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3-D Dynamic Interaction Between Two Rigid Foundations Resting on Layered Soil Parte I: Homogenous Soil

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Abstract: This study dynamic interaction between surface rigid foundations in a homogeneous viscoelastic soil limited by a substratum. The vibrations come from masseles rigid foundations placed in soil layer and subjected to harmonic loads of translation, swing and torsion. Required dynamic response of rigid surface foundations constitute the solution of the waves equations obtained by taking account of the conditions of interaction structure-soil-structure.The solution being formulated in terms of boundary integral equation calculated in the discretized field for which the Green's functions are known for each element. This study allows the establishing of a mathematical model enabling us to determine the impedance (compliance) functions of adjacent foundations according to theirs different distances, the depth of the substratum, the dynamic properties of the soil and the frequency of excitation.

Key words: Dynamic, soil-foundation interaction, Green's functions, boundary integral equation, 3-D analysis

INTRODUCTION

The dynamic foundation-soil-foundation interaction phenomenon has long been recognised as an important factor in the seismic and machine vibration response of critical facilities and other closely spaced structures or portions of a structure.

Rotative machines foundation constitutes a source of vibrations which are transmitted to the surrounding soil. Depending on the energy communicated to the medium, this disturbance may affect greatly either the soil or the adjacent structure. Rational analysis of the phenomenon requires taking into account the dynamic nature of the interaction between the soil and the foundation. This is essentially a wave propagation problem with mixed boundary condition (i.e. rigid body displacement under the foundation and none traction elsewhere).

Although, a solution of a foundation-soil-foundation interaction problem in most cases involves a straightforward application of any of the well established soil-structure interaction methods, a relatively small number of 3-D investigations have appeared in the related literature. This is probably due to the substantial computational effort required by the Finit Element Method and the usual straightforward Boundary Element method formulations. Furthermore, there is a noticeable absence of simplified discrete models which is due, perhaps, to the general lack of rigorous results which would be used for the verification and calibration of such models.

The complicated geometries, loadings and soil conditions, have dicouraged, in general, the development of analytical solutions. Luco[1] determined the impedance functions for a rigid circular disk on layered elastic and viscoelastic medium using an integral equation approch. Apsel and Luco[2] used the integral equation approch based on the green's functions for layered soil media reported in Apsel[3] for the determination of the impedance functions of embedded foundations. Using this approch Wong and Luco[4] studied the dynamic interaction between rigid foundations in a layered half-space. Using a semi-analytical formulation Gazetas and Roesset[5] analysed the 2-D problem of strip foundations on a layered half-space, Boumekik *et al.*[6] studied the 3-D problem of embedded foundations on a layered substratum, on the other hand Sbartai and Boumekik[7-11] analysed the transmission of the wave in the soil layer and interaction between two structures caused by machine foundation. The Finit Element Method has also been applied by Gonzalez[12], Kausel *et al.*[13]; Kausel and Roesset[14] ; Lin *et al.*[15] in determinig the behaviour of rigid foundations placed or embedded in a stratum over bedrock. A frequency domain Boudary Element Method formulation has been developed to treat wave propagation problems, soil-structure problems and structure-soil-structure problems, which limit the discretization at the soil foundations interface. In this approach the field displacement is formulated as an integral equation in terms of Green's functions Beskos[16-21].

In this study, the solution is formulated in frequency domain Boundary Integral Equation Method. Only the foundation-soil-foundation interface and a free

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surface between adjacent foundations are discritised. Within the discritised medium, the Green's functions (displacement of the i^{th} element due to harmonic unit force applied on the j^{th} element) are calculated using the Thin Layers Method[22]. Using this approach we established a mathematical model enabling us to determine the impedance (compliance) functions of adjacent foundations.

