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Decay of Temperature Fluctuations in Dusty Fluid MHDTurbulence Before the Final Period in a Rotating System

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Abstract: Using Deissler's approach the decay of temperature fluctuations in MHD turbulence before the final period in a rotating system in presence of dust particle is studied. We have considered correlations between fluctuating quantities at two and three points. In this case two and three points correlations equations in rotating system is obtained and the set of equation is made to determinate by neglecting the quadruple correlations in comparison to the second and third order correlations. The correlation equations are converted to spectral form by taking their Fourier-transforms. Finally, integrating the energy spectrum over all wave numbers, the solution is obtained and this solution gives the energy decay law of temperature fluctuations in dusty fluid MHD turbulence before the final period in a rotating system.

Key words: Deissler's method, dusty fluid MHD turbulence, rotating system, temperature fluctuation

INTRODUCTION

Deissler[1,2] developed a theory for homogeneous turbulence, which was valid for times before the final period. Using Deissler's theory Loeffler and Deissler[3] studied the temperature fluctuations in homogeneous turbulence before the final period. Following Deissler's approach Sarker and Islam[4] also studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the ease of multi-point and multi-time. Sarker and Rahman[5] studied the decay of temperature fluctuations in MHD turbulence before the final period. Islam and Sarker[6] studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Kumar and Patel[7] also studied on fist-order reactant in homogeneous turbulence before the final period of decay for the case of multipoint and multi-time. Sarker and Islam[8] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Sarker and Kishore[9] had been done further work along this same line for the case of multi-point and single time. They considered two and three-point correlations after neglecting higher order correlation terms compared to the second-and third-order correlation terms. Also Kishore and Dixit[10], Kishore and Singh[11] discussed the effect of coriolis force on acceleration covariance in ordinary and MHD turbulence. Shimomura and Yoshizawa[12], Shimomura[13,14] also discussed the statistical analysis of turbulent viscosity, turbulent scalar flux and turbulent shear flows respectively in a rotating system by two-scale direct interaction approach. Sarker and Islam[15] studied the decay of dusty fluid turbulence before the final period in a rotating system.

By analyzing the above theories we have studied the decay of temperature fluctuations in dusty fluid MHD turbulence before the final period in a rotating system. Here two-and three-point correlation equations have been considered after neglecting fourth-order correlation terms in comparison to the second-and third-order correlation terms. Finally, the energy decay law of temperature fluctuations in MHD dusty fluid turbulence before the final period in a rotating system is obtained.

BASIC EQUATIONS

The equation of motion and continuity for viscous, incompressible MHD dusty fluid turbulent flow in a rotating system are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}\left(u_i u_k-h_i h_k\right)=-\frac{\partial w}{\partial x_i}+ \nu\frac{\partial^2 u_i}{\partial x_k \partial x_k}-2\in_{mki}\Omega_m u_i+f(u_i-v_i) \tag{1}$$

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$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k)=\frac{\nu}{P_M}\frac{\partial^2 h_i}{\partial x_k \partial x_k} \quad (2)$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_k}=-\frac{k}{m_s}(v_i-u_i) \quad (3)$$

with

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial v_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}=0 \quad (4)$$

and the equation of energy for an incompressible fluid with constant properties and for negligible frictional heating

$$\frac{\partial T}{\partial t}+u_i\frac{\partial T}{\partial x_i}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2 T}{\partial x_i \partial x_i} \quad (5)$$

The subscripts can take on the values 1, 2 or 3.

Here, u_i, turbulent velocity component; h_i, magnetic field fluctuation component,

$$W(\hat{x},t)=\frac{p}{\rho}+\frac{1}{2}\langle h^2\rangle+\frac{1}{2}\left|\hat{\Omega}\times\hat{x}\right|^2,$$

v_i, dust velocity component total MHD pressure inclusive of potential and centrifugal force;

$p(\hat{x},t)$ = hydrodynamic pressure,

ρ = fluid density,

$p_M=\frac{\nu}{\lambda}$, magnetic prandtl number,

$p_r=\frac{\nu}{\gamma}$, prandtl number,

ν = kinematic viscosity,

$\gamma=\frac{K}{\rho c_p}$, thermal diffusivity,

$\lambda=(4\pi\mu\sigma)^{-1}$, magnetic diffusivity,

c_p = heat capacity at constant pressure,

Ω_m = constant angular velocity components,

$\in_{mki}$ = alternating tensor,

$$f=\frac{kN}{\rho},$$

dimension of frequency; N, constant number density of dust particle,

$$m_s=\frac{4}{3}\pi R^3{}_s\rho_s,$$

mass of single spherical dust particle of radius R_s,

ρ_s = constant density of the material in dust particle,

x_k = Space co-ordinate, the subscripts can take on the values 1, 2 or 3.

