International Journal of Systems Signal Control and Engineering Application
MEDWELL PUBLICATIONS SINCE 2000

Tracking Control of Quadrotor using NonLinear Quadratic Tracking With Extended Kalman Filter

[1, 2]Mohammad Nur, [1]Trihastuti Agustinah and [1] A.K. Rusdhianto Effendi
[1]*Department of Electrical Engineering, Institute Teknologi Sepuluh, Nopember, Surabaya, Indonesia*
[2]*Department of Electrical Industry, Faculty of Engineering, Politeknik Negeri Madura, Surabaya, Indonesia*

Key words: Quadrotor, nonlinear quadratic tracking, extended Kalman filter, UAV, NLQT

Corresponding Author:
Mohammad Nur
Department of Electrical Engineering Institute Teknologi Sepuluh, Nopember, Surabaya, Indonesia

Page No.: 147-152
Volume: 10, Issue 6, 2017
ISSN: 1997-5422
International Journal of Systems Signal Control and Engineering Application

Abstract: Quadrotor is one of the Unmanned Aerial Vehicle (UAV) which is a MIMO system and has a non-linear dynamics. The nonlinearity properties of rotational motion and translational motion of quadrotor are very high and the control inputs interact each other. The interaction between the control inputs lead to system instability. This characteristic causes difficulties in tracking quadrotor automatically. Quadratic Nonlinear Tracking (NLQT) is used to overcome the problem of tracking in quadrotor with maintaining the linear nature of the matrix B. NQLT is developed from Linear Quadratic control method Tracking (LQT). The Extended Kalman Filter (EKF) is used as a state estimator to overcome the noise measurement. Based on the test results before the addition of the EKF, the proposed method provides the excellent performance of quadrotor in tracking. Quantitatively, the quadrotor can track the given reference signal with the position errors of quadrotor are 0.009 on the x-axis and 0.0099 m on the y-axis 0.0095 m for the measurement noise with zero mean and variance of 0.009. The addition of the EKF on the control system and by using the same noise properties, the position error along the x-axis and y-axis, respectively are 0.0062 and 0.0062 m.

INTRODUCTION

In the last decade, the world of robotics has developed rapidly in the presence of unmanned aerial vehicles called Unmanned Aerial Vehicle (UAV). The use of this UAV can be categorized quite extensive as for military purposes, security and others (Knight *et al*., 2008). One type of UAV that widely studied today is quadrotor, mini helicopter type UAV that uses four motors, as main driving force. Quadrotor movement and speed are determined by the speed of each rotor. Besides the need to control the speed of each motor, must be considered as well as to control in terms of attitude of quadrotor include motion acceleration (throttle), roll motion (roll), nod motion (pitch) and a circular motion (yaw) quadrotor as its function, must have a good balance when flying, especially, on rotational and translational motion that affect quadrotor flying conditions (Moonumca *et al*., 2013).

Various studies about quadrotor have been conducted quadrotor. One of study using neuro adaptive PID controllers that perform updates gain Kd automatically (Fatan *et al*., 2013). Other research is done using LQR control scheme with a full-order observer for all their state is used as a control in rotational motion but the angle signal angulernya system is not able to follow the track well (Panomrattanarug *et al*., 2013). Linear Quadratic Control Tracking (LQT) re-used in research entitled "Optimal Path Tracking Control o Quadrotor UAV" to control the tracking path quadrotor. Nonlinear models of quadrotor linearized only on around hover condition. Linearized model that has been used to solve the problems of optimal control. The advantages of this study is that the controller can perform tracking well but at the time of the system by outside interference responses showed that when given the disturbances on the z axis, the system can not handle distractions well (Suicmez and Kutay, 2014). This study proposed the use of nonlinear quadratic controllers to deal with tracking problems and extended Kalman filter to overcome the state of measurement errors due to noise.

MATERIALS AND METHODS

System modelling

Quadrotor: The basic theory used in this study include the type quadrotor an unmanned AUV that uses four motors with a pattern of a plus (+). Two motors are opposite each other to form a pair, in which the first pair (motor 1 and the motor 3) the direction of movement is set in line with the x-axis and rotates clockwise while the second pair (motor 2 and the motor 4) direction of movement is set in line with the y-axis and rotate anti clockwise. Configuring the opposite direction of the motor this motor can replace the need to have a propeller on the tail of the standart helicopter, that is used to set the direction of movement. Lifting force produced is proportional to the speed of four motors (Fig. 1).