Basic equations

Model of calculation: The model of calculation is represented on the Fig. 1. The two foundations considered are supposed to be rigid, of form rectangular (square) placed at the surface or partially embedded in homogeneous soil limited by a substratum. The soil at height H, is supposed to be viscoelastic linear characterized by its mass density ρ, its shear modulus G, its damping coefficient β and Poisson's ratio ν. The foundations are subjected to three harmonic external forces Px, Py, Pz and at three harmonic moments Mx, My, Mz. It is assumed that the time dependence of the excitation is of the type $e^{i\omega t}$ in which ù denotes the frequency. For brevity, this time factor will be omitted in the sequel. The goal being to obtain the impedance or compliance functions of two foundations.

Displacements in an unspecified point α of the soil may be obtaining from the solution of the wave Equation:

$$((C^2_p - C^2_s)u_{j,ik} + C^2_s u_{k,ji} + C^2_p \omega^2 u_k)\rho = 0 \qquad (1)$$

where:
C_s, C_P are the celerities of the waves of shearing and compression, ω the angular frequency of excitation and ρ the mass density of the soil. The solution of Eq. 1 can be formulated by following boundary integral Eq:

$$u\alpha = \int_S U\ \alpha\beta . t\beta . dS\ \ \beta \qquad (2)$$

with:
$U_{\alpha\beta}$ are the Green's functions which represent displacements in a point α had with a load harmonic unit (vertical and horizontal) applied in another point β of soil and t_β represent a harmonic load distributed on a surface of soil dS_β.

As long as the medium is continuous, this last relation remains very difficult to evaluate. However, if the solid mass of the soil is discretized in an adapted way, this relation can be made algebraic and displacement can then be calculated.

Discretization of the model: In this approach, the principle of the discretization of the solid mass of soil is represented on the Fig. 2. It is based on two types of horizontal discretization one and the other vertical.

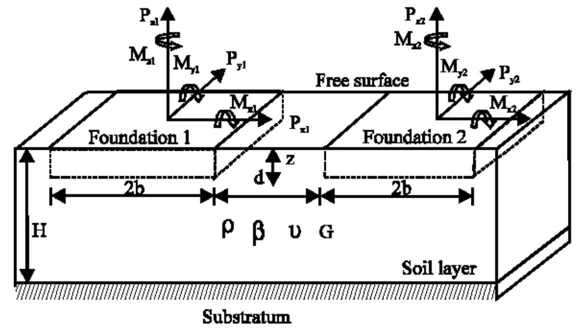

Fig. 1: Model of calcul

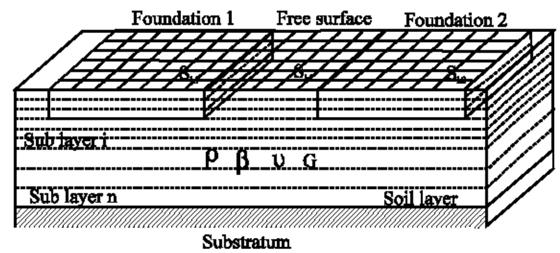

Fig. 2: Horizontal and vertical iscretization

horizontal discretization consists in subdividing any horizontal section of the solid mass of the soil in square elements of S_k sections. The average displacement of the element is replaced by displacement of its center and on which the distribution of the constraints is supposed to be uniform. The vertical discretization consists in subdividing the solid mass of the soil in under layers (Infinite Elements in the horizontal direction) rather low thickness compared to the wavelength of Rayleigh (λ/10) to be able to linearize the displacement of under layer to the other. This discretization being characterized by the embedding of the foundation and the depth of the substratum.

In the discretized model, the Eq. 2 is expressed in algebraic form as follows:

$$d\ \alpha = \sum_{\beta=1}^{NRT} \int_S U\,\alpha\beta . t\beta\ .dS\ \beta \qquad (3)$$

where:
NRT represents the total number of elements discretizing the soil between two foundations and the interface soil-foundations.

