

Cylindrical Teeth's Gear in Developpante of Circle

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Abstract: In the design projects involving the teeth gears, the designer may find difficulties caused by the fact that the gear's coefficient is not known. This leads to year arbitrary choice of these coefficients which may not respond to all the boundary condition: Interference in generation and gearing, teeth generation, gearing continuity. The investigation presented consists of varying the number of teeth, the angle of inclination and the transmission ratio at the same time. With the optimization of all these parameters, the choice limits of the deport coefficients are materialized by boundary curves represented by coordinates (X^1 and X^2).

Key words: Gears, contact, coefficient of offset, interference, correction

INTRODUCTION

The gears are characterized by the number of teeth, the standardized geometrical parameters, the system of calculation of the diameters of head (conservation or not of the standardized or standard radial play), the angle of inclination and the coefficients of $X1$ offset and $X2$ par variation only of $X1$ and $X2$ and the conservation of all the other parameters, one can obtain an infinity of gears with very different geometries.

The field of the existence of the gears with some coefficients which characterize engaging, can be presented using the curves plotted for each concrete combination of the number of tooth $Z1$ and $Z2$ in the field of the frame of reference ($X1, X2$). this construction of the diagrams emanates from M. B.GRAMAN^[1].

For real gears, one cannot use all the points of the fields of co-ordinates ($X1, X2$); because of existence of the limits to avoid the interference with cutting, the grinding of the $Z1$ teeth and $Z2$ the interference of engaging, as well as a report/ratio of control supporting a continuity of engaging.

The whole of the lines which limit the choice of the coefficient of offset by which separate the fields from the acceptable values of $X1$ and $X2$ and inadmissible is called "contour of blocking". This last is proposed by B.A GRAVILENKO^[2].

For the gears with skew teeth, "the CONTOUR OF BLOCKING" changes form in dependence of the value

taken beforehand of the angle of inclination β ; and each value of β , "the contour of blocking" is built separately,^[3].

At first approximation, it is recommended according to T.BOTOLOVSKA^[3], for the tilted gears with teeth the use of "CONTOURS OF BLOCKING" of right teeth (with $Z1$ and $Z2$ realities), but to use the equivalent number of the teeth, determined by the formula.

$$Z_{v1,2} = \frac{Z_{1,2}}{\cos \beta} \quad (1)$$

Practically in the case of the use of "CONTOURS DEBLOCAGE" of the gears with right teeth, one obtains rather large values which leave the limits of the combinations $Z1$ and $Z2$ given according to M.GRAMAN^[1], from where one will have difficulties for the calculation of the gears with skew teeth during the use of the rather large values of the angle of inclination β . According to M.GRAMAN^[1] are represented the gears with the number of teeth maximum $Z1 = 100$ and maximum $Z2 = 190$. For example if $\beta = 45$, the equivalent number of teeth is approximately 2.9 times larger than the real number.

In the study of a project of gears with skew teeth, the manufacturer finds difficulties, since the field of the existence of the gears is unknown for him. This imposes the choice of $X1$ and $X2$ arbitrarily, of with the hazardous choice; after that one makes a checking in extreme cases (quoted previously). Often in spite of a significant

computational load, it is that one leads to a negative result: The point chooses is in the zone external of the "CONTOUR OF BLOCKING". If only one of the limits is not respected, it is sufficient to obtain this case.

Even if the result is positive (combination of X1, X2, Z1, Z2 and β) the manufacturer cannot obtain any image in the neighbourhoods of the limiting curves against the interference. The existence of the "CONTOUR OF BLOCKING" excludes from the tests in the case of the choice of the coefficient of offset, the checking of the four limits becomes useless and calculation is reduced.

Analytical resolution of the phenomenon of the interference: Analytically let us solve the phenomenon of the interference of the gears with tilted teeth.

The trajectory of the top of the tooth of the opposite wheel of the gears is in contact with the involute at the low point P of the active part of the profile appears (1). If the angle of profile α_p is higher than the angle of profile α_i , at the point there limits is no interference, appears Fig. 1a. On the tooth shape there is part of the involute PL which does not take part in engaging. In this case $\alpha_p = \alpha_i$ the points P and L are confused and the gears are in extreme cases of wedging. Thus the case $\alpha_p < \alpha_i$ there exists interference which is inadmissible Fig. 1b. Thus the condition of inexistence of the interference can arise as follows:

$$\alpha_p \geq \alpha_i \quad (1)$$

The real danger is the wedging of the gears because of the contact of the top of the tooth of the driven gear Z2 with to the base of the tooth of the driving wheel Z1. In this case it is necessary to observe the condition: $\alpha_p \geq \alpha_i$. We will use the known relations^[4], valid for the gears with tilted teeth.

