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Scattering by Random Rough Metallic Surfaces

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Abstract: In order to study the problems of scattering by rough metallic surfaces, we have used Maxwell's equations in covariant form within the framework of a non-orthogonal coordinates system adapted to the geometry of the problem. Electromagnetic fields are written in Fourier's integral form. The solution is found by using a perturbation method applied to the smooth surface problem, this is fully justified when the defects are of small magnitude. For the direct problem, the mean value of diffraction intensity is obtained for random rough surfaces of finite conductivity by computer simulation. In the case of the inverse problem, the reconstruction of the profile of the metal surface from values of the diffraction intensity, obtained by simulation, is found using an iterative algorithm.

Key words: Scattering waves, rough surface, perturbation

INTRODUCTION

The problem of the diffraction of an electromagnetic wave by a rough surface has become during these last years a significant topic of research in electromagnetism[1-7] because of its various applications in optics, acoustics, propagation radio waves and radar techniques. The diffraction of the electromagnetic waves by thin layers used in optics, the surface of the sea or irregular grounds is the object of several applications.

Beside practical problems, various techniques are employed: the Stylet with diamond point (risk of surface deterioration and unreliability of the results), electronic microscopy (requiring expensive equipment). The current tendency is to widen the use of the optical methods based on the study of the diffracted wave generated by surface of interest when it is illuminated by light. Several researchers studied the case of the diffraction of a plane wave by a periodically deformed surface[8-10].

The problem that we will evoke in this article relates to diffraction by a random metallic surface of finite conductivity. One seeks, on the one hand, to characterize the diffracted wave knowing the characteristics of the diffracting profile (direct problem) and on the other hand, to determine the surface topographic characteristics from the diffracted wave characteristics (inverse problem).

The method suggested here is based on the use of the covariant form of Maxwell's equations written in the framework of non-orthogonal coordinate systems adapted to the limits of the structure[11,12]. The deformation is limited in space so that a representation in Fourier integral is possible. The solution is found by using a perturbation method applied to the smooth surface problem, this is fully justified when the defects are of small averaged magnitude (less than 1/10 of the wavelength λ)._One advantage of this method is that it may lead to analytical calculations and thus it allows numerical simulations.

FORMALISM

Problem definition: We consider a surface whose generator in parallel to the oz axis is supported by the curve y= a(x), where a(x) is a function that is, at least, twice derivable of the variable x which represents the form of the surface profile (Fig. 1). We consider this condition for the types of studied surfaces.

A random rough surface is a surface whose profile is not known explicitly but it is defined by its statistical characteristics (average height of roughness, correlation function, etc...).

The metallic surface characterized by its complex index ν is illuminated by a plane wave with a wavelength l where the vector of wave $\vec{k}$ ($|\vec{k}| = 2\pi/\lambda$), located in the xoy plane, forms the incidence angle θ_0 with oy axis, θ_ being the angle of diffraction.

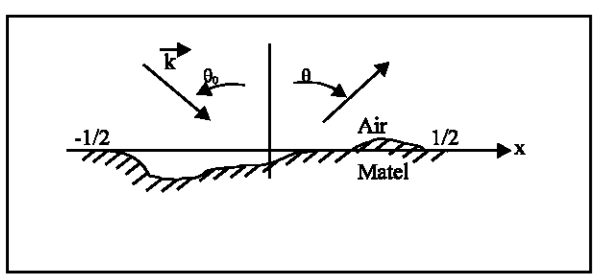

Fig. 1: Rough surface profile

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We show that this problem of diffraction is reduced to the study of two fundamental cases of polarization noted TE (electric transverse) and TM (magnetic transverse), according to whether the electric or magnetic incident field is parallel to the oz[8,12]. The total electric field for the case TE, or the total magnetic field for the case TM, remains parallel to the oz axis. It is the same for both of the electric and magnetic diffracted field.

We will then indicate by F the complex amplitude of the electric field in the TE case or of the magnetic field in the TM case (by adopting a temporal dependence on exp (jωt)).

