Comparison of Six Different Mathematical Models to the Lactation Curve of Simmental Cows Reared in Kazova State Farm

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Abstract: The aim of this study was to investigate the fitness of Incomplete Gamma (WD), Quadratic (Q), Cubic (C), Exponential (WIL), Mixed Log (ML) and Polynomial Regression (AS) models to the lactation curve of Simmental Cows. Data were collected from 752 Simmental cows raised on the Kazova state farm in Turkey. Milk yield was recorded monthly. Total milk yields estimated by the 6 models were very close to real total milk yield. The 6 models were found to be adequate for estimation of milk yield. ML model underestimated the peak yield significantly. The differences between peak yields of the models and real peak yield were statistically insignificant ($p > 0.05$) and they were ranged from 13.19-18.59 l. Cubic models forecasted peak time earlier than real peak time. The differences between the persistency values of the 6 models were insignificant ($p > 0.05$). The 6 model's persistency value was nearly equal to real persistency value. R^2 values of the 6 models changed from 92.69-95.14%. The highest value of R^2 was found in AS model. Furtermore, least value of MSPE was found in AS model. Consequently, the entire model's showed the best fit to the lactation data of Simmental cows and allowed a suitable definition of the lactation curve.

Key words: Simmental, cows, lactation curve, milk yield, mathematical model

INTRODUCTION

In animal breeding, explanation of yields with mathematical models ensures early estimation of yields for both one yield period and during animal's lifetime. First mathematical model which explanated lactation curves was informed by Brody et al. (1923). This model is:

$$Y_{(w)} = ae^{-cw}$$

where,

$Y_{(w)}$: Milk yield in wth week.

a and c : Paramaters of lactation curve.

e : Logarithm bottom.

After this model, parabolic upper function ($Y_{(w)} = ae^{(bw-cw2)}$) developed by Sikka (1950), reverse polinomial model ($Y_{(w)} = w/(a + bw + cw^2)$) developed by Nelder (1966), Gamma function modelm ($Y_{(w)} = aw^b e^{-cw}$) used commonly and declared by Wood (1967). Modificated Gamma function model ($Y_{(w)} = awe^{-cw}$) declared by Jenkins and Ferrell (1984). Square model ($Y_{(w)} = a + bw + cw^2$) declared by Dave (1971) were used, respectively (Landete-Castillejos and Gallego, 2000; Keskin and Tozluca, 2004).

The purpose of the present study was to determine mathematical model which is the most accurately explanated lactation curves of Simmental cows reared in Kazova State Farm.

MATERIALS AND METHODS

Research materials consisted of lactation record of 752 heads of Simmental cattle herd of Kazova state farm belonging to the period of 1992-2001 years. In this farm, the total milk yield was estimated from test milk yields collected once a month during all lactation periods using the Fleischmann method as below:

$$TMY = y_1 t_1 + \sum \left((y_i + y_{i+1})/2(t_{i+1} - t_i) \right)$$

where,

TMY = Total milk yield.

y_1 = Yield at first milk record.

t_1 = Interval between calving and first recording.

y_i = Yield of the ith record.

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t_i = Interval between the ith record and the record $(i + 1)$ $(i = 1, ..., k)$ (Ruiz *et al.*, 2000).

In this study, to explanate lactation curves, 6 different mathematical model was used. These models are as follows:

- Incomplete gamma (WD) (Wood, 1967):

$$Y_{(t)} = at^b e^{-ct}$$

- Quadratic (Q) (Dave, 1971):

$$Y_{(t)} = a + bt + ct^2$$

- Cubic (C):

$$Y_{(t)} = a + bt + ct^2 + dt^3$$

- Exponential (WIL) (Wilmink, 1987):

$$Y_{(t)} = a + be^{-kt} + ct$$

(Which was fitted with the parameter k fixed at 0.61).

- Mixed log (ML) (Guo and Swallve, 1995):

$$Y_{(t)} = a + bt^{1/2} + c \log t$$

- Polynomial Regression (AS) (Ali and Schaffer, 1987):

$$Y_{(t)} = a + bt + ct^2 + d \lg o t + e \log t^2$$

For all models, Y_t is daily milk yield at tth lactation week, e is the base of natural logarithm, a, b, c, d and e are parameters which characterize the shape of the curve and which were estimated from a nonlinear regression analysis.

Really Peak Yield (PY) was assumed as the maximum test day milk yield and really Peak Time (PT) was accepted as the test time at which daily milk yields were maximum. PT values of the models were calculated by equalizing the first partial derivations of the functions to zero. And PY values were found by replacing PT values in the functions.

Persistency (P) was calculated as:

$$P\,(\%) = \frac{\sum_{i=1}^{k} (pi+1)/pi}{k} \times 100$$

where,
p_i = Yield of the ith record that starts at peak time.
k = The record number from peak time to the end of lactation.

RESULTS AND DISCUSSION

Means of milk yield in first test day was 17.40 kg. Means of milk yield decreased with number of test day. Means of milk yield in last test day or 10 test days decreased to 10.55 kg.

In this study, milk yield estimated with 6 different mathematical models was found rather near to real test day milk yield (Fig. 1).

Calculating of lactation curve parameters for 6 different mathematical models was presented in Table 1.

In this study, a parameter which was explained as beginning milk yield was found the lowest value as 16.182 ± 2.2194 in AS model. However, a parameter was found the highest as 23.299 ± 0.8478 in ML model. Value of a parameter was agrement with Orhan and Kaygısız (2002). In other models, a parameter was similar to one another in this study.

In this study, b was parameter which was explained as slope until accession of the value of highest milk yield. This value was found as the highest in AS model and as the lowest in ML model.