Quadrotor body parts are connected to the frame inertia by the vector position (x, y, z) for translation system and Euler angles (θ, φ, ψ) for the rotation system. The dynamics equations of quadrotor translational motion expressed by Eq. 1-3 and the rotation is written in Eq. 4-6 (Bresciani, 2008):

$$\ddot{x} = (\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi)\frac{U_1}{m} \quad (1)$$

$$\ddot{y} = (-\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi)\frac{U_1}{m} \quad (2)$$

$$\ddot{z} = -g + (\cos\phi\cos\theta)\frac{U_1}{m} \quad (3)$$

$$\ddot{\phi} = \frac{I_{yy} - I_{zz}}{I_{xx}}qr + \frac{J_r}{I_{xx}}q\Omega + \frac{U_2}{I_{xx}} \quad (4)$$

$$\ddot{\theta} = \frac{I_{zz} - I_{xx}}{I_{yy}}pr + \frac{J_r}{I_{yy}}p\Omega + \frac{U_3}{I_{yy}} \quad (5)$$

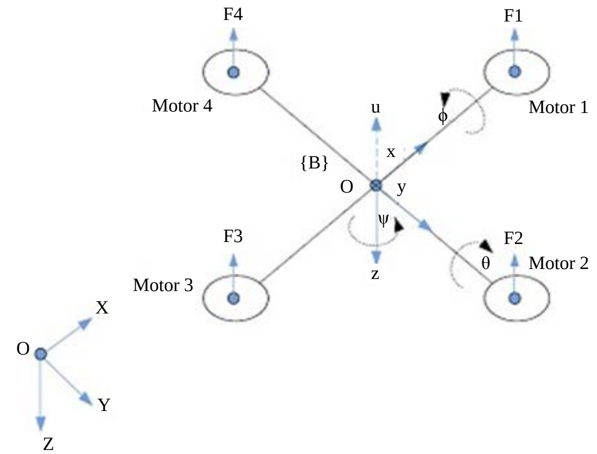

Fig. 1: Quadrotor scheme

Table 1: The parameter values of quadrotor (Bresciani, 2008)

Parameters	Names	Values
m	Mass	0.530
l	Length	0.232
d	Constant of drag	1.516×10^{-7}
b	Constant of thrust	2.247×10^{-6}
J_r	Inersia rotor	1.125×10^{-7}
I_{xx}	Inertia of x	6.228×10^{-3}
I_{yy}	Inertia of y	6.228×10^{-3}
I_{zz}	Inertia of z	1.125×10^{-2}

$$\psi = \frac{I_{xx} - I_{yy}}{I_{zz}}pq + \frac{U_4}{I_{zz}} \quad (6)$$

Quadrotor has a 6 Degree of Freedom (DoF) with 12 outputs, 6 outputs of these 12 output determines the attitude of quadrotor. These variables are as follows:

x = Quadrotor position of the Xe axis
y = Quadrotor position of the Ye axis
z = Quadrotor position of the Ze axis
u = Xb axis velocity of quadrotor
v = Yb axis velocity of quadrotor
w = Zb axis velocity of quadrotor
φ = Roll angle to the axis Xe
θ = Pitch angle to the axis Ye
ψ = Yaw angle to the axis Ze
p = Roll angle velocity of quadrotor
q = Pitch angel velocity of quadrotor
r = Yaw angel velocity of quadrotor

The parameter values of Quadrotor are given in Table 1.