Displacement matrix of discretized soil: The total matrix displacement of the soil is obtained by successive application of the loads units distributed on the constituent elements of the solid mass of the discretized soil. This matrix includes the terms of flexibilities of the soil which will be occupied by the first foundation (medium1), the soil which will be occupied by the second foundation (medium 2) and those of the coupling between the two mediums which can be written as follows:

$$[F_t] = \begin{bmatrix} F_1 & F_2 \\ F_{21} & F_2 \end{bmatrix} \quad (4)$$

where:
F_1, $(3N_1 x 3N_1)$ is the flexibility matrix of medium 1. F_2, $(3N_2 X 3N_2)$ is the flexibility matrix of medium 2. F_{12}, $(3N_1 X 3N_2)$ is the flexibility matrix of coupling of medium 1 on the medium 2. F_{21}, $(3N_2 X 3N_1)$ is the flexibility matrix of coupling of medium 2 on the medium 1. N_1 and N_2 are, respectively the number of elements discretizing medium 1 and medium 2.

The Displacements in the two mediums are expressed then by:

$$\{d_1\} = [F_1]\{t_1\} + [F_{12}]\{t_2\} \quad (5)$$

$$\{d_2\} = [F_2]\{t_2\} + [F_{21}]\{t_1\} \quad (6)$$

where:
$\{t_1\} = \{t_{11}, t_{12},..., t_{1k},..., t_{1N1}\}^t$ represent the vector charges of the medium 1 in which $\{t_{1k}\} = \{h,t,n\}^t_k$ is the under-vector charges applied to the disk k, where h, t and n are the harmonic loads distributed according to respective directions x, y and z.
$\{t_2\} = \{t_{21}, t_{22},..., t_{2k},..., t_{2N2}\}^t$ represent the vector charges of the medium 2 in which $\{t_{2j}\} = \{h,t,n\}^t_j$ is the under-vector charges applied to the disk j, where h, t and n are the harmonic loads distributed according to respective directions x, y and z.
$\{d_2\} = \{d_{11}, d_{12},..., d_{1k},..., d_{1N1}\}^t$ represent the vector displacements of the medium 1 in which $\{d_{1j}\} = \{u,v,w\}^k_j$ is the under-vector displacements applied to the disk k.
$\{d_2\} = \{d_{21}, d_{22},..., d_{2k},..., d_{2N2}\}^t$ represent the vector displacements of the medium 2 representing the same characteristics as the vector $\{d_1\}$.

Condition of compatibility and equilibrium: When the two foundations are in place, they impose their displacements on the various sections which will be constrained to move like a rigid body. For all the elements of the model, one can write the following relations:

$$\{d_1\} = [R_1]\{D_1\} \quad (7)$$

$$\{d_2\} = [R_2]\{D_2\} \quad (8)$$

with:
$\{D_1\} = \{\Delta_x, \Delta_y, \Delta_z, \varphi_x, \varphi_y, \varphi_z\}^t_1$ the vector displacement of the first foundation for the 6 degrees of freedom considered; $\{D_2\} = \{\Delta_x, \Delta_y, \Delta_z, \varphi_x, \varphi_y, \varphi_z\}^t_2$ the vector displacement of the second foundation for the 6 degrees of freedom considered and $[R_1] = [R_1, R_2,..., R_k,..., R_{N1}]^t$ is a matrix of transformation of dimension $(3N_1 x 6)$, depending only on the geometrical characteristics of the discretized volume of the soil of the first foundation where under matrix is given by:

$$[R_1]_k = \begin{bmatrix} 1 & 0 & 0 & 0 & z & -y \\ 0 & 1 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix}_k \quad (9)$$

in which x_k, y_k and z_k are the co-ordinates of the element k compared to the center of the foundation; $[R_2] = [R_1, R_2,..., R_k,..., R_{N2}]^t$ is also a matrix of transformation of dimension $(3N_2 x 6)$, depending only on the geometrical characteristics of the discretized volume of the soil of the second foundation where under matrix $[R_2]^j$ is similar to that of the relation (9) in which x_j, y_j and z_j are the co-ordinates of the element j compared to the center of the second foundation.

