^2 + \frac{2S + DM_1^2[c(1-r)I_c - pI_d]}{D[h+c(1-r)I_c]}\right\} + cD(1-r)(1-M_1I_c)$$

$$= \frac{D[h+c(1-r)I_c]}{2T}\left\{T - \sqrt{\frac{2S + DM_1^2[c(1-r)I_c - pI_d]}{D[h+c(1-r)I_c]}}\right\}^2$$

$$+\{\sqrt{D[h+c(1-r)I_c]\{2S + DM_1^2[c(1-r)I_c - pI_d]\}} + cD(1-r)(1-M_1I_c)\}. \quad (5)$$

Equation 5 represents that the minimum of $TVC_1(T)$ is obtained when the quadratic non-negative term, depending on T, is made equal to zero. Therefore, the optimum value T_1^* is

$$T_1^* = \sqrt{\frac{2S + DM_1^2[c(1-r)I_c - pI_d]}{D[h+c(1-r)I_c]}} \text{ if } 2S + DM_1^2[c(1-r)I_c - pI_d] > 0. \quad (6)$$

Therefore, Eq. 5 has a minimum value for the optimal value of T_1^* reducing $TVC_1(T)$ to

$$TVC_1(T_1^*) = \sqrt{D[h+c(1-r)I_c]\{2S + DM_1^2[c(1-r)I_c - pI_d]\}} + cD(1-r)(1-M_1I_c). \quad (7)$$

Similarly, we can derive $TVC_2(T)$ without derivatives as follows.

$$TVC_2(T) = \frac{S}{T} + \frac{hDT}{2} + c(1-r)D - pI_dD(M_1 - \frac{T}{2})$$

$$= \frac{D(h+pI_d)}{2T}\left[T^2 + \frac{2S}{D(h+pI_d)}\right] + D[c(1-r) - pM_1I_d]$$

$$= \frac{D(h+pI_d)}{2T}\left[T - \sqrt{\frac{2S}{D(h+pI_d)}}\right]^2 + \{\sqrt{2SD(h+pI_d)} + D[c(1-r) - pM_1I_d]\}. \quad (8)$$

Equation 8 represents that the minimum of $TVC_2(T)$ is obtained when the quadratic non-negative term, depending on T, is made equal to zero. Therefore, the optimum value T_2^* is

$$T_2^* = \sqrt{\frac{2S}{D(h+pI_d)}} \quad (9)$$

Therefore, Eq. 8 has a minimum value for the optimal value of T_2^* reducing $TVC_2(T)$ to

$$TVC_2(T_2^*) = \sqrt{2SD(h+pI_d)} + D[c(1-r) - pM_1I_d]. \quad (10)$$

Likewise, we can derive $TVC_3(T)$ algebraically as follows.

$$TVC_3(T) = \frac{S}{T} + \frac{hDT}{2} + cD + \frac{cI_cD(T-M_2)^2}{2T} - \frac{pI_dDM_2^2}{2T}$$

$$= \frac{2S + DM_2^2(cI_c - pI_d)}{2T} + \frac{DT(h+cI_c)}{2} + cD(1-M_2I_c)$$

$$= \frac{D(h+cI_c)}{2T}\left[T - \sqrt{\frac{2S + DM_2^2(cI_c - pI_d)}{D(h+cI_c)}}\right]^2$$

$$+\{\sqrt{D(h+cI_c)[2S + DM_2^2(cI_c - pI_d)]} + cD(1-M_2I_c)\}. \quad (11)$$

Equation 11 represents that the minimum of $TVC_3(T)$ is obtained when the quadratic non-negative term, depending on T, is made equal to zero. Therefore, the optimum value T_3^* is

$$T_3^* = \sqrt{\frac{2S + DM_2^2(cI_c - pI_d)}{D(h+cI_c)}} \text{ if } 2S + DM_2^2(cI_c - pI_d) > 0. \quad (12)$$

Therefore, Eq. (11) has a minimum value for the optimal value of T_3^* reducing $TVC_3(T)$ to

$$TVC_3(T_3^*) = \sqrt{D(h + cI_c)[2S + DM_2^2(cI_c - pI_d)]} + cD(1 - M_2I_c). \quad (13)$$

And Last, we also derive $TVC_4(T)$ without derivatives as follows.