Non-Linear Quadratic Tracking (NLQT) the nonlinear system:

$$\dot{x}(t) = f(x, u) \quad (7)$$

$$y(t) = Cx \quad (8)$$

Performance index:

$$J(t_0)=\frac{1}{2}(Cx-r)^T P(Cx-r)+\frac{1}{2}\int_{t_0}^{T}\left\{(Cx-r)^T Q(Cx-r)+u^T Ru\right\}dt \tag{9}$$

assuming that P≥0, Q≥0, R>0, P, Q, R symmetry. Final state tracking error is expressed in the equation:

$$\phi=\frac{1}{2}\left[\left(Cx(T)-r(T)\right)^T P\left(Cx(T)-r(T)\right)\right] \tag{10}$$

Optimal controller: Hamilton function:

$$H(x, u, t)=\frac{1}{2}\left\{(Cx-r)^T Q(Cx-r)+u^T Ru\right\}+\lambda^T\left(f(x, u)\right) \tag{11}$$

From Hamiltonian functions in Eq. 11 it is obtained state equation:

$$\dot{x}=\frac{\partial H}{\partial \lambda} f(x,u), t\geq t_0 \tag{12}$$

Costate equation:

$$\begin{aligned}-\dot{\lambda}&=\frac{\partial H}{\partial x}=\frac{\partial f^T}{\partial x}\lambda+\frac{\partial L}{\partial x}, t\geq T\\ -\dot{\lambda}&=C^TQ(Cx-r)+A^T\lambda\end{aligned} \tag{13}$$

Stationary condition:

$$0=\frac{\partial H}{\partial u}=\frac{\partial f^T}{\partial u}\lambda+\frac{\partial L}{\partial u}$$

$$0=Ru+B^T\lambda^T$$

$$u=-R^{-1}B^T\lambda^T \tag{14}$$

Boundary condition:

$$\left(\phi_x+\psi_x^T v-\lambda\right)^T\Big|_T dx(T)+\left(\phi_t+\psi_t^T v+H\right)\Big|_T dT=0 \tag{15}$$

which Ψ = 0 and T is fixed, so that, dT = 0, then:

$$\left(\phi_x-\lambda\right)^T\Big|_T dx(T)=0 \tag{16}$$

By solving the above equation for all t<T, the equation:

$$\lambda(t)=C^TPCx(t)-C^T Pr(t) \tag{17}$$

or the purposes of simplification can asumtion C^TPC = S(t) and $C^TPr(t)$ and $C^TPr(t)$ = v(t), then:

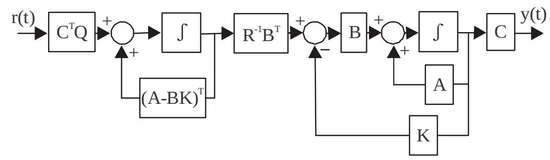

Fig. 2: NLQ tracking with following model

$$\lambda(t)=S(t)x(t)-v(t) \tag{18}$$

derivative of Eq. 17 and by substituting $\dot{x}$ then the trivial solution is obtained as the following equation:

$$-\dot{S}=A^TS+SA-SBR^{-1}B^TS+C^TQC$$

and

$$-\dot{v}=\left(A^T-SBR^{-1}B^T\right)v+C^TQr \tag{19}$$

Next, define that:

$$K(t)=R^{-1}B^TS(t) \tag{20}$$

As Kalman gain, then Eq. 21 be:

$$-\dot{v}=(A-BK)^T v+C^TQr \tag{21}$$

Substitution Eq. 18 and 20 to Eq. 14:

$$u(t)=-K(t)x(t)+R^{-1}B^Tv(t) \tag{22}$$

And then do the simulation by using Eq. 21 and 22 (Fig. 2).

Translation and rotation system controller: Simplification of Eq. 1-6 can be done to turn it into a state equation such as the following first order:

$$\begin{bmatrix}\dot{x}\\ \dot{u}\\ \dot{y}\\ \dot{v}\\ \dot{z}\\ \dot{w}\end{bmatrix}=\begin{bmatrix}u\\ (\cos\phi\sin\psi+\cos\phi\sin\theta\cos\psi)\frac{U_1}{m}\\ v\\ (-\sin\phi\sin\psi+\cos\phi\sin\theta\cos\psi)\frac{U_1}{m}\\ w\\ -g+(\cos\phi\cos\theta)\frac{U_1}{m}\end{bmatrix} \tag{23}$$