If one notes P_{i1} and M_{i1} the components of the vector charges applied to the first foundation, the equilibre between the latter and the forces distributed on the elements discretizing the volume of the foundation are expressed for the loads of translations and rotations by:

$$P_{i1} = \sum_{k=1}^{N1} t_k = \sum_{k=1}^{N1} \begin{pmatrix} h \\ t \\ n \end{pmatrix}_k \quad (i = x,y,z) \quad (10)$$

$$M_{i1} = \sum_{k=1}^{N_1} \begin{pmatrix} y \cdot n - z \cdot t \\ z \cdot h - x \cdot n \\ x \cdot t - y \cdot h \end{pmatrix}_k \quad (i = x,y,z) \quad (11)$$

These two last expressions can be put in the following matrix form:

$$\{P_1\} = [R_1]^t . \{t_1\}$$

Same manner, one can write the same thing for the second foundation one has then:

$$P_{i2} = \sum_{j=1}^{N_2} t_j = \sum_{j=1}^{N_2} \begin{pmatrix} h \\ t \\ n \end{pmatrix}_j \quad (i = x,y,z) \quad (13)$$

$$M_{i2} = \sum_{j=1}^{N_2} \begin{pmatrix} y \cdot n - z \cdot t \\ z \cdot h - x \cdot n \\ x \cdot t - y \cdot h \end{pmatrix}_j \quad (i = x,y,z) \qquad (14)$$

These two last expressions can be put in the following matrix form:

$$\{P_2\} = [R_2]^t . \{t_2\} \qquad (15)$$

Response of the model: The relation binding the vector directly charges external $\{P_1\}$ applied to the centre of gravity of the foundation with the vectors displacements $\{D_1\}$ and $\{D_2\}$ can be expressed starting from the relations[5-8,12]by:

$$\{P_1\} = [K_1].\{D_1\} + [K_{12}].\{D_2\} \qquad (16)$$

where:

$$[K_1] = [R_1]^t.[A]^{-1}.[R_1] \qquad (17)$$

is the dynamic stifness matrix of the firts foundation, with

$$[A] = [F_1] - [F_{12}].[F_2]^{-1}.[F_{21}] \qquad (18)$$

$$[K_{12}] = [R_1]^t.[A]^{-1}.[F_{12}].[F_2]^{-1}.[R_2] \qquad (19)$$

is the coupling matrix of the first foundation on the second foundations.

Same manner, one can obtain the relation binding the vector charges external applied to the centre of gravity of the second foundation to the vectors displacements starting from the relations[5-8,15] by:

$$\{P_2\} = [K_2] \cdot \{D_2\} + [K_{21}] \cdot \{D_1\} \qquad (20)$$

where:

$$[K_2] = [R_2]^t \cdot [F_2]^{-1} \cdot [M] \cdot [R_2] \qquad (21)$$

is the Dynamic stifness matrix of the second foundation, with:

$$[M] = [I] + [F_{21}] \cdot [A]^{-1} \cdot [F_{12}] \cdot [F_2]^{-1} \qquad (22)$$

$$[K_{21}] = -[R_2]^t \cdot [F_2]^{-1} \cdot [F_{21}] \cdot [A]^{-1} \cdot [R_1] \qquad (23)$$

is the coupling matrix of the second foundation on the first foundation.

If as supposed the second foundation does not exist, matrix [A] is then equal to the flexibility matrix of the first foundation $[F_1]$ and the expression of the dynamic stifness of the foundation (17) becomes:

$$[K_1] = [R_1]^t \cdot [F_1]^{-1} \cdot [R_1] \qquad (24)$$

who represents the dynamic stifness matrix of single foundation placed on the surface or partially embedded in a mono or multilayer soil.