$$\text{tg} \alpha_{pi} = \text{tg} \alpha_{tw} - \frac{Z_2}{Z_1} (\text{tg} \alpha_{a2} - \text{tg} \alpha_{tw}) \quad (2)$$

$$\text{tg} \alpha_{i1} = \text{tg} \alpha_i - \frac{4(h_i^* - h_a^* - x_i) \cos \beta}{z_1 \sin 2\alpha_i} \quad (3)$$

$$y = \frac{a_w - a}{m} = x_{\Sigma} - \Delta y \quad (4)$$

$$\alpha_w = m \frac{Z_1 + Z_2 \cos \alpha_i}{2 \cos \beta \cos \alpha_{tw}} \quad (5)$$

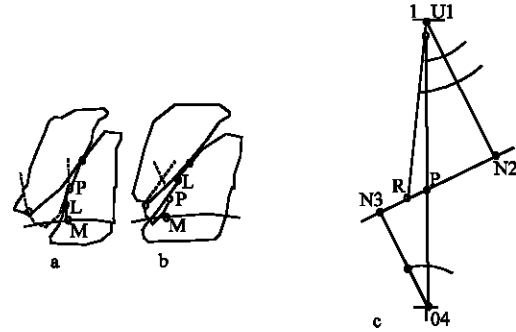


Fig. 1: Interference of the teeth

For this condition in the article one obtains the following results and after a series of transformation one obtains:

$$r_{a2} = m(h_a^* - x_1 + \frac{\alpha_w}{m} - \frac{Z_1}{2 \cos \beta}) \quad (6)$$

One considers the case of limit $\text{tg} \alpha_{pi} = \text{tg} \alpha_{ti}$ dans the frame of reference (X1 and X2), the line which corresponds to this relation is the line of limit which separates the plan in two parts: it is allowed the use of the part in which there is not wedging ($\text{tg} \alpha_{pi} \geq \text{tg} \alpha_{ti}$); and it is inadmissible to use the other part because of the interference ($\text{tg} \alpha_{pi} < \text{tg} \alpha_{ti}$). From 2 and 3, it follows:

$$\text{tg} \alpha_{tw} - \frac{Z_2}{Z_1} (\text{tg} \alpha_{a2} - \text{tg} \alpha_{tw}) = \text{tg} \alpha_i - \frac{h_i^* - h_a^* - x_1}{z_1 \sin 2\alpha_i} \cos \beta \quad (7)$$

For the triangle P1 POz on the figure (c) the theorem of the cosine is applied.

$$r_{a2}^2 = p_1 p^2 - (\frac{d_{w2}}{2}) - 2p_1 p \frac{d_{w2}}{2} \sin \alpha_{tw} \quad (8)$$

A series of relations are used for the transformation of the expression:

$$p_1 p = 0.5 d_{b2} (\text{tg} \alpha_2 - \text{tg} \alpha_{tw}) \quad (9)$$

$$p_1 p = \frac{z_1 m \cos \alpha}{2 \cos \beta} (\text{tg} \alpha_{tw} - \text{tg} \alpha_i + 4 \frac{h_i^* - h_a^* - x_1}{z_1 \sin 2\alpha_i} \cos \beta) \quad (10)$$

One knows according to L.BARTOCHEV,^[5] that:

$$\frac{d_{w2}}{2} = \frac{m Z_2 \cos \alpha_i}{2 \cos \beta \cos \alpha_{tw}} \quad (11)$$

After replacement of (6) and (11) in (8) one obtains:

$$m2(h_{\alpha}^* - X_1 + \frac{\alpha_w}{m} \frac{Z_1}{2\cos\beta})2 =$$

$$(\frac{mZ_1 \cos\alpha_t}{2\cos\beta})(\text{tg}\alpha_{tw} - \text{tg}\alpha_1 + \frac{4(h_1^* - h_{\alpha}^* - X_1)\cos\beta}{Z_1 \sin 2\alpha_t})^2 +$$

$$(\frac{mZ_2 \cos\alpha_t}{2\cos\beta \cos\alpha_{tw}})^2 + 2\frac{mZ_1 \cos\alpha_t}{2\cos\beta}[\text{tg}\alpha_{tw} - \text{tg}\alpha_t +$$

$$\frac{4(h_1^* - h_{\alpha}^* - X_1)\cos\beta}{Z_1 \sin 2\alpha_t}] \frac{mZ_2 \cos\alpha_t}{2\cos\beta \cos\alpha_{tw}} \sin\alpha_{tw}$$
(12)

It is noted that for the established condition there is no interference for any value of the module $m > 0$ (since $m \leq 0$ is physically unreal).

After a series of transformation and the fact that for standardized and not standardized starting contours respects the relation $h_1^* = 2h_{\alpha}^*$ one obtains definitively:

$$\frac{4}{\text{tg}_2\alpha_t} X_1^2 + [8\frac{\alpha_w}{m} 8\frac{h_{\alpha}^*}{\text{tg}_2\alpha_t} - \frac{4(Z_1 + Z_2)\text{tg}\alpha_{tw}}{\text{tg}\alpha}] X_1 +$$

$$\frac{\cos^2\alpha_t}{\cos^2\alpha\beta} [Z_1 + Z_2]\text{tg}\alpha_{tw} - Z_1\text{tg}\alpha_t + \frac{4h_{\alpha}^* \cos\beta}{\sin 2\alpha_t}]^2 +$$

$$Z_2^2 [2\frac{\alpha_w}{m} - \frac{Z_1}{\cos\beta} + 2h_{\alpha}^*]^2 = 0$$
(13)

The equation obtained compared to X_1 is an equation of the form $ax^2 + bx + c$ the limitation avoiding the interference of the wheel z_1 is composed of two curves. These last are cut for the value of the coefficient of total offset $x_E = 0$ and $\alpha_{tw} = \alpha_t$.