As it is known that the model translation systems can be controlled independently of the matrices A and B can be broken down for each translational motion x-axis, y-axis and z-axis as follows:

$$A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} \tag{24}$$

$$B = \begin{bmatrix} 0 \\ b_i \end{bmatrix}$$

$$C = [\ 1\ 0]$$

Where:

$$b_x = (\cos\phi \cos\theta \cos\psi)\frac{U_1}{m}$$

$$b_y = (-\cos\phi\cos\psi - \sin\phi\sin\theta\sin\psi)\frac{U_1}{m} \quad (25)$$

$$b_z = (\cos\phi\cos\theta)\frac{1}{m}$$

with i = x, y, z. for each of the x-axis, y-axis and z-axis and input for Euler angels (θ, φ) is obtained from the invers function of model quadrotor. Being the value of the angle, ψ rated constant zero, so that, the inverse function can be expressed in Eq. 11-12:

$$\theta = a\sin\left(\frac{\frac{m}{U_1}u_x}{\cos\phi}\right) \quad (26)$$

$$\phi = \sin\left(-\frac{mv_x}{U_1}\right) \quad (27)$$

The controller that will be used for rotation systems is different with controllers on the system translation. The controller used is linear quadratic tracking. Model Eq. 4-6 can be simplified into one order state equation becomes:

$$\begin{bmatrix} \dot{\phi} \\ \dot{p} \\ \dot{\theta} \\ \dot{q} \\ \dot{\psi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} p \\ \frac{I_{yy}-I_{zz}}{I_{xx}}qr - \frac{Jr}{I_{xx}}q\Omega + \frac{U_2}{I_{xx}} \\ q \\ \frac{I_{zz}-I_{xx}}{I_{yy}}pr + \frac{Jr}{I_{yy}}p\Omega + \frac{U_3}{I_{yy}} \\ r \\ \frac{I_{xx}-I_{yy}}{I_{zz}}pq + \frac{U_4}{I_{zz}} \end{bmatrix} \quad (28)$$

State that is needed can be obtained by simply taking three state to control the position of the speeds. State to be used only p, q dan r. as shown in the following equation:

$$\begin{bmatrix} \dot{p} \\ q \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{I_{yy}-I_{zz}}{I_{xx}}qr - \frac{Jr}{I_{xx}}q\Omega + \frac{U_2}{I_{xx}} \\ \frac{I_{zz}-I_{xx}}{I_{yy}}pr + \frac{Jr}{I_{yy}}p\Omega + \frac{U_3}{I_{yy}} \\ \frac{I_{xx}-I_{yy}}{I_{zz}}pq + \frac{U_4}{I_{zz}} \end{bmatrix} \quad (29)$$

To get the matrix A it is going to do linearized around the point of equilibrium point. That is linearized at point p = 0, q = 0 dan r = 0. Thus, obtained the matrix A and matrix B below:

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (30)$$

$$B = \begin{bmatrix} \frac{1}{I_{xx}} & 0 & 0 \\ 0 & \frac{1}{I_{yy}} & 0 \\ 0 & 0 & \frac{1}{I_{zz}} \end{bmatrix}$$

Selection matrix C are given based on the need of the matrix to be measured, it is:

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The matrix C are chosen to show the value of state that needed in the measurement. A brief explanation to obtain a gain value EKF and updates weight in the algorithm can be expressed as follows (Wang and Yang, 2012):

Extended Kalman filter

Predict cycle:

$$\hat{x}(k+1|k) = A_k(\hat{x}(k|k)) + BU_k + \omega_k$$

$$P(k+1|k) = \bar{A}(k)P(k|k)\overline{A^T}(k) + Q(k)$$

Filtered cycle:

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)\left[y_{k+1} - \bar{C}_{k+1}(\hat{x}(k+1))\right]$$

$$K(k+1) = P(k+1|k)\bar{C}^T(k+1)\left[\bar{C}(k+1)P(k+1|k)\ \bar{C}^T(k+1) + R(k+1)\right]^{-1}$$

$$P(k+1|k+1) = \left[I - K(k+1)\bar{C}(k+1)\right]P(k+1|k)$$

with $\bar{A}(k)$ and $\bar{C}(k)$ is jacobian matrix of A(x) and C(x) around equilibrium point.