$$TVC_4(T) = \frac{S}{T} + \frac{hDT}{2} + cD - pI_dD(M_2 - \frac{T}{2})$$

$$= \frac{D(h + pI_d)}{2T}\left[T^2 + \frac{2S}{D(h + pI_d)}\right] + D(c - pM_2I_d)$$

$$= \frac{D(h + pI_d)}{2T}\left[T - \sqrt{\frac{2S}{D(h + pI_d)}}\right]^2 + [\sqrt{2SD(h + pI_d)} + D(c - pM_2I_d)]. \quad (14)$$

Equation 14 represents that the minimum of $TVC_4(T)$ is obtained when the quadratic non-negative term, depending on T, is made equal to zero. Therefore, the optimum value T_4^* is

$$T_4^* = \sqrt{\frac{2S}{D(h + pI_d)}}. \quad (15)$$

Therefore, Eq. 14 has a minimum value for the optimal value of T_4^* reducing $TVC_4(T)$ to

$$TVC_4(T_4^*) = \sqrt{2SD(h + pI_d)} + D(c - pM_2I_d). \quad (16)$$

From Eq. 9 and 15, we can find $T_2^* = T_4^*$.

DETERMINATION OF THE OPTIMAL CYCLE TIME T*

From Section 2, Eq. 6 implies that the optimal value of T for the case of $T \geq M_1$, that is $T_1^* > M_1$. We substitute Eq. 6 into $T_1^* > M_1$, then we can obtain the optimal value of T

$$\text{if and only if } -2S + DM_1^2(h + pI_d) < 0. \quad (17)$$

Likewise, Eq. 9 implies that the optimal value of T for the case of $T \leq M_1$, that is $T_2^* < M_1$. We substitute Eq. 9 into $T_2^* < M_1$, then we can obtain the optimal value of T

$$\text{if and only if } -2S + DM_1^2(h + pI_d) > 0. \quad (18)$$

Of course, if $T_1^* = T_2^* = M_1$, we can obtain that

$$\text{if and only if } -2S + DM_1^2(h + pI_d) = 0. \quad (19)$$

Similar disscussion, we can obain following results.

$$T_3^* > M_2 \text{ if and only if } -2S + DM_2^2(h + pI_d) < 0, \quad (20)$$

$$T_4^* < M_2 \text{ if and only if } -2S + DM_2^2(h + pI_d) > 0 \quad (21)$$

and

$$T_3^* = T_4^* = M_2 \text{ if and only if } -2S + DM_2^2(h + pI_d) = 0. \quad (22)$$

Furthermore, we let

$$\Delta_1 = -2S + DM_1^2(h + pI_d) \quad (23)$$

and

$$\Delta_2 - 2S + DM_2^2(h + pI_d). \quad (24)$$

Since $M_1 < M_2$, we can get $\Delta_1 < \Delta_2$ from Eq. 23 and 24. Summarized above Eq. 17 to Eq. 24, the optimal cycle time T* can be obtained as follows.

Theorem 1

- If $\Delta_1 \geq 0$, then $TVC(T^*) = \min\{TVC_2(T_2^*), TVC_4(T_4^*)\}$. Since $T_2^* = T_4^*$, therefore $T^* = T_2^* = T_4^*$.
- If $\Delta_1 < 0$ and $\Delta_2 \geq 0$, then $TVC(T^*) = \min\{TVC_1(T_1^*), TVC_4(T_4^*)\}$. Hence T^* is T_1^* or T_4^* associated with the least cost.
- If $\Delta_2 < 0$, then $TVC(T^*) = \min\{TVC_1(T_1^*), TVC_3(T_3^*)\}$. Hence T^* is T_1^* or T_3^* associated with the least cost.

Theorem 1 is an effective procedure to find the optimal cycle time T* by easy judgment the numbers Δ_1 and Δ_2. Theorem 1 is really very simple.

NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

The proposed approach is applied in above section to efficiently solve the following numerical examples. In addition, the percentage cost penalty (PCP) is investigated between our solution and Chang's solution[21] in changing the parameter of the cash discount rate r. Furthermore, the sensitivity analysis is conducted for a problem with 2 parameters-the period of cash discount M_1 and the unit selling price p.