The incident part F^i of the total field F for a plane wave is expressed by:

$$F^i = \exp j(\beta_0 x - \chi_0 y) \qquad (1)$$

With: $\beta_0 = k\sin\theta_0$ et $\chi_0 = k\sin\theta_0$
In the air, the diffracted field F^d is defined by:

$$F^d(x,y) = F(x,y) - F^i(x,y) \qquad (2)$$

In any diffraction problem, three conditions must be satisfied:

- Maxwell's equations in each medium,
- The condition of radiation ad infinitum,
- The boundary conditions on the surface of separation between the two mediums

From these three conditions, we show that, for all $y > y_m$ wherey_m is the maximum value of a(x), the diffracted field F^d is the sum of planar waves propagating towards $y > y_m$.

$$F^d(x,y) = \int_{-\infty}^{+\infty} R(\beta)\exp(-j(\beta x + \chi y))d\beta \qquad (3)$$

with: $\chi = \sqrt{k^2 - \beta^2}$ si $|\beta| \le k$

$$\chi = j\sqrt{\beta^2 - k^2} \quad \text{si} \quad |\beta| > k \qquad (4)$$

R(β) being the complex amplitude of the diffracted wave in the direction θ such that:

$$\beta = k\sin\theta \qquad (5)$$
$$\chi = k\cos\theta$$

The part of the integral (3), for $|\beta| > k$ corresponds to the evanescent waves.

Equations formulation: The problem can be formulated by using Maxwell's equations in covariant form, written in a coordinated system known as the translation system, which is defined from the Cartesian system (x,y,z) by posing u = y-a(x), a(x) characterizes the deformation (Appendix). The diffracting surface is supposed to be infinite in oz's direction: we consider the problem with two dimensions $(\partial/\partial z = 0)$ and a harmonic wave $(\partial/\partial t = j\omega)$.

As we have already shown[11,12], the general form of Maxwell's equations allowing the simultaneous calculation of the amplitude of the fields in each medium for both modes TE and TM is:

$$\left.\begin{aligned} &\frac{\partial^2 F}{\partial x^2} + k^2 F + (1+a'^2)\frac{\partial^2 F}{\partial u^2} - 2a'\frac{\partial^2 F}{\partial x \partial u} - a''\frac{\partial F}{\partial u} = 0 \\ &kG = j(1+a'^2)\frac{\partial F}{\partial u} - ja'\frac{\partial F}{\partial x} \end{aligned}\right\} \qquad (6)$$

with: $a' = \partial a(x)/\partial x$; $a'' = \partial^2 a(x)/\partial x^2$

For the mode TE: $\begin{cases} F = E_z; & E_x = E_u = ZH_z = 0 \\ G = ZH_x; & ZH_u \ne 0. \end{cases}$

And for the mode TM: $\begin{cases} F = ZH_z; & E_z = ZH_x = ZH_u = 0 \\ G = E_x; & E_u \ne 0. \end{cases}$

λ: is the wavelength in the considered medium.
Z: is the impedance of the medium.

When the deformation is limited in space, the functions a(x), F and G can be expressed in Fourier integral form. Let A(β), f(β,u) and g(β,u) be their Fourier transforms respectively. The use of the convolution property makes it possible to obtain, by applying the Fourier transform of to the system (6), the following system:

$$\left.\begin{aligned} &\frac{\partial^2 f(\beta,u)}{\partial u^2} - \chi^2 f(\beta,u) = \int_{-\infty}^{+\infty}(\alpha^2 - \beta^2)A(\beta-\alpha)\frac{\partial f(\alpha,u)}{\partial u}d\alpha + \\ &\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\gamma(\beta-\alpha-\gamma)A(\beta-\alpha-\gamma)A(\gamma)\frac{\partial^2 f(\alpha,u)}{\partial u^2}d\alpha d\gamma \\ &kg(\beta,u) = j\frac{\partial f(\beta,u)}{\partial u} + j\int_{-\infty}^{+\infty}\alpha A(\beta-\alpha)f(\alpha,u)d\alpha - \\ &j\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\gamma(\beta-\alpha-\gamma)A(\beta-\alpha-\gamma)A(\gamma)\frac{\partial f(\alpha,u)}{\partial u}d\alpha d\gamma \end{aligned}\right\} \qquad (7)$$

