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Age Associated with Expectation of Life (E_x) for Male of Bangladesh: Application of Polynomial Model

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Abstract: The purpose of the present study is to build the model to expectation of life (e_x) associated with age for male of Bangladesh. For this, the expectation of life (e_x) for male population of Bangladesh in the census years of 1974, 1981 and 1991 have been used in this study. A few mathematical models have been tried to fit to e_x for male of Bangladesh and it is found that they follow polynomial models. The Cross Validity Prediction Power (CVPP) and F-test are also applied to these models to check the validity and significance of the models.

Key words: Expectation of life (e_x) Mathematical modeling coefficient of determination (R^2) Cross-Validity Prediction Power (CVPP) F-test

INTRODUCTION

It is to be mentioned here that mathematical modeling in Population Studies especially in Demography (Mortality, Fertility, Migration, Nuptiality etc.) in Bangladesh have been worked very limited scale. In the process of globalization, mathematical models are very realistic and sophisticated tools to express data in Mathematics. Mathematical model is of great help to demographers in realizing the process in differentiating among various variables to find out the functional relationships among various demographic phenomenons. Mathematical models in demography are basically two groups: stochastic and deterministic. Deterministic model has been studied in the present study. Traditionally, one can draw graphs of the demographic parameters but very few of us know in the context of Bangladesh which models are more appropriate for the parameters.

The reviews of literature on mathematical modeling in this study are as follows:

Age structure, Age Specific Death Rates (ASDRs) and the number of persons surviving at an exact age x (l_x) for male population of Bangladesh in 1991 follow modified negative exponential model, 4th and 3rd degree polynomial model, respectively[1,2]. In[3] it was found that Age Specific Fertility Rates (ASFRs) follows slightly modified biquadratic polynomial model where as forward and backward cumulative ASFRs follow quadratic and cubic polynomial model, respectively in the rural area of Bangladesh. In this study an effort has been given attention to observe the pattern of e_x values for male related to age in mathematics. For this polynomial model is selected to see this pattern. A brief discussion on polynomial model is in the following:

A general expression of the form

$$y = f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_nx^n\ (a_n \neq 0)^{[4]}$$

where a_0 is the constant term; a_i is the coefficient of (i =1, 2, 3, ..., n) but a_1, a_2,..., a_n are also constants but these belong to a field (field means a nonempty set in which group for addition, group for multiplication and left as well as right distributive law hold) and n is the positive integer, is called a polynomial of degree n and the symbol x is called an indeterminate. If n = 0 then it becomes constant function. If n = 1 then it is polynomial of degree 1 i.e. simple linear function. If n = 2 then it is polynomial of degree 2 i.e. quadratic polynomial, etc.[5].

So, the main aims and objectives of the present study are as follows:

- to fit some mathematical models to e_x for male of Bangladesh and
- to apply Cross-Validity Prediction Power (CVPP) to check the validity of the models.

MATERIALS AND METHODS

In the present study e_x values for male population of Bangladesh have been taken from Islam[1] in which six abridged life tables constructed by Widowhod method using marital status information of the censuses of 1974 BBS[6], 1981 BBS[7] and 1991 BBS[8]. These e_x values have been demonstrated in Table 1.

In this study the meaning of expectation of life at age x (e_x) is the average number of years of a survivor to age x is expected to live. It is given by

Table 1: The e_x values for male of Bangladesh

Age group (x)	1974	Predicted (1974)	1981	Predicted (1981)	1991	Predicted (1991)
0	45.15313	55.06246	48.33692	55.7782	55.13	59.15708
1	53.91502	55.02795	55.06998	55.7405	59.62	59.11242
2	56.42797	54.92669	57.00176	55.6298	60.57	58.98164
3	57.30306	54.76196	57.57019	55.4501	60.6	58.76947
4	57.44855	54.53703	57.53177	55.2048	59.65	58.48053
5	57.20167	54.25505	57.17022	54.8976	59.06	58.11933
10	53.71899	52.09578	53.48003	52.5533	54.95	55.37995
15	49.29753	48.94637	48.99051	49.1539	50.31	51.44961
20	44.91232	45.12491	44.54143	45.0571	45.71	46.77219
25	40.56737	40.90588	40.13022	40.5706	41.15	41.72476
30	36.27837	36.52017	35.7744	35.9512	36.63	36.61758
35	32.06113	32.15502	31.48515	31.4055	32.16	31.6941
40	27.97157	27.95407	27.32583	27.0892	27.82	27.13097
45	24.06867	24.01734	23.36027	23.108	23.66	23.03803
50	20.39195	20.40124	19.63775	19.5166	19.77	19.45832
55	17.02981	17.11856	16.26108	16.3196	16.25	16.36807
60	13.93288	14.13846	13.19187	13.4709	13.12	13.6767
65	11.23996	11.38653	10.57707	10.8738	10.56	11.22682
70	8.900003	8.744682	8.355919	8.3814	8.51	8.794245
75	6.912444	6.051261	6.525223	5.796	7.06	6.087975
80	2.5000	3.100974	2.500032	2.86953	2.50	2.750208

$$e_x = \frac{T_x}{l_x}$$

; where T_x is the total person-years lived at age x and l_x is the number of persons surviving at an exact age x[9].

The expectation of life associated with age x (e_x) for male of Bangladesh have been plotted (Fig. 1-3). It appears that e_x can be fitted by polynomial model for different ages. Therefore, an nth degree polynomial model is treated and the model of the nth degree polynomial function is

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u \quad [9]$$

where, x is age group; y is e_x for male; a_i is the constant; is the coefficient of x_i (i =1, 2, 3, ..., n) and u is the stochastic error term of the model. Here we have to select a suitable n so that the error sum of square is minimum.