TWO-POINT CORRELATION AND SPECTRAL EQUATIONS

The induction equation of a magnetic field at the point p is

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=(\frac{\nu}{p_M})\frac{\partial^2 h_i}{\partial x_k \partial x_k} \quad (6)$$

and the energy equation at the point P\` is

$$\frac{\partial T'_j}{\partial t}+u'_k\frac{\partial T'_j}{\partial x'_k}=(\frac{\nu}{p_r})\frac{\partial^2 T'_j}{\partial x'_k \partial x'_k} \quad (7)$$

Multiplying Eq (6) by T'_j and (7) by h_i, adding and taking ensemble average, we get

$$\frac{\partial\langle h_i T'_j\rangle}{\partial t}+u_k\frac{\partial\langle h_i T'_j\rangle}{\partial x_k}+u'_k\frac{\partial\langle h_i T'_j\rangle}{\partial x^1{}_k}-h_k\frac{\partial\langle u_i T'_j\rangle}{\partial x'_k}=\nu\left[\frac{1}{P_M}\frac{\partial^2\langle h_i T'_j\rangle}{\partial x_k \partial x_k}+\frac{1}{P_r}\frac{\partial^2\langle h_i T'_j\rangle}{\partial x'_k \partial x'_k}\right] \quad (8)$$

Angular bracket $\langle.............\rangle$ is used to denote an ensemble average and the continuity equation is

$$\frac{\partial u_k}{\partial x_k}=\frac{\partial u'_k}{\partial x'_k}=0 \quad (9)$$

Substituting Eq. (9) in to Eq.u (8) yields

$$\frac{\partial\langle h_i T'_j\rangle}{\partial t}+\frac{\partial\langle u_k h_i T'_j\rangle}{\partial x_k}+\frac{\partial\langle u'_k h_i T'_j\rangle}{\partial x'_k}-\frac{\partial\langle u_i h_k T'_j\rangle}{\partial x_k}=\nu\left[\frac{1}{p_M}\frac{\partial^2\langle h_i T'_j\rangle}{\partial x_k dx_k}+\frac{1}{p_r}\frac{\partial^2\langle h_i T'_j}{\partial x'_k \partial x'_k}\right] \quad (10)$$

Using the transformations

$$\frac{\partial}{\partial r_k} = -\frac{\partial}{\partial x_k} = \frac{\partial}{\partial x'_k}$$

and the Chandrasekhar relation[16].

$$\langle u_k h_i T'_j \rangle = -\langle u'_k h_i T'_j \rangle \cdot$$

Equation (10) become

$$\frac{\partial}{\partial t}\langle h_i T'_j \rangle + 2\frac{\partial}{\partial r_k}\langle u'_k h_i T'_j \rangle + \frac{\partial \langle u_i h_k T'_j \rangle}{\partial r_k} = \nu\left[\frac{\partial^2 \langle h_i T'_j \rangle}{\partial r_k \partial r_k}\left(\frac{1}{P_M}+\frac{1}{p_r}\right)\right] \quad (11)$$

Now we write Eq. (10) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms.

$$\langle h_i T'_j(\hat{r}) \rangle = \int_{-\infty}^{\infty} \langle \psi_i \tau'_j(\hat{K}) \rangle \exp\left[i(\hat{K},\hat{r})\right] d\hat{K} \quad (12)$$

$$\langle u_i h_k T'_j(r) \rangle = \int_{-\infty}^{\infty} \langle \phi_i \psi_k \tau'_j(\hat{K}) \rangle \exp\left[\hat{i}(\hat{K},\hat{r},)\right] d\hat{K} \quad (13)$$

$$\langle u'_k h_i T'_j(r) \rangle = \langle u_k h_i T'_j(-\hat{r}) \rangle = \int_{-\infty}^{\infty} \langle \phi_k \psi_i \tau'_j(-\hat{k}) \rangle \exp\left[\hat{i}(\hat{k},\hat{r})\right] d\hat{K} \quad (14)$$

Equation (14) is obtained by interchanging the subscripts i and j and then the points p and P`.