Considering: $\gamma^2 = k^2 - \beta^2$

When the average amplitude of the deformation is low compared to the wavelength, a good approximation can be made by seeking the solutions f and g in the form of entire series of a parameter t (0< τ < 1) which is taken to be equal to h/λ (h being the average height of roughness). This choice is very suitable for the majority of the real world problems.

$$f(\beta,u)=\sum_{p=0}^{+\infty}\tau^p f_p(\beta,u); \quad g(\beta,u)=\sum_{p=0}^{+\infty}\tau^p g_p(\beta,u) \tag{8}$$

By substituting the expressions (8) into the differential system (7) and identifying the terms having the same degree in t, we find to the pth order ✖ the following system:

$$\left.\begin{aligned}&\frac{\partial^2 f_p(\beta,u)}{\partial u^2}-\chi^2 f_p(\beta,u)=\int_{-\infty}^{+\infty}\frac{A(\beta-\alpha)}{\tau}(\alpha^2-\beta^2)\frac{\partial f_{p-1}(\alpha,u)}{\partial u}d\alpha+\\&\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\gamma(\beta-\alpha-\gamma)\frac{A(\beta-\alpha-\gamma)}{\tau}\frac{A(\gamma)}{\tau}\frac{\partial^2 f_{p-2}(\alpha,u)}{\partial u^2}d\alpha d\gamma\\&kg_p(\beta,u)=j\frac{\partial f_p(\beta,u)}{\partial u}+j\int_{-\infty}^{+\infty}\alpha(\beta-\alpha)\frac{A(\beta-\alpha)}{\tau}f_{p-1}(\alpha,u)d\alpha-\\&j\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\gamma(\beta-\alpha-\gamma)\frac{A(\beta-\alpha-\gamma)}{\tau}\frac{A(\gamma)}{\tau}\frac{\partial f_{p-2}(\alpha,u)}{\partial u}d\alpha d\gamma\end{aligned}\right\} \tag{9}$$

SOLUTION

To study the problem of diffraction by a rough metal surface of finite conductivity, we are brought to solve the system (9) in the two mediums (air, conductor). This is done by recurrence starting from the undisturbed case corresponding to a planar surface and whose solutions are known. We define a quantity known as " diffracted intensity " of light as the ratio of the power diffracted by the rough part in the direction θ_ (β = k sinθ) with the incident power:

$$I^d(\theta)=\frac{P^d(\theta)}{P^i}=\frac{\mathrm{Re}\left[f^d(\beta,u).g^{d*}(\beta,u)\right]}{\mathrm{Re}\left[f^i(\beta_0,u).g^{i*}(\beta_0,u)\right]} \tag{10}$$

Re [S] indicates the real part of S. The terms in u are simplified in the expression (10). With a first order disturbance, we find the following expression:

$$I^d(\theta)=\vartheta(\beta,\beta_0)\left|A(\beta-\beta_0)\right|^2 \tag{11}$$

with:

$$\vartheta(\beta,\beta_0)=\begin{cases}\left|2k\chi_0\dfrac{\chi_0-\chi'_0}{\chi+\chi'}\right|^2\dfrac{\chi}{\chi_0} & \text{mode TE}\\ \left|2k\chi_0\dfrac{(1-\nu^2)(\chi'_0\chi'-\nu^2\beta\beta_0)}{(\chi'_0+\nu^2\chi_0)(\chi'+\nu^2\chi)}\right|^2\dfrac{\chi}{\chi_0} & \text{mode TM}\end{cases}$$

and: $\nu^2=\varepsilon_r-j\dfrac{\sigma}{\omega\varepsilon_0}$, $k'=\nu k$ (ν medium index) ,

$$\chi_0=\sqrt{k^2-\beta_0^2};\quad \chi'_0=\sqrt{k'^2-\beta_0^2}=\chi'_{01}+j\chi'_{02}$$

$$\chi=\sqrt{k^2-\beta^2};\quad \chi'=\sqrt{k'^2-\beta^2}=\chi'_1+j\chi'_2$$

Where l is the width of the deformation.