Using the software STATISTICA, the mathematical models have been estimated. The results on model fittings have been demonstrated in result section.

Checking model validation: The Cross Validity Prediction Power (CVPP), ρ^2_{cv}, is applied to check how much these models are stable. Here

$$\rho^2_{cv} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1-R^2)$$

where, n = the number of cases, k is the number of predictors in the model and the cross validated R is the correlation between observed and predicted values of the dependent variable[10]. The shrinkage of the model is denoted by ζ and is defined by ζ is equal to the absolute value of (R^2-ρ^2_{cv}). The stability of R^2 of the model is denoted by ηand is defined by η=1-ζ. The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 is launched in result section.

F-test: The mathematical formula for F-test is given below:

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)}$$

where n = the number of cases, k = the number of parameters is to be estimated in the model and R^2 is the coefficient of determination in the model[11].

RESULTS AND DISCUSSION

The polynomial model is assumed for the age associated with expectation of life (e_x) for male population of Bangladesh and the fitted models are given below:

$y = 55.06246-0.03507x^2+0.000570x^3 -0.0000029x^4$ in 1974 … (1)

t-stats- (52.60488) (-6.03302) (3.176091) (-2.08266)

p-value- (0.00000) (0.00001) (0.005524) (0.05270)

with proportion of variance explained (R^2)= 0.98234 and is 0.974153.

$y = 55.77821-0.03837x^2+0.000646x^3 -0.0000034x^4$ in 1981 … (2)

t-stats- (69.95744) (-8.66476) (4.726718) (-3.16589)

p-value- (0.00000) (0.00000) (0.000195) (0.00565)

providing R^2 = 0.99018 and = 0.985628.

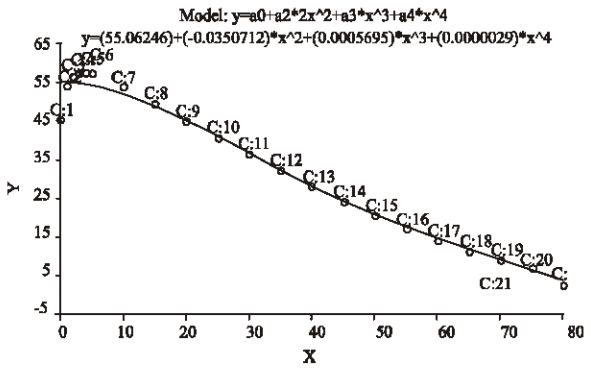

Fig. 1: Observed and fitted e_x values for male of Bangladesh in 1974

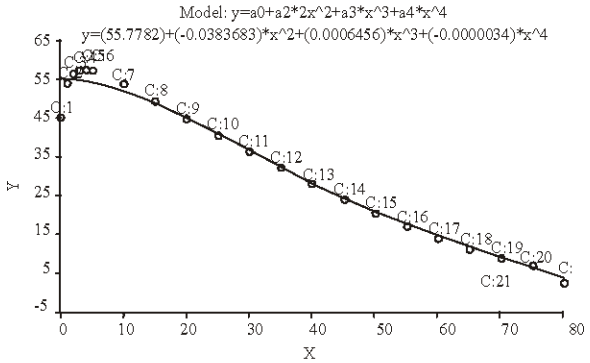

Fig. 2: Observed and fitted e_x values for male of Bangladesh in 1981

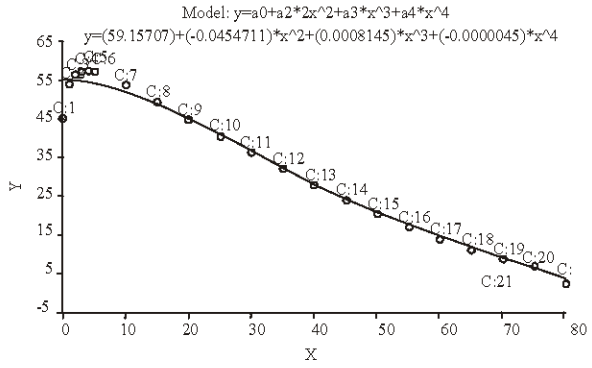

Fig. 3: Observed and fitted e_x values for male of Bangladesh in 1991

$y = 59.1571-0.0455x^2+0.000815x^3 -0.0000045x^4$ in 1991 … (3)

t-stats- (116.3068) (-16.0971) (9.347905) (-6.56480)

p-value- (0.00000) (0.00000) (0.00000) (0.00000)

giving $R^2 = 0.99648$ and is 0.994848.

It is to be noted that the parameter i.e. the coefficient of x is insignificant, so, the term containing x is kept out in the fitted model of e_x. The estimated or predicted values of e_x are presented in Table 1.

From these statistics it is seen that all the parameters of the fitted models are highly significant with significant proportion of variance explained. It is seen that all the fitted (1)-(3) are highly cross-validated and their shrinkages are 0.008187, 0.004552 and 0.0016329, respectively. These imply that the fitted models (1)-(3) will be stable more than 97, 98 and 99%, respectively. Furthermore, the stability of R^2 of these models are more than 99%.

Moreover, in this study F-test is also applied to verify the measure of the overall significance of the model as well as the significance of R^2. The calculated values of F-test of models (1)-(3) are 315.2091, 571.387 and 1604.182 with (3, 17) degrees of freedom (d.f.), respectively. But, the tabulated values of F-test of these models are only 5.18 with (3, 17) d.f. at 1% level of significance. So, from this statistics we see that the models and R^2 of the models are highly statistically significant.

CONCLUSIONS

In this study it is seen that the patern or distribution of e_x associated with age follow polynomial model of degree 4, i.e. biquadratic polynomial model but in which the term containing x is excluded because of the insignificant of the parameter.

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