Substituting of Eq. (12) to (14) in to Eq. (11) leads to the Spectral equation

$$\frac{\partial \langle \psi_i \tau_j \rangle}{\partial t} + iK_k\left[2\langle \phi_k \psi_i \tau'_j(-\hat{K}) + \langle \phi_i \psi_k \tau'_j(\hat{K}) \rangle\right] = -\nu\left[\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2 \langle \psi_i \tau'_j(\hat{K}) \rangle\right] \quad (15)$$

The tensor Eq. (15) be comes a scalar equation by contraction of the indices i and j

$$\frac{\partial \langle \psi_i \tau'_i(\hat{K}) \rangle}{\partial t} + iK_k\begin{bmatrix} 2\langle \phi_k \psi_i \tau'(-\hat{K}) \rangle + \\ \langle \phi_i \psi_k \tau'_i(\hat{K}) \rangle \end{bmatrix} = -\nu\begin{bmatrix} \left(\frac{1}{P_M}+\frac{1}{P_r}\right) \\ k^2 \langle \psi_i \tau'_i(\hat{k}) \rangle \end{bmatrix} \quad (16)$$

THREE-POINT CORRELATION AND SPECTRAL EQUATIONS

Similar Procedure can be used to find the three points correlation equation. For this purpose we take the momentum equation of MHD turbulence at the point P, the induction equation at the point P` and the energy Equation at P`` as

$$\frac{\partial u_i}{\partial t} + u_k\frac{\partial u_i}{\partial k_k} - h_k\frac{\partial h_i}{\partial x_k} = -\frac{\partial w}{\partial x_i} + \nu\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\in_{mki}\Omega_m u_i + f(u_i - v_i) \quad (17)$$

$$\frac{\partial h'_i}{\partial t} + u'_k\frac{\partial h'_i}{\partial x_k} - h'_k\frac{\partial u'_i}{\partial x'_k} = \left(\frac{\nu}{P_M}\right)\frac{\partial^2 h'_i}{\partial x'_k \partial x'_k} \quad (18)$$

and

$$\frac{\partial T''_j}{\partial t} + u''_k\frac{\partial T''_j}{\partial x''_k} = \left(\frac{\nu}{P_r}\right)\frac{\partial^2 T''_j}{\partial x''_k \partial x''_k} \quad (19)$$

where

$$W(\hat{x},t) = \frac{P}{\rho} + \frac{1}{2}\langle h^2 \rangle + \frac{1}{2}\left|\hat{\Omega}\times\hat{X}\right|^2,$$

total MHD pressure inclusive of potential and centrifugal force $P(\hat{x},t)$, hydrodynamic pressure; Ωm, constant angular velocity components; $\in_{mki}$, alternating tensor,

$$f = \frac{kN}{\rho},$$

dimension frequency; N, constant number density of dust particle.

Multiplying Eq. (19) by $h'_i T''_j$, (18) by $u_i T''_j$ and (19) by $u_i h'_i$, adding and taking ensemble average, one obtains

$$\frac{\partial \langle u_i h'_i T''_j \rangle}{\partial t} + \frac{\partial \langle u_i u_k h'_i Tj \rangle}{\partial x_k} - \frac{\partial \langle h_i h_k h'_i T''j \rangle}{\partial x''_k} + \frac{\partial \langle u_i u'_k h'_i T''j \rangle}{\partial x'_k} - \frac{\partial \langle u_i u'_i h'_k T''_j \rangle}{\partial x'_k} + \frac{\partial \langle u_i h' u''_k T''_i \rangle}{\partial x''_k} = -\frac{\partial \langle w h'_i T''_i \rangle}{\partial x_i} + \nu\frac{\partial^2 \langle u_i h'_i T''_j \rangle}{\partial x_k \partial x_k}$$
$$+\nu\left[\frac{1}{P_M}\frac{\partial^2 \langle u_i h'_i T''_j \rangle}{\partial x'_k \partial x'_k} + \frac{1}{P_r}\frac{\partial^2 \langle u_i h'_i T''_j \rangle}{\partial x''_k \partial x''_k}\right] - 2\in_{mki}\Omega_m \langle u_i h'_i T''_j \rangle + f(\langle u_i h''_i T''_j \rangle - \langle v_i h'_i T''_j \rangle) \quad (20)$$