Simple letters correspond to the vacuum, whereas the letters with prime correspond to metal medium.

DIRECT PROBLEM AND RESULTS

The random surface a(x) will not be defined analytically. It will be represented by its statistical characteristics. From the expression of the diffraction intensity (11) and by carrying out an average of a great number of surfaces, we obtain the average value of the diffraction intensity given by:

$$\overline{I^d(\theta)}=\vartheta(\beta,\beta_0)\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{1}{2}-x}^{\frac{1}{2}-x}\overline{a(x).a(x+\kappa)}.\exp(j(\beta-\beta_0)\kappa)dxd\kappa \tag{12}$$

By recalling that a(x) is null apart from the interval [-1/2, 1/2] and by supposing that the width of the transition zones between the plane zones and the modulated zone of the surface (to ensure the continuity of the first and second derivatives) is small compared to the width of the surface, $\overline{a(x).a(x+\kappa)}$ can be replaced by φ (K) for k $\in$ [-1,1], which gives:

$$\overline{I^d(\theta)}=\vartheta(\beta,\beta_0)\int_{-1}^{1}(1-|\kappa|)\varphi(\kappa).\exp(j(\beta-\beta_0)\kappa)d\kappa \tag{13}$$

According to the theorem of convolution, we find that:

$$\overline{I^d(\theta)}=\vartheta(\beta,\beta_0).l^2.\phi(\frac{\beta-\beta_0}{2\pi})*\mathrm{sinc}^2(\frac{\beta-\beta_0}{2\pi}l) \tag{14}$$

whereϕ is the spectral density of the surface and (*) represents the convolution product.

The average value of the diffraction intensity is obtained from the expression (12), applied numerically to five hundred samples of random rough surfaces length l (l=20λ generated by simulation methods (Fig. 2) for the various values of the average coefficient of roughness (lT=1, 10, 30, 100 with lT=l/T. T being the distance of correlation). The numerical procedure used to generate such a surface is a spectral method exposed by Thorsos [13]. Every graph has traced for the following numerical values: wavelength λ=0,6330mm, average height of roughness h=0,05 L, metal: Silver, angle of incidence_θ_0 = 30°.

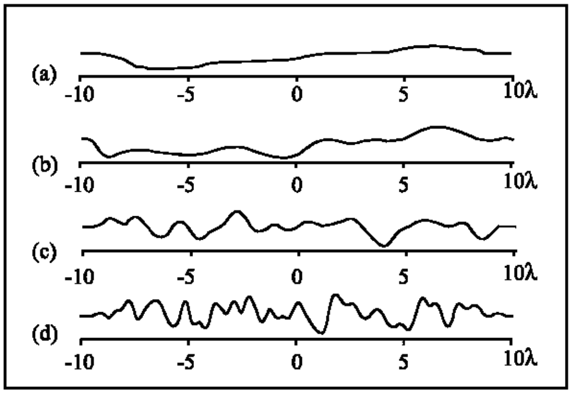

Fig. 2:Profile of the random surface
(a) lT=1, (b) lT=10, (c) lT=30, (d)lT=100.

For lT=1, the curve presents a maximum in the specular direction. This is due to the fact that the surface is almost flat. As soon as we increase the average roughness coefficient, this phenomenon tends to disappear and oscillating curves show up corresponding to a successive band of bright and dark zones which is a well-known phenomenon to opticians. A secondary maximum appears on the graphs in the direction of incidence: it is the phenomenon of Backscattering. It is noted that with five hundred (500) samples, the average value of the intensity is close to the theoretical value given by the expression (14). This result will be better if a larger number of samples is used.