If the second foundation is unloaded ($P_2 = 0$), the Eq. (16) and (20) becomes:

$$\{P_1\} = [K_1] \cdot \{D_1\} + [K_{12}] \cdot \{D_2\} \qquad (25)$$

$$\{0\} = [K_2] \cdot \{D_2\} + [K_{21}] \cdot \{D_1\} \qquad (26)$$

From there system one can write:

$$[C_{11}] = \left[[K_1] - [K_{12}] \cdot [K_2]^{-1} \cdot [K_{21}] \right]^{-1} \qquad (27)$$

is the compliance matrix of the loaded foundation and

$$[C_{12}] = -[K_2]^{-1} \cdot [K_{21}] \cdot [C_{11}] \qquad (28)$$

is the coupling compliance matrix of the unloaded foundation.

In the following, these two last relations are used to analyses the dynamic interaction between two surface square rigid footings placed on homogenous soil where only one foundation is loaded.

RESULTS

Due the space limitations only the vertical compliances of the foundations are considered according to theirs different separations, depth of substratum, the dynamic properties of the soil and the frequency of excitation.

Validation: The results of this work will be validated while comparing results them obtained by the present study at those obtained by the 3-D frequency domain BEM formulation of Karabalis *et al.*[20]. The comparison relates to the case of a square foundations placed at the surface of a viscoelastic and isotropic semi-infinite soil having the following characteristics: $\rho = 1$, $G = 1$, $s = 1$, $\nu = 0.333$, $\beta = 0.05$, $H/b = 16$ (to approach the semi-infinite one) with $b = 1/2$ is the half wide of the foundations. Only the first footing is loaded with the unit vertical force $P_z = 1$, however the second footing is unloaded. The dimensionless vertical compliance C_v is defined as:

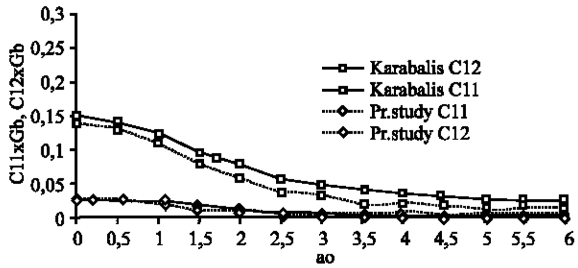

Fig. 3: Validation of the model

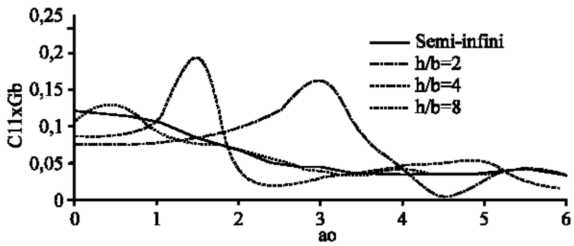

Fig. 4: Vertical compliance C $_{11}$

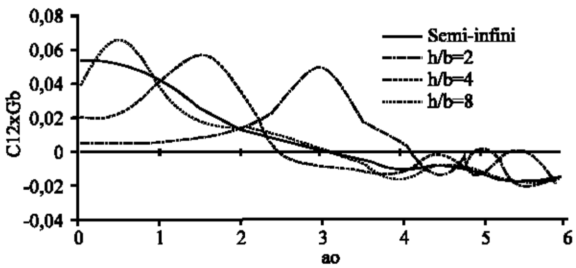

Fig. 5: Vertical coupling compliance C $_{12}$

$$C_v = Gb \cdot K_v^{-1} \tag{22}$$

The soil is discretized horizontally in 9 quadrilateral constant elements on the soil-footings interfaces and 9 quadrilateral constant elements on the free surfaces between the footings. For the vertical discretization the depth of the substratum will be subdivided in 10 under layers. The compliances are calculated at relative distance d/b=2 between two footings versus different dimensionless frequency a_0. The results thus presented on Fig. 3 are practically comparable.