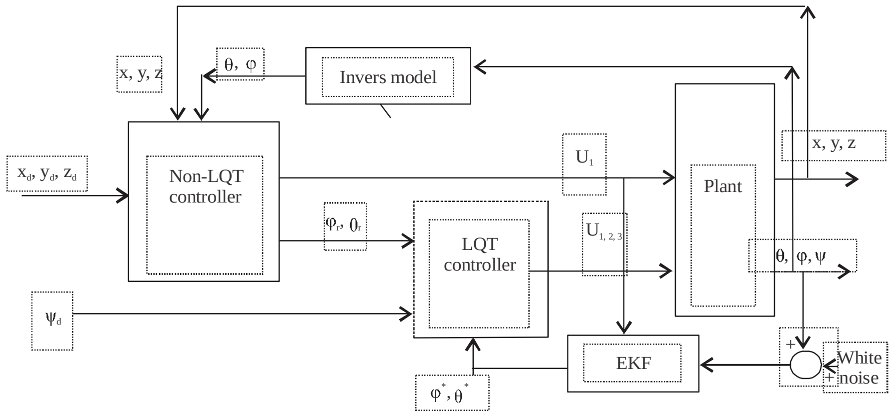

Fig. 3: Conceptual diagram of the system

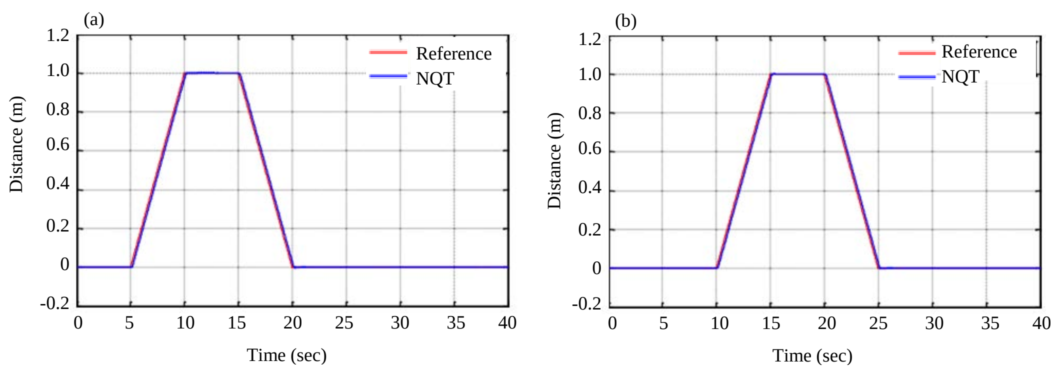

Fig. 4(a, b): x and y position

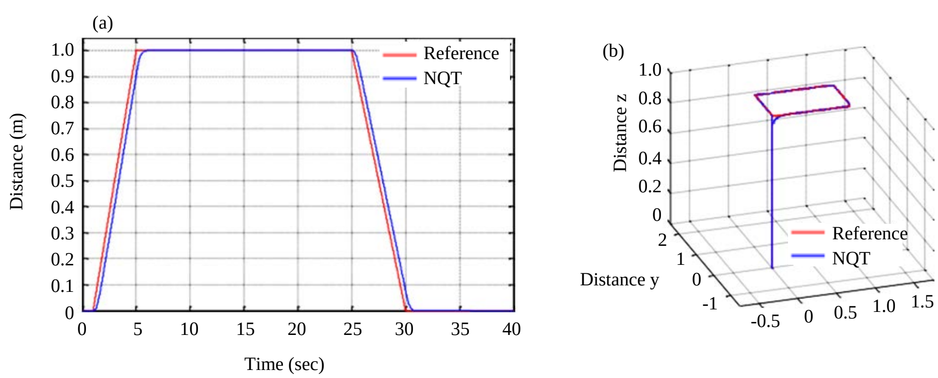

Fig. 5(a, b): z position and xyz position

RESULTS AND DISCUSSION

Experiments: While the conceptual scheme of the system expressed in Fig. 3. In this study, NLQT shows the simulation results using NLQT control design. It is seen that the output follows the reference in Fig. 4-6.