Table 1 represents the average value of the total diffracted power (integrated in all the directions above the deformed surface, only the contribution of the deformed part is represented). For each polarization, the wave arrives under an angle of 30°. Four types of metals (infinitely conducting, Aluminium, Silvery and Gold) are represented.

The remarks drawn from this table are as follows:

- In TE mode, Silver has a coefficient of reflectivity larger than that of Gold and Aluminium and close to the infinitely conducting case.
- In TM mode, Gold has a coefficient of reflectivity larger than that of Silver and Aluminium and close to the infinitely conducting case.
- In TM mode, the value of the diffracted power is clearly more significant than in TE mode.

Table 1: Average value of the total diffracted intensity

		Inf. Cond.	Aluminium	Silver	Gold
Mode	TE	$1{,}632\text{x}10^{-3}$	$1{,}025\text{x}10^{-3}$	$1{,}415\text{x}10^{-3}$	$1{,}210\text{x}10^{-3}$
	TM	$2{,}734\text{x}10^{-3}$	$2{,}124\text{x}10^{-3}$	$2{,}321\text{x}10^{-3}$	$2{,}405\text{x}10^{-3}$

Angle of incidence $\theta_0 = 30°$, LT=30, NE=500.
Metal: Aluminium $\nu = 1{,}2099\text{-j}6{,}4299$
Silver $\nu = 0{,}06699\text{-j}4{,}040$
Gold $\nu = 0{,}1619\text{-j}3{,}2103$.

Inverse problem and results

For the inverse problem, the theoretical difficulties are larger than those of the direct problem. The search for the solution encounters instability complications. The studied inverse problem is the following: knowing the intensity of diffraction, rebuild the profile of rough surface. At the beginning, one is given the intensity of diffraction I_m measured for a certain range of the diffraction angle as well as a coarse estimate of the interval L. In fact, in the study presented here, intensity I_m consists of measurements simulated using a computer in the following way: a certain profile a_m is a priori selected; I_m is then the intensity corresponding to this profile calculated a computer using a direct program. As the process continues, a_m "is forgotten ", and one tries to rebuild it starting from I_m .

The intensity of diffraction corresponding to an unspecified profile a(x) depends obviously on a(x); it is noted. $I(\theta,a)$ This correspondence is not linear. Initially, a natural idea consists of restricting the study to small variations δI and δa for I (θ) and a(x). With a first order approximation in $\delta a\_$, there is a linear relation between δI and δa expressed by:

$$\delta I(\theta) = \int_{-l/2}^{l/2} N(\beta,a,x).\delta a(x)dx \qquad (15)$$

where the function $N(\beta,a,x)$ is called as kernel.

From the relation (15), linear to the first order between the intensity of diffraction I and the deformation a(x), we can solve the inverse problem using an iterative algorithm. That is to say I (θ) intensity of diffraction is given a priori, and a(x) a profile of the associated rough surface that we admit the existence. Let us also suppose that a first approximation $a_1(x)$ (even coarse) of a(x) is known. With the help of the algorithm associated with the direct problem, one can calculate the intensity of diffraction I_1 associated to a_1 .If a_1 is not too far away from the solution a, the functions $\delta a=a-a_1$ and $\delta I=I-I_1$ are small variations roughly bound by the relation (15).

Let us calculate the function a_2 solution of the equation:

$$I - I_1 = \int_{-l/2}^{l/2} N(\beta,a_1,x).(a_2 - a_1)dx \qquad (16)$$

The function $N(\beta,a_1,x)$ is associated with known and calculable surface $a_1(x)$.