In this study, c was parameter which was explained as slope after the value of highest milk yield. This value was found as the highest in ML model and as the lowest in AS model.

Total Milk Yield (TMY), maximum milk yield (PY), accession time to maximum milk yield (PT), Persistency (P), determination coefficient (R^2) and the Mean Square Prediction Error (MSPE) was presented in Table 2.

As be seen in Table 2, total milk yield in all models was nearly similar. Differences among 6 models was statistically insignificant (p>0.05). This result determines that 6 models can be enough in estimation of total milk yield.

Value of persistency for real data was calculated as 86.62, value of persistency was calculated as 86.39 in AS model and as 86.58 in ML model. Differences among models were statistically insignificant (p>0.05).

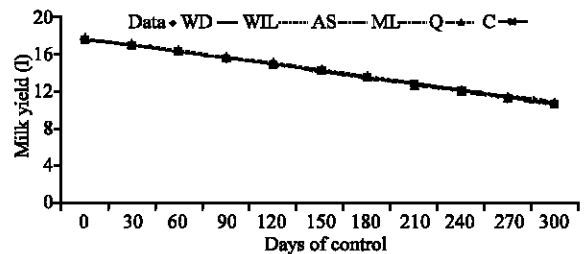Orman and Okan (1999) studied to determine the best lactation curve with data collected from the dairy cattle

Fig. 1: Shape of lactation curve according to the models

Table 1: Estimates of the model parameters

Models	Model parameters				
	a	b	c	d	e
WD	18.435±0.4155	0.0521±0.02183	0.06107±0.00663	---	---
QM	18.050±0.4110	-0.6235±0.07914	-0.00589±0.00679	---	---
CM	18.162±0.5205	-0.7163±0.30137	0.01262±0.05694	-0.00103±0.00300	---
WIL	18.341±0.5228	-0.3892±0.71236	-0.71031±0.05813	---	---
ML	23.299±0.8478	-5.9644±0.71254	2.96607±0.61737	---	---
AS	16.182±2.2194	1.2895±2.37850	-0.95841±0.06672	-1.19904±1.78046	1.74089±2.04495

Table 2: Comparison of the models for estimating Total Milk Yield (TMY), Peak Yield (PY), time to Peak Yield (PT), P (persistency) and goodness-of-fit statistics (R^2 and MSPE values)

	Fleischmann (Really)	Models					
		WD	Q	C	WIL	ML	AS
TMY (l)	4281±69.03[ns]	4282±68.75[ns]	4281±69.03[ns]	4281±69.03[ns]	4281±69.03[ns]	4281±69.03[ns]	4281±69.03[ns]
PY (l)	17.48±0.358[A]	17.14±0.439[A]	18.04±0.400[A]	18.59±0.642[A]	17.47±0.367[A]	13.19±0.570[B]	17.50±0.361[A]
PT (day)	1.55±0.276[AB]	1.29±0.862[AB]	0.003±0.003[B]	-0.16±0.064[B]	0.40±0.315[AB]	2.22±0.799[A]	1.15±0.082[AB]
P (%)	86.62±0.271[ns]	86.63±0.220[ns]	86.40±0.256[ns]	86.38±0.262[ns]	86.45±0.237[ns]	86.58±0.230[ns]	86.39±0.264[ns]
R^2 (%)		92.75±2.355[ns]	92.69±2.642[ns]	94.43±2.050[ns]	93.04±2.396[ns]	92.73±2.380[ns]	95.14±1.765[ns]
MSPE		0.35±0.117[ns]	0.36±0.137[ns]	0.26±0.093[ns]	0.34±0.123[ns]	0.35±0.121[ns]	0.23±0.078[ns]

[A, B]: $p < 0.01$ (in the same row), [ns]: insignificant

breeding unit at Ceylanpınar State Farm. Three different lactation curve models (Wood, Schaeffer, Glasbey) were applied to all data. The values of the determination coefficients of three lactation curve models were estimated at 70.62-79.47%. Determination coefficients estimated in this study were higher than reported values by Orman and Okan (1999). Although, Determination coefficient (R^2) in models changed between 92.69 and 95.14%, differences between models was found statistically insignificant ($p>0.05$).

Value of MSPE in all models changed between 0.23 and 0.36. Differences among models according to values of MSPE were statistically insignificant ($p>0.05$).

Maximum milk yield (PY) was found between 13.19 and 18.59. The value was the lower than value reported as 21.63 for Holstein cows by Orman and Okan (1999). While, maximum milk yield (PY) was estimated as the lowest in ML model, similar results were found in other models. Differences among models according to values of PY were statistically significant ($p<0.01$). Other models except for ML model can be used for estimation of maximum milk yield.

The test time at which daily milk yields were maximum (PT) was calculated the highest (2.22th test day) in ML model. In Cubic model and Quadratic model, the test time at which daily milk yields were maximum (PT) was in more early time. Also, value of PT in cubic model was negative. Values of PT in other models were similar. Differences among models according to value of PT was found statistically significant ($p<0.01$). Other models except for ML model and cubic model can be used in estimation for value of PT. This value reported for Holstein cows by Orman and Okan (1999) was higher than value estimated in this study.

CONCLUSION

In this study, Total Milk Yield (TMY), Persistency (P), Determination Coefficient (R^2) and Mean Square Predicted Error (MSPE) were similar in all models. However, Peak Yield (PY) in ML model was lower than real values. PT in ML model was higher than real values. Values of PT in Cubic model (C) and Quadratic Model (Q) were found as in earlier time than real time.

According to this result, It can be said that tree model (WD, WIL and AS) among 6 models in this study can be used in determination with mathematical models of milk yield of Simmental cows.

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