Using the transformations

$$\frac{\partial}{\partial x_k} = -(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r'_k}), \frac{\partial}{\partial x'_k} = \frac{\partial}{\partial r_k}, \frac{\partial}{\partial x''_k} = \frac{\partial}{\partial r'_k}$$

in to Eq. (20)

$$\frac{\partial\langle u_i h'_i T''_j\rangle}{\partial t} - \nu\left[\begin{array}{c}(1+\frac{1}{p_M})\frac{\partial^2\langle u_i h'_i T''_j\rangle}{\partial r_k\partial r_k} + (1+\frac{1}{p_r}) \\ \frac{\partial^2\langle u_i h'_i T''_j\rangle}{\partial r'_k\partial r'_k} + 2\frac{\partial^2\langle u_i h'_i T''_j\rangle}{\partial r_k\partial r'_k}\end{array}\right] = \frac{\partial\langle u_i u_k h'_i T''_j\rangle}{\partial r_k} + \frac{\partial\langle u_i u_k h'_i T''_j\rangle}{\partial r'_k} - \frac{\partial\langle h_i h_k h'_i T''_j\rangle}{\partial r_k} - \frac{\partial\langle h_i h_k h'_i T''_j\rangle}{\partial r'_k} - \frac{\partial\langle u_i u'_k h'_i T''_j\rangle}{\partial r_k} + \frac{\partial\langle u_i u'_i h'_k T''_j\rangle}{\partial r_k} - \frac{\partial\langle u_i u''_k h'_i T''_j\rangle}{\partial r'_k} + \frac{\partial\langle w h'_i T''_j\rangle}{\partial r_i} + \frac{\partial\langle w h'_i T''_j\rangle}{\partial r'_i} - 2\in_{mki}\Omega_m\langle u_i h'_i T''_j\rangle + f(\langle u_i h'_i T''_j\rangle - \langle v_i h'_i T''_j\rangle) \quad (21)$$

In order to write the Eq. (21) to spectral form, we can define the following six dimensional Fourier transforms:

$$\langle u_i h'_i(\hat{r}) T''_j(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \beta'_i(\hat{k})\theta''_j(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad (22)$$

$$\langle u_i u_k h'_i(\hat{r}) T''_j(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \varphi_k \beta'_i(\hat{k})\theta''_j(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad (23)$$

$$\langle h_i h_k h'_i(\hat{r}) T''_j(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \beta_i \beta_k \beta'_i(\hat{k})\theta''_j(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad (24)$$

$$\langle u_i u'_k h'_i(\hat{r}) T''_j(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \phi_k(\hat{k})\beta''_i(\hat{k})\theta''_j(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad (25)$$

$$\langle u_i u'_i(\hat{r}) h'_k(\hat{r}) T''_j(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \phi'_i(\hat{k})\beta'_i(\hat{k})\theta'_k \hat{k})\theta''_j(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad (26)$$

$$\langle w h'_i(\hat{r}) T''_j(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \gamma \beta'_i(\hat{k})\theta''_j(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad (27)$$

$$\langle v_i h'_i(\hat{r}) T''_j(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \mu_i \beta'_i(\hat{k})\theta''_j(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad (28)$$

Interchanging the points p` and p`` along with the indices i and j result in the relations

$$\langle u_i u''_k h'_i T''_j\rangle = \langle u_i u'_k h'_i T''_j\rangle .$$

By use of this fact and Eq. (22)-(28), the Eq. (21) may be transformed as

$$\frac{\partial\langle \phi_i \beta'_i \theta''_j\rangle}{\partial t} + \nu\left[\begin{array}{c}(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + \\ 2k_k k'_k + \frac{2\in_{mki}\Omega_m}{\nu} - \frac{f}{\nu}\end{array}\right]\langle \phi_i \beta'_i \theta''_j\rangle = i(k_k + k'_k)\langle \phi_i \phi_k \beta'_i \theta''_j\rangle - i(k_k + k'_k)\langle \beta_i \beta_k \beta'_i \theta''_j\rangle - i(k_k + k'_k)\langle \phi_i \phi'_k \beta'_i \theta''_j\rangle + ik_k\langle \phi_i \phi'_i \beta'_k \theta''_j\rangle + i(k_i + k'_i)\langle \gamma \beta'_i \theta''_j\rangle - f\langle \mu_i \beta'_i \theta''_j\rangle \quad (29)$$