For the analogical method and from $\alpha_{p2} \geq \alpha_{t2}$, one obtains the relation which expresses the condition of inexistence of the interference between the base of the tooth of the driven gear Z_2 and the top of the driving wheel Z_1 .

$$\frac{4}{\text{tg}_2\alpha_t} X_2^2 + [8\frac{\alpha_w}{m} - 8\frac{h_{\alpha}^*}{\text{tg}_2\alpha_t} - \frac{4(Z_1 + Z_2)\text{tg}\alpha_{tw}}{\text{Tga}}] X_2 +$$

$$\frac{\cos^2\alpha_t}{\cos^2\beta} [Z_1 + Z_2]\text{tg}\alpha_{tw} - Z_2\text{tg}\alpha_t + \frac{4h_{\alpha}^* \cos\beta}{\sin 2\alpha_t}]^2 +$$

$$Z_2 - (2\frac{\alpha_w}{m} - \frac{Z_2}{\cos\beta} + 2h_{\alpha}^*)^2 = 0$$
(14)

The two curves of limitation avoiding the interference are also cut for the value of the total coefficient $x_E = 0$ and $\alpha_{tw} = \alpha_t$.

By the Eq. 13 and 14, with values given of X_1 or α_{tw} for z_1 and Z_2 known and an arbitrary choice of β ; one can

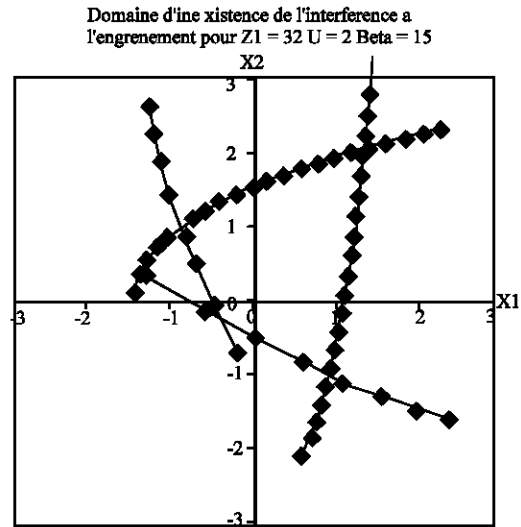
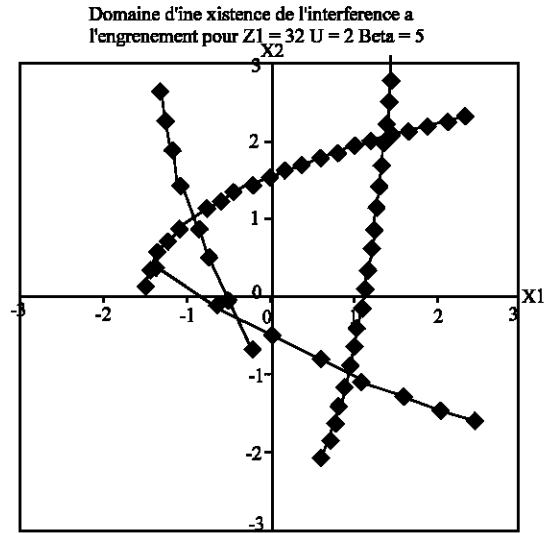


Fig. 2: Acceptable field of the interference to engaging

determine the values of X_1 and respectively X_2 ; one will be able to then build the lines of limit to avoid the interference.

According to the relations (13) and (14), it was elaborate a program for the calculation of X_1 and X_2 for various values of β with the combination of Z_1 , Z_2 and β like with a starting contour standardized (c.a.d; $\alpha = \beta^\circ$; $h_{\alpha}^* = 1$). Once the values obtained one builds the convenient diagrams with the practical use; Fig. 2a, b.

CONCLUSION

We fixed in our research, the choice of the coefficients of offsets (X_1 and X_2) since with the

variations of these coefficients, one can obtain an infinity of gears with different geometries. In our research one realized that one cannot choose X_1 and X_2 on fields also broad since there is risk of the following phenomena:

Interference with cutting and engaging;
Grinding of the teeth Z_1 and Z_2 ;
Report/ratio of control $\varepsilon_\alpha < 1$.

For that:

- One made theoretical studies which determine the limits with the respect of the condition X_Σ .
- One deduced the equations from the limiting curves compared to the interference;
- One worked out a research programme for the establishment of the field of the existence of the cylindrical gear with involute tooth of circle with an arbitrary combination of the geometrical parameters;
- One built graphs for a possible practical use with limiting curves avoiding the interference.

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