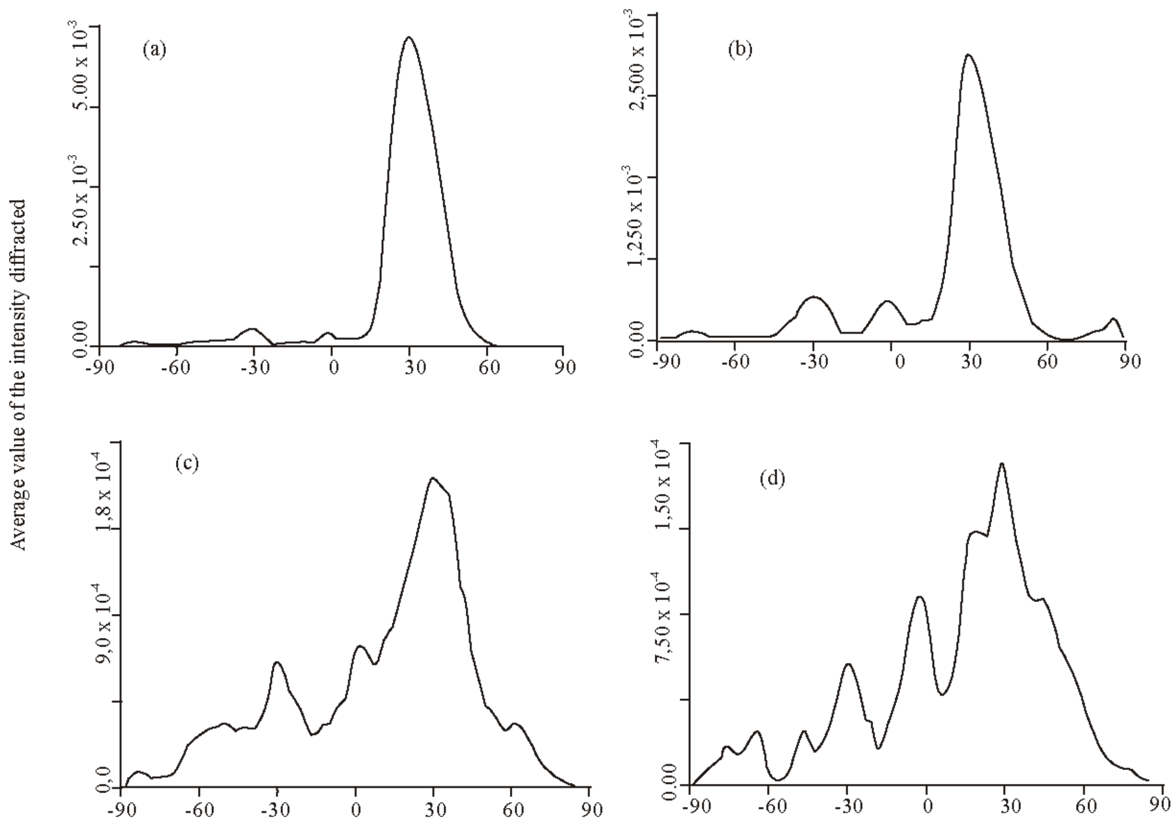

Fig. 3: Average value of the diffracted intensity Angle of incidence θ_0 = 30° metal: Money, Polarization TE, number of sample NE=500. (a) lT=1, (b) lT=10, (c) lT=30, (d) lT=100