The tensor Eq. (29) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\langle \partial\langle \phi_i \beta'_i \theta''_i\rangle}{\partial t} + \nu\left[\begin{array}{c}(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 \\ +2k_k k'_k + 2\frac{\in_{mki}\Omega_m}{\nu} - \frac{f}{\nu}\end{array}\right]\langle \phi_i \beta'_i \theta''_i\rangle = i(k_k + k'_k)\langle \phi_i \phi_k \beta'_i \theta''_i\rangle - i(k_k + k'_k)\langle \beta_i \beta_k \beta'_i \theta''_i\rangle - i(k_k + k'_k)\times\langle \phi_i \phi_k \beta'_i \theta''_i\rangle + ik_k\langle \phi_i \phi'_i \beta'_k \theta''_i\rangle + i(k_i + k'_i)\langle \gamma_i \beta'_i \theta''_i\rangle - f\langle \mu_i \beta'_i \theta''_i\rangle \quad (30)$$

If derivative with respect to x_i is taken of the momentum Eq. (17) for the point p, the equation multiplied through by $h_i T_j$" and time average taken, the resulting equation

$$-\frac{\partial^2\langle wh_i' T_j''\rangle}{\partial x_i \partial x_i} = \frac{\partial^2}{\partial x_i \partial x_k}\left(\langle u_i u_k h_i' T_j''\rangle - \langle h_i h_k h_i' T_j''\rangle\right) \quad (31)$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_i \partial r_i} + 2\frac{\partial^2}{\partial r_i \partial r_i'} + \frac{\partial^2}{\partial r_i' \partial r_i'}\right]\langle wh_i' T_j''\rangle = \left[\frac{\partial^2}{\partial r_i \partial r_k} + \frac{\partial^2}{\partial r_i' \partial r_k} + \frac{\partial^2}{\partial r_i \partial r_k'} + \frac{\partial^2}{\partial r_i' \partial r_k'}\right] \times\left(\langle u_i u_k h_i' T_j''\rangle - \langle h_i h_k h_i' T_j''\rangle\right). \quad (32)$$

Now taking the Fourier transforms of Eq. (32)

$$-\langle \gamma \beta_i' \theta_j''\rangle = \frac{\left(k_i k_k + k_i' k_k + k_i k_k' + k_i' k_k'\right)\left(\langle \phi_i \phi_k \beta_i' \theta_j''\rangle - \langle \beta_i \beta_k \beta_i' \theta_j''\rangle\right)}{k_i k_i + 2k_i' k_i + k_i' k_i'} \quad (33)$$

Equation (33) can be used to eliminate $\langle \gamma \beta_i' \theta_j''\rangle$ from Eq. (29).

SOLUTION FOR TIMES BEFORE THE FINAL PERIOD

It is known that equation for final period of decay is obtained by considering the two-point correlations after neglecting the 3rd order correlation terms. To study the decay for times before the final period, the three point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms.

Equation (33) shows that term $\langle \gamma \beta_i' \theta_j''\rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of Eq. (30)

$$\frac{\partial\langle \phi_i \beta_i' \theta_i''\rangle}{\partial t} + \nu\left[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2k_k k_k' + \frac{2\in_{mki}\Omega_m}{\nu} - \frac{fS}{\nu}\right]\langle \phi_i \beta_i' \theta_i''\rangle = 0 \quad (34)$$

where $\langle \mu_i \beta_i' \theta_i''\rangle = R\langle \phi_i \beta_i' \theta_i''\rangle$ and 1-R=S, here R and S are arbitrary constant.

Integrating the Eq. (34) between t_o and t with inner multiplication by k_k and gives

$$k_k\langle \phi_i \beta_i' \theta_i''\rangle = k_k\left[\phi_i \beta_i' \theta_i''\right]_0 \exp\left[-\nu\left\{(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2kk'\cos\theta + \frac{2\in_{mki}\Omega_m}{\nu} - \frac{fS}{\nu}\right\}(t-t_0)\right] \quad (35)$$

where θ is the angle between k and k\` and $\langle \phi_i \beta_i' \theta_i''\rangle_0$ is the value of $\langle \phi_i \beta_i' \theta_i''\rangle$ at t= t_o.