Table 1: Comparisons of our solution and Chang's solution (in changing the parameter r)

Let S=\$200/order, D=1000units/year, c=\$100/unit, p=\$110/unit, h=\$1/unit/year, Ic=\$0.15/\$/year, Id=\$0.1/\$/year, M_1=0.01year and M_2=0.1year

			Chang's solution		Our solution		
r	Δ_1	Δ_2	T*	$TVC_C(T^*)$	T*	$TVC_O(T^*)$	PCP*
0.1	<0	<0	T_1=0.158193	92276.97	T_1*=0.166143	92274.07	0.003139
0.2	<0	<0	T_1=0.158193	82172.85	T_1*=0.175434	82160.64	0.014865
0.3	<0	<0	T_1=0.158193	72068.73	T_1*=0.186489	72039.63	0.040399
0.4	<0	<0	T_1=0.158193	61964.61	T_1*=0.199950	61909.50	0.089020
0.5	<0	<0	T_1=0.158193	51860.49	T_1*=0.216836	51768.10	0.178470
0.6	<0	<0	T_1=0.158193	41756.37	T_1*=0.238896	41612.27	0.346292
0.7	<0	<0	T_1=0.158193	31652.26	T_1*=0.269461	31437.03	0.684610
0.8	<0	<0	T_1=0.158193	21548.14	T_1*=0.315911	21233.65	1.481097

*PCP(percentage cost penalty)={[$TVC_C(T^*)-TVC_O(T^*)]/TVC_O(T^*)$}*100%

Table 2: The optimal cycle time with various values of M_1 and p

Let S=\$200/order, D=2000units/year, c=\$50/unit, h=\$5/unit/year, Ic=\$0.15/\$/year, Id=\$0.05/\$/year, r=0.05 and M_2=0.2year.

	M_1=0.05 year				M_1=0.1 year				M_1=0.15 year			
p=\$/unit	Δ_1	Δ_2	T*	TVC(T*)	Δ_1	Δ_2	T*	TVC(T*)	Δ_1	Δ_2	T*	TVC(T*)
100	<0	>0	T_1*=0.13013	97443	<0	>0	T_1*=0.13508	96851	>0	>0	T_2*=0.14142	96328
150	<0	>0	T_1*=0.12813	97395	<0	>0	T_1*=0.12722	96660	>0	>0	T_2*=0.12649	95912
200	<0	>0	T_1*=0.12610	97346	<0	>0	T_1*=0.11884	96457	>0	>0	T_2*=0.11547	95464

In Table 1, the optimal cycle time in Chang's solution will not change when cash discount rate is increasing in this example. Since the optimal cycle time (T_1) in Chang's solution is independent of the cash discount rate r. In addition, it is found that the percentage cost penalty is increasing when the cash discount rate is increasing. Therefore, it may cause significant errors and penalties using wrong model when the cash discount rate is larger.

To study the effects M_1 and p on the optimal cycle time and the annual total cost for the retailer derived based on the proposed method, the example is solved and shown in Table 2 with various values of M_1 and p. The following inferences can be made based on Table 2. First of all, the retailer will adopt the short payment period, optimal cycle time is T_1* or T_2*, to obtain the minimized annual total cost in this example. Secondly, both T* and TVC(T*) for the retailer are decreasing when p is increasing. This result implies that the retailer will order less quantity to take the benefits of the delay payments more frequently when the unit selling price is higher than the unit purchasing price more and more. Thirdly, the annual total cost for the retailer is decreasing when M_1 is increasing, but the optimal cycle time for the retailer is increasing in lower p and decreasing in higher p. It implies that the retailer will order more quantity to obtain more cash discount in lower p. In higher p situation, the benefits of earned interest more frequently ordering small lot-size are larger than the cash discount ordering big lot-size. Therefore, the retailer will adopt big lot-size policy in lower unit selling price and small lot-size policy in higher unit selling price. Nevertheless, the retailer will take the benefits from the annual total cost reduced when the supplier offers the longer period of cash discount.

CONCLUSION

This study corrects Chang's paper[21] that discussed the economic order quantity under the conditions of cash discount and payment delay using algebraic approach. Using this approach presented in this paper, we can find the optimal cycle time without using differential calculus. This should also mean that this algebraic approach is a more accessible approach to ease the learning of basic inventory theories for younger students who lack the knowledge of differential calculus. Then, we provide one theorem to efficiently determine the optimal cycle time depending on the numbers of Δ_1 and Δ_2. Finally, we conclude that the decision-maker may cause significant errors and penalties using wrong model when the cash discount rate is larger.

The proposed model can be extended in several ways. For instance, we may generalize the model to allow for shortages, time value of money, finite time horizon, finite replenishment rate and others.