Parametric analysis: In the application, only one foundation is subjected to unit vertical force $P_z = 1$ for different dimensionless frequency $a_0 = \omega b/2C_s$ The soil is discretized in 9 quadrilateral constant elements to the interface soil-foundations and in 9 quadrilateral constant elements to the free surface and is characterised by $\rho = 1$, $G = 1$, $\nu = 0.333$, $\beta = 0.05$. For this, the dimensionless vertical compliance $C_{11}xGb$ for loaded foundation and vertical coupling compliance of the unloaded foundation $C_{12}xGb^2$ have been studied for different cases of relative depth layer stratum (H/b = 2, 4, 8) on relative

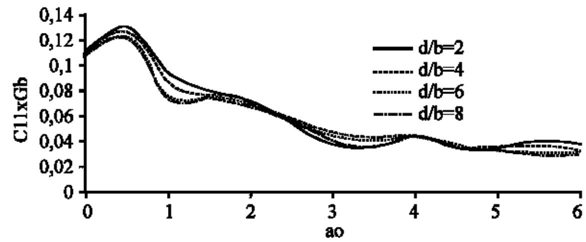

Fig. 6: Vertical compliance C $_{11}$

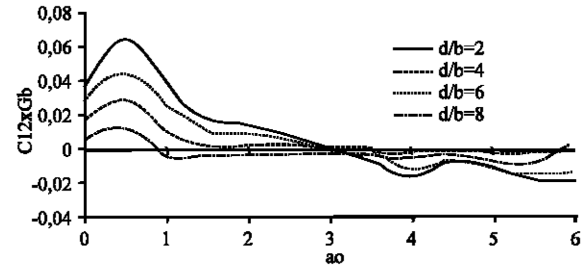

Fig. 7: Vertical coupling compliance C $_{12}$

frequency a_0. In Fig. 4 and 5 the effect of the soil layer depth versus frequency is examined while the foundations are massless, the distance ration between foundations d/b = 2. The dimensionless vertical compliance $C_{mn}(\omega)$ indicate the vertical compliance of the foundation n when the foundation m is loaded with vertical force. While varying the depth of the substratum according to the frequency we noted:

- The static response increase when the layer depth increases.
- The response of the foundation on the stratum approached the semi-infinite solution at layer depth increase.
- A remarkable shift in the resonant frequencies.
- A variation in the peaks of resonance.
- An important variation in the magnitude on the level of the resonant frequencies with the smaller depth of the substratum.

The behaviour of the unloaded foundation is similar to that described above for the loaded foundation, the only difference being that the magnitude of the resonant peak increase with the layer depth increase. The dimensionless vertical compliance $C_{11}xGb$ for loaded foundation and vertical coupling compliance of the unloaded foundation $C_{12}xGb$ have been studied for different cases of distance between two foundations (d/b = 2, 4, 6, 8). In Fig. 6 and 7, the influence of the distance between foundations versus frequency $a_0 = \omega b/2C_s$ is examined while the foundations are massless, the distance ration between foundations H/b = 8 and the damping level is kept constant at $\beta = 0.05$.

It is noticed that the variation magnitude of the compliance of the charged footing is appreciably affected in the vicinity of the maximum values with a reduction magnitudes as the distance between the two footings increases. Concerning coupling compliance of the no charged footing, we noted the same remark but with magnitude much less important. So, we noted that the coupling compliance is more affected by interaction phenomena that the charged footing. If d/b>8, we notice that the first foundation does not influence almost any more the second foundation and of this fact the effect of the interaction between the two foundations cancels itself.

CONCLUSION

In this study, the dynamic interaction between two surface rigid foundations resting on homogenous viscoelastic soil subjected to vertical harmonic external force excitation has been developed and fully tested. The solution is formulated in frequency domain Boundary Element Method in conjunction with the Kausel-Peek Green's function for a layered stratum and quadrilateral constant element to study the dynamic interaction between adjacent footings with which the parameters of interaction structure-soil-structure in a soil layer profile will be numerically given. The advantage of the method used lies in limited the enough number of elements used in the discretization of the model on the one hand and the taking into account of the heterogeneity of the soil on the other hand. This last case will be to study later on in other publications. This study shows well us the great importance of the interaction foundation-soil-foundation which proves to be different from the interaction soil-foundation (single foundation). To this end, we recommend to take into account this phenomenon in account for the study of any structure.

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