By letting r\` = 0 in Eq. (22) and comparing with Eq. (13) and (14), we get

$$\langle \phi_i \psi_k \tau_i'(\hat{k})\rangle = \int_{-\infty}^{\infty}\langle \phi_i \beta_i' \theta_i''\rangle d\hat{k}' \quad (36)$$

$$\langle \phi_i \psi_i \tau_i'(-\hat{k})\rangle = \int_{-\infty}^{\infty}\phi_k \beta_i'(-\hat{k})\theta_i''(-\hat{k}')d\hat{k}' \quad (37)$$

Substituting Eq. (35), (36) and (37) in Eq. (16), we get

$$\frac{\partial\langle \psi_i \tau_i'(\hat{k})\rangle}{\partial t} + \nu(\frac{1}{p_M} + \frac{1}{p_r})k^2\langle \psi_i \tau_i'(\hat{k})\rangle = -\int_{-\infty}^{\infty} ik_k\left[\langle \phi_i \beta_i' \theta_i''\rangle + 2\langle \phi_k \beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\right]_0 \times\exp\left[-\nu(t-t_o)\{(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2kk'\cos\theta + 2\frac{\in_{mki}\Omega_m}{\nu} - \frac{fS}{\nu}\}\right]d\hat{k}' \quad (38)$$

Now, $d\hat{k}'$ can be expressed in terms of k\` and θ as $-2\pi k'^2 d(\cos\theta)dk'$ (Deissler[1]).

Hence.

$$d\hat{k}' = -2\pi k'^2 d(\cos\theta)d\hat{k}' \quad (39)$$

Putting Eq. (39) in Eq. (38) yields

$$\frac{\partial\langle\psi_i\tau_i'(\hat{k})\rangle}{\partial t}+\nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2\langle\psi_i\tau_i'(\hat{k})\rangle=-\int_{-\infty}^{\infty}2\pi i k_k\left[\langle\phi_i\beta_i'\theta_i''\rangle+2\langle\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\right]_0 k'^2 \times\left[\int_{-1}^{1}\exp\left\{\begin{matrix}-\nu(t-t_o)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2\\ +2kk'\cos\theta+2\frac{\in_{mki}\Omega_m}{\nu}-\frac{fS}{\nu}]\end{matrix}\right\}\right] d(\cos\theta)d\hat{k}' \quad (40)$$

In order to find the solution completely and following Loeffler and Deissler[3] we assume that

$$ik_k\begin{bmatrix}\langle\phi_i\beta_i'(\hat{k})\theta_i''(\hat{k}')\rangle+\\ 2\langle\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\end{bmatrix}_0=\frac{\beta_0}{(2\pi)^2}\left(k^2k'^4-k^4k'^2\right) \quad (41)$$

where β_0 is a constant depending on the initial conditions. Substituting Eq. (41) into Eq. (40) and completing the integration with respect to $\cos\theta$ one obtains

$$\frac{\partial 2\pi\langle\psi_i\tau_i'(\hat{k})\rangle}{\partial t}+\nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2 2\pi\langle\psi_i\tau_i'(\hat{k})\rangle=-\frac{\beta_0}{2\nu(t-t_o)}\int_0^{\infty}\left(k^3k'^5-k^5k'^3\right)[\exp\{-\nu(t-t_o)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2-2kk'+\frac{2\in_{mki}\Omega_m}{\nu}-\frac{fS}{\nu}]\}+-\frac{\beta_0}{2\nu(t-t_o)}\int_0^{\infty}\left(k^3k'^5-k^5k'^3\right)\exp\{-\nu(t-t_o)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2kk'+2\frac{\in_{mki}\Omega_m}{\nu}-\frac{fS}{\nu}]dk' \quad (42)$$

Multiplying both sides of Eq. (42) by k^2, we get

$$Q=2Q\pi k^2\langle\psi_i\tau'(\hat{k})\rangle$$

$$\frac{\partial Q}{\partial t}+\nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2Q=F \quad (43)$$

where,

$$Q=2Q\pi k^2\langle\psi_i\tau'(\hat{k})\rangle \quad (44)$$

Q is the Magnetic energy Spectrum function.
and

$$F=-\frac{\beta_0}{2\nu(t-t_0)}\int_0^{\infty}(k^3k'^5-k^5k'^3)\exp\left\{\begin{matrix}-\nu(t-t_0)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})\\ k'^2-2kk'+\frac{2\in_{mki}\Omega_m}{\nu}-\frac{fS}{\nu}]\end{matrix}\right\}+\frac{\beta_0}{2\nu(t-t_0)}\int_0^{\infty}(k^3k'^5-k^5k'^3)\exp\left\{\begin{matrix}-\nu(t-t_0)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2\\ +2kk'+\frac{2\in_{mki}\Omega_m}{\nu}-\frac{fS}{\nu}]\end{matrix}\right\}d\hat{k}' \quad (45)$$