Table 2: RMS for state

Varian	x	y	z	Roll	Pitch	Yaw
0	0.0055	0.0053	0.0091	2.3×10^{-5}	2.8×10^{-5}	0
0.001	0.0055	0.0053	0.0122	4.9×10^{-3}	4×10^{-5}	0
0.005	0.0056	0.0053	0.0123	1.1×10^{-2}	7×10^{-5}	0
0.009	0.0062	0.0062	0.123	1.5×10^{-2}	9×10^{-5}	0

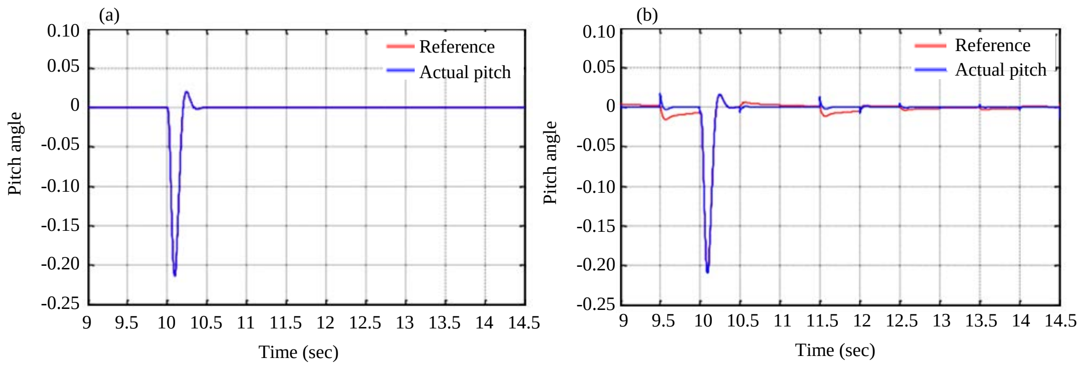

Fig. 6(a, b): Rol angle

CONCLUSION

Adapting from LQT method to gain constant K has developed a method of NLQT (Nonlinear quadratic tracking) to maintain the value of the nonlinear matrix B. In maintaining the value of the matrix B making the value of K nonlinear gain anyway. The NLQT control method works relatively well. Although, the use of the method only be done on a translation system, it is seen from the results of the simulation which is able to follow a expected track. The RMSE obtained by the noise variance for 0009 which is only 0.0099 for the motion of the x-axis and 0.0095 for the y-axis motion. The addition EKF estimator effect to reduce noise. This is evident from the resulting RMSE of noise with the same variant becomes 0.0062 to the motion of the x-axis and y-axis

REFERENCES

Bresciani, T., 2008. Modelling, identification and control of a quadrotor helicopter. M.Sc. Theses, Lund University, Lund, Sweden.

Fatan, M., B.L. Sefidgari and A.V. Barenji, 2013. An adaptive neuro pid for controlling the altitude of quadcopter robot. Proceedings of the 2013 18th International Conference on Methods & Models in Automation & Robotics (MMAR), August 26-29, 2013, IEEE, Miedzyzdroje, Poland, pp: 662-665.

Knight, A.S., P.L. Artem, M.M. George, D.G. Jonathan and R.J.D. Mark, H.D. Maximillian and J.M. Michael, 2008. Design optimization of a quad-rotor capable of autonomous flight. Worcester Polytechnic Institute, Worcester, Massachusetts.

Moonumca, P., Y. Yamamoto and N. Depaiwa, 2013. Adaptive PID for controlling a quadrotor in a virtual outdoor scenario: Simulation study. Proceedings of the 2013 IEEE International Conference on Mechatronics and Automation, August 4-7, 2013, IEEE, Takamatsu, Japan, pp: 1080-1086.

Panomrattanarug, B., K. Higuchi and F. Mora-Camino, 2013. Attitude control of a quadrotor aircraft using LQR state feedback controller with full order state observer. Proceedings of the SICE Annual Conference 2013, September 14-17, 2013, IEEE, Nagoya, Japan, pp: 2041-2046.

Suicmez, E.C. and A.T. Kutay, 2014. Optimal path tracking control of a quadrotor UAV. Proceedings of the 2014 International Conference on Unmanned Aircraft Systems (ICUAS), May 27-30, 2014, IEEE, Orlando, Florida, pp: 115-125.

Wang, S. and Y. Yang, 2012. Quadrotor aircraft attitude estimation and control based on Kalman filter. Proceedings of the 31st Chinese Control Conference, July 25-27, 2012, IEEE, Hefei, China, pp: 5634-5639.