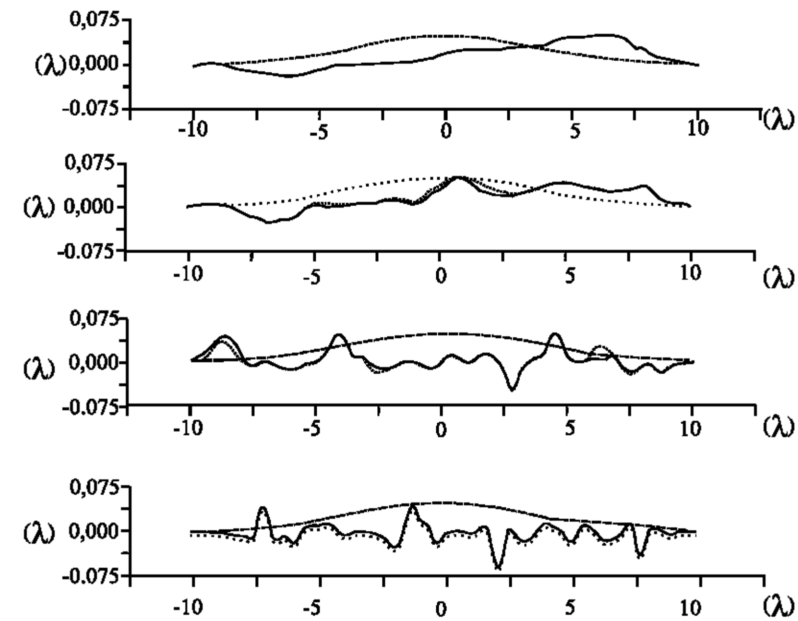

Fig. 4: Reconstruction profile of random surface (a) IT= 1 (b) IT=10 © IT= 30, (d) IT= 100 (______) True profile (_ _)Estimate, (...........) Reconstrution profile

However, a solution without some precautions may lead after the first iterations to instability. To avoid this problem, it is necessary to find a method of regularization. The regularization consists in replacing the solution of (15) by the following variational problem: find the function δa which produces the minimum of the expression:

$$J(\delta a) = J_1(\delta a) + rJ_2(\delta a) \quad (17)$$

with: $J_1(\delta a) = \left\| \int_{-1/2}^{1/2} N(\beta,a,x).\delta a(x)dx - \delta I(\beta) \right\|^2$;

$J_2(\delta a) = \|\delta a(x)\|^2$. According to (11), the kernel N (β, a, x) will be given as:

$$N(\beta,a,x) = 2\vartheta(\beta,\beta_0)\int_{-1/2}^{1/2} a(x')\times\cos\left[(\beta-\beta_0)(x'-x)\right]dx' \quad (18)$$

If δa carries out the minimum of J(δa), so will be $\delta J(\delta a_r)=0$..One obtains a first order Fredholm integral equation which should be solved numerically to determine δa_r The determination of the parameter of r is performed in order to ensure stability. All of the study is restricted to the TM case, but the same method would be quite applicable to the TE case. The first estimate of surface to be simulated is Gaussian.

Figure 4 shows an example of reconstructing random surfaces with weak height, illuminated under the incidence angle θ_0 = 60° by a planar wave length λ = 0,6330 m m and presenting a modulated area of a width 20λ_ with Silver as a metal. The smaller is the average number of roughness; the better is the reconstruction of the profile. As soon as lT is increased, some difficulties emerge in the rebuilding of the profile.

CONCLUSIONS

With an aim of studying in a precise way the problem of diffraction by a rough metal surface, we used Maxwell's equations in covariant form and the reference frame of translation to write in a simple analytical way the boundary conditions. By considering the function a(x) which represents the deformation as a disturbing function, we could seek the solutions by a method of perturbation.

The study of the direct problem enabled us to find the analytical expressions of the intensities of diffraction in the general case (the form and the diffracting surface metal). We can generalize this method for the case of rough dielectric surfaces of one or several layers used in integrated optics.

The algorithm used for the inverse problem gave us a good reconstructed profile diffracting in the field of resonance.

APPENDIX

Maxwell's equations under covariant form in translation coordinate.

In an arbitrary frame of reference (orthogonal or not) of a three-dimensional space , the Maxwell's equations in covariant form, between the covariant components E^i and H^i of the vectors $\vec{E}$ and $\vec{H}$ and the contravariant components D^i and B^i of the pseudo vectors $\vec{D}$ and $\vec{B}$ are written as follows[14]:

$$\begin{aligned} \xi^{ijk}\partial_j E_k &= -\frac{\partial B^i}{\partial t}, \\ \partial_i B^i &= 0, \\ \xi^{ijk}\partial_j H_k &= \frac{\partial D^i}{\partial t}, \\ \partial_i D^i &= \rho, \end{aligned} \quad (19)$$

i,j,k=1,2,3. where ξ^{ijk} represents the Levi–Civita` indicator'

The analytical form of these equations is not affected by the frame of reference (these are the affine equations) contrary to the classical case ($\mathrm{rot}\ \vec{E} = -\partial\vec{B}/\partial t$).