Integrating Eq (45) with respect to k`, we have

$$F=-\frac{\beta_0\sqrt{\pi}p_r^{5/2}}{2\nu^{3/2}(t-t_o)(1+p_r)^{5/2}}\exp\left[-\left\{\frac{2\in_{mki}\Omega_m}{\nu}-\frac{fS}{\nu}\right\}(t-t_o)\right]\times\exp\left[-\nu(t-t_o)\left(1+\frac{1}{p_M}-\frac{p_r}{1+p_r}\right)k^2\right]\begin{bmatrix}\frac{15p_rk^4}{4\nu^2(t-t_o)^2(1+p_r)}+\left\{\frac{5p_r^2}{(1+p_r)^2}-\frac{3}{2}\right\}\\ \frac{k^6}{\nu(t-t_o)}+\left\{\frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{(1+p_r)}\right\}k^8\end{bmatrix} \quad (46)$$

The series of Eq. (46) contains only even powers of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three points at a time.

It is interesting to note that if we integrate Eq (46) over all wave numbers, we find that

$$\int_0^{\infty}Fdk=0 \quad (47)$$

which indicates that the expression for F satisfies the condition of continuity and homogeneity.

The linear Eq. (43) can be solved to give

$$Q=\exp\left[\frac{-\nu k^2(\frac{1}{p_M}+\frac{1}{p_r})}{(t-t_o)}\right]\int F\exp\left[\frac{\nu k^2(\frac{1}{p_M}+\frac{1}{p_r})}{(t-t_o)}\right] \quad (48)$$

$$dt+J(k)\exp\left[-\nu k^2(\frac{1}{p_M}+\frac{1}{p_r})(t-t_o)\right]$$

where

$$J(K)=\frac{N_0k^2}{\pi}$$

is a constant of integration. Substituting the values of F from Eq. (46) in to Eq. (48) and integrating with respect to t, we get

$$Q(\hat{k},t)=\frac{N_0k^2}{\pi}\exp\left[-\nu k^2(\frac{1}{p_M}+\frac{1}{p_r})(t-t_o)\right]+\frac{\beta_0\sqrt{\pi}p_r^{3/2}}{2\nu^{3/2}(1+p_r)^{7/2}}$$

$$\times\exp[-\{2\in_{mki}\Omega_m-fS\}(t-t_o)]\exp\left[\begin{matrix}-\nu k^2(t-t_o)\\ \left\{\frac{1+p_r+p_M}{p_M(1+p_r)}\right\}\end{matrix}\right]$$

$$\left[\frac{3p_rk^4}{2\nu^2(t-t_o)^{5/2}}+\frac{p_r(7p_r-6)k^6}{3\nu(1+p_r)(t-t_o)^{3/2}}-\frac{4(3p_r^2-2p_r+3)k^8}{3(1+p_r)^2(t-t_o)^{1/2}}\right.$$

$$\left.+\frac{8\sqrt{\nu}(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{5/2}\sqrt{p_r}}N(\omega)\right] \quad (49)$$

where

$$N(\omega)=e^{-\omega^2}\int_0^{\omega}e^{x^2}dx,$$

and

$$\omega=k\sqrt{\frac{\lambda(t-t_o)}{p_r(1+p_r)}}$$

The function has been calculated numerically and tabulated in[5].