REFERENCES

1. Goyal, S.K., 1985. Economic order quantity under conditions of permissible delay in payments. J. Operational Res. Soc., 36: 35-38.
2. Chung, K.J., 1998. A theorem on the determination of economic order quantity under conditions of permissible delay in payments. Comput. Operations Res., 25: 49-52.
3. Aggarwal, S.P. and C.K. Jaggi, 1995. Ordering policies of deteriorating items under permissible delay in payments. J. Operational Res. Soc., 46: 658-662.

4. Chang, H.J., C.Y. Dye and B.R. Chuang, 2002. An inventory model for deteriorating items under the condition of permissible delay in payments. Yugoslav J. Operations Res., 12: 73-84.
5. Liao, H.C., C.H. Tsai and C.T. Su, 2000. An inventory model with deteriorating items under inflation when a delay in payment is permissible. Intl. J. Prod. Econom., 63: 207-214.
6. Sarker, B.R., A.M.M. Jamal and S. Wang, 2000. Supply chain model for perishable products under inflation and permissible delay in payment. Comput. Operations Res., 27: 59-75.
7. Jamal, A.M.M., B.R. Sarker and S. Wang, 1997. An ordering policy for deteriorating items with allowable shortages and permissible delay in payment. J. Operational Res. Soc., 48: 826-833.
8. Chang, H.J. and C.Y. Dye, 2001. An inventory model for deteriorating items with partial backlogging and permissible delay in payments. Intl. J. Sys. Sci., 32: 345-352.
9. Chang, H.J., C.H. Hung and C.Y. Dye, 2001. An inventory model for deteriorating items with linear trend demand under the condition of permissible delay in payments. Production Planning and Control, 12: 274-282.
10. Hwang, H. and S.W. Shinn, 1997. Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. Comput. Operations Res., 24: 539-547.
11. Jamal, A.M.M., B.R. Sarker and S. Wang, 2000. Optimal payment time for a retailer under permitted delay of payment by the wholesaler. Intl. J. Prod. Econom., 66: 59-66.
12. Sarker, B.R., A.M.M. Jamal and S. Wang, 2000. Optimal payment time under permissible delay in payment for products with deterioration. Production Planning and Control, 11: 380-390.
13. Teng, J.T., 2002. On the economic order quantity under conditions of permissible delay in payments. J. Operational Res. Soc., 53: 915-918.
14. Shinn, S.W. and H. Hwang, 2003. Optimal pricing and ordering policies for retailers under order-size-dependent delay in payments. Comput. Operations Res., 30: 35-50.
15. Chung, K.J. and Y.F. Huang, 2003. The optimal cycle time for EPQ inventory model under permissible delay in payments. Intl. J. Prod. Econom., 84: 307-318.
16. Huang, Y.F., 2003. Optimal retailer's ordering policies in the EOQ model under trade credit financing. J. Operational Res. Soc., 54: 1011-1015.
17. Huang, Y.F. and K.J. Chung, 2003. Optimal replenishment and payment policies in the EOQ model under cash discount and trade credit. Asia-Pacific J. Operational Res., 20: 177-190.
18. Chang, C.T., Ouyang and J.T. Teng, 2003. An EOQ model for deteriorating items under supplier credits linked to ordering quantity. Apply Math. Modelling, 27: 983-996.
19. Chung, K.J. and J.J. Liao, 2004. Lot-sizing decisions under trade credit depending on the ordering quantity. Comput. Operations Res., 31: 909-928.
20. Huang, Y.F., 2004. Optimal retailer's replenishment policy for the EPQ model under supplier's trade credit policy. Production Planning and Control, 15: 27-33.
21. Chang, C.T., 2002. Extended economic order quantity model under cash discount and payment delay. Intl. J. Inform. Manage. Sci., 13: 57-69.
22. Ouyang, L.Y., M.S. Chen and K.W. Chuang, 2002. Economic order quantity model under cash discount and payment delay. Intl. J. Inform. Manage. Sci., 13: 1-10.
23. Grubbström, R.W. and A. Erdem, 1999. The EOQ with backlogging derived without derivatives. Intl. J. Prod. Econom., 59: 529-530.
24. Cárdenas-Barrón, L.E., 2001. The Economic Production Quantity (EPQ) with shortage derived algebraically. Intl. J. Prod. Econom., 70: 289-292.
25. Yang, P.C. and H.M. Wee, 2002. The economic lot size of the integrated vendor-buyer inventory system derived without derivatives. Optimal Control Applications and Methods, 23: 163-169.
26. Wu, K.S. and L.Y. Ouyang, 2003. An integrated single-vendor single-buyer inventory system with shortage derived algebraically. Production Planning and Control, 14: 555-561.