For an unspecified medium these equations are accompanied by the relations of medium, they depend on the frame of reference (metric relations):

$$D^i = \sum_j \varepsilon^{ij} E_j, \qquad B^i = \sum_j \mu^{ij} H_j. \quad (20)$$

For an isotropic homogeneous medium of permittivity ϵ_ and permeability μ_θ the pseudo-tensors of medium ε^{ij} and μijare written:

$$\varepsilon^{ij} = \varepsilon\sqrt{g}g^{ij}, \qquad \mu^{ij} = \mu_0\sqrt{g}g^{ij}. \quad (21)$$

$g^{ij}$and g_{ij} are the contravariant or covariant components metric tensor of the frame of reference and G is the determinant of this same metric tensor.

$$g = \det(g_{ij}) \quad (22)$$

From these and relation Maxwell's equations of medium and for a monochromatic wave ($\frac{\partial}{\partial t} = j\omega t$), it is then possible to write the following equations of propagation:

$$\xi^{ijk}\xi^{lmn}\partial_j \frac{g_{kl}}{\mu\sqrt{g}}\partial_m \begin{matrix} E_n \\ H_n \end{matrix} - \left\{k^2\sqrt{g}g^{ij}\right\}_{H_j}^{E_j} = 0 \quad (23)$$

with: k= ω/c and $\varepsilon_0\ \mu_0\ c^2 = 1$.

The coordinated translation system x,u,z is defined starting from the Cartesian system x, y,z by letting:

$$(g_{ij}) = \begin{bmatrix} 1+a'^2 & a' & 0 \\ a' & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ And } (g^{ij}) = \begin{bmatrix} 1 & -a' & 0 \\ -a' & 1+a'^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

With: $a' = \frac{da(x)}{dx}$

In the case of the problems with two dimensions ($\frac{\partial}{\partial z} \equiv 0$), there are two types of solution (or polarization):

TE = Transverse Electric, $E_z \neq 0$, $H_x = 0$,
TM = Transverse Magnetic, $E_z = 0$, $H_x \neq 0$,

It is possible to find a single system of equation for the two types of polarization, by letting:

$$\text{TE} \quad \begin{array}{c} E_1 = E_x = 0,\ E_2 = E_u = 0,\ H_1 = H_x = \sqrt{\varepsilon/\mu}G, \\ H_2 = H_u \quad 0, \\ E_3 = E_z = F,\ H_3 = H_z = 0. \end{array} \qquad (24)$$

$$\text{TM} \quad \begin{array}{c} E_1 = E_x = -G,\ E_2 = E_u \quad 0,\ H_1 = H_x = 0, \\ H_2 = H_u = 0, \\ E_3 = E_z = 0,\ H_3 = H_z = \sqrt{\varepsilon/\mu}F. \end{array} \qquad (25)$$

If a surface with u=constant separates two mediums, then F and G are continuers on this surface.

A first relation for F is given by the equation of propagation (23) for I =3:

$$\frac{\partial^2 F}{\partial x^2} + k^2 F + (1+a'^2)\frac{\partial^2 F}{\partial u^2} - 2a'\frac{\partial^2 F}{\partial x \partial u} - a''\frac{\partial F}{\partial u} = 0 \qquad (26)$$

One second equation for G is obtained from (19), for example for TE polarization:

$$\xi^{ijk}\partial_j E_k = -\frac{\partial B^i}{\partial t} = -\ j\omega\mu\sqrt{g}g^{ij}H_j \qquad (27)$$

By taking into account the metric tensor, this will give for i = 1 and I =2, the following expressions:

$$\begin{array}{l} \partial_2 E_3 = -\ j\omega\mu H_1 + j\omega\mu a' H_2, \\ -\partial_1 E_3 = j\omega\mu a' H_1 - j\omega\mu(1+a'^2)H_2. \end{array} \qquad (28)$$

After linear combination and using the functions F and G, we can write:

$$kG = j(1+a'^2)\frac{\partial F}{\partial u} - ja'\frac{\partial F}{\partial x} \qquad (29)$$

It is also possible to obtain the same relation for the modes TM.

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