By setting and

$$\hat{r}=0,\quad j=i,\quad d\hat{K}=-2\pi k^2d(\cos\theta)dk$$

and $Q=2\pi k^2\left\langle\psi_i\tau_i'(\hat{K})\right\rangle$ in Eq. (12), we get the expression for temperature energy decay as

$$\frac{\langle T^2\rangle}{2}=\frac{T_iT_i'}{2}=\int_0^{\infty}Q(\hat{k})d\hat{k} \quad (50)$$

Substituting Eq (49) in to (50) and after integration, we get

$$\frac{\langle T^2\rangle}{2}=\frac{N_oP_r^{3/2}p_M^{3/2}(t-t_o)^{-3/2}}{4\sqrt{\pi}\nu^{3/2}(p_r+p_M)^{3/2}}+\exp\left[-\{2\in_{mki}\Omega_m-fS\}\right]$$

$$\frac{\beta_0\pi p_r^{7/2}p_M^{5/2}(t-t_o)^{-5}}{2\nu^6(1+p_r)(1+p_r+p_M)^{5/2}}\times$$

$$\left\{\begin{matrix}\frac{9}{16}+\frac{5p_M(7p_r-6)}{16(1+p_r+p_M)}-\frac{35p_M^2(3p_M^2-2p_r+3)}{8p_r(1+p_r+p_M)^2}\\ +\frac{8p_M^3(3p_r^2-2p_r+3)}{3.2^6.p_r^2(1+p_r+p_M)^3}\sum_{n=0}^{\infty}\frac{1.3.5....(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\end{matrix}\right\}$$

or

$$\frac{\langle T^2\rangle}{2}=\frac{N_0P_r^{3/2}p_M^{3/2}(t-t_o)^{-3/2}}{4\sqrt{\pi}\nu^{3/2}(p_r+p_M)^{3/2}}+ \quad (51)$$
$$\beta_0z\nu^{-6}(t-t_o)^{-5}\times\exp\left[-\{2\in_{mki}\Omega_m-fS\}\right]$$

where,

$$Z=\frac{\pi p_r^{7/2}p_M^{5/2}}{2(1+p_r)(1+p_r+p_M)^{5/2}}\cdot$$

$$\left[\begin{matrix}\frac{9}{16}+\frac{5P_M(7P_r-6)}{16(1+P_r+p_M)}-\frac{35p_M^2(3p_r^2-2p_r+3)}{8p_r(1+p_r+p_M)^2}+\\ \frac{8p_M^3(3p_r^2-2p_r+3)}{3.2^6p_r^2(1+p_r+p_M)^3}\sum_{n=0}^{\infty}\frac{1.3.5....(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\end{matrix}\right]$$

Thus the energy decay law for temperature field fluctuation of dusty fluid MHD turbulence in a rotating system before the final period may be written as

$$\langle T^2\rangle=X(t-t_o)^{-3/2}+\exp\left[-\{2\in_{mki}\Omega_m-fS\}\right]Y(t-t_o)^{-5} \quad (52)$$

where

$$X=\frac{N_0P_r^{3/2}p_M^{3/2}}{2\sqrt{\pi}\nu^{3/2}(p_r+p_M)}$$

$$Y = 2\beta_0 Z \nu^{-6}$$

$\langle T^2 \rangle$ is the total energy (the mean square of the temperature fluctuations) t is the time, x and t_0 are constants determined by the initial conditions. The constant Y depends on both initial conditions and the fluid Prandlt number.

CONCLUSIONS

In Eq. (52) we obtained the decay law of temperature fluctuations in MHD turbulence before the final period in a rotating system in presence of dust particle considering three-point correlation equation after neglecting quadruple correlation terms. If the fluid is clean and the system is non-rotating then f = 0, Ω = 0 the Eq. (52) becomes.

$$\langle T^2 \rangle = X(t - t_0)^{-3/2} + Y(t - t_0)^{-5} \quad (53)$$

which was obtained earlier by Sarker and Rahman[5]

In the absence of a magnetic field, magnetic Prandtl number coincides with the Prandtl number (i.e. $p_r = p_M$) and the system is non-rotating with clean fluid the Eq. (51) becomes

$$\frac{\langle T^2 \rangle}{2} = \frac{N_0 p_r^{3/2}}{8\sqrt{2\pi}\nu^{3/2}(t - t_0)^{3/2}} + \frac{\beta_0 Z}{\nu^6 (t - t_0)^5} \quad (54)$$

which was obtained earlier by Loeffler and Deissler[3].

Here we conclude that due to the effect of rotation in presence of dust particles in the flow field, the turbulent energy decays more rapidly than the energy for non-rotating clean fluid.

The 1st term of the right hand side of Eq. (52) corresponds to the temperature energy for two-point correlation and second term represents temperature energy for three-point correlation. For large times the last term in the Eq. (52) becomes negligible, leaving the-3/2power decay law for the final period. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the Eq. (52).

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