International Journal of Systems Signal Control and Engineering Application

MEDWELL PUBLICATIONS SINCE 2000

A New Optimal Method to Tune PI Controllers using Evolutionary Polynomial Regression

Arman Sharifi and Hooman Sadjadian
Department of Electrical Engineering, Iran University of Science and Technology, Tehran, Iran

Key words: Tuning methods, PI controller, frequency-domain criterion, evolutionary polynomial regression, FOPDT

Corresponding Author:
Arman Sharifi
Department of Electrical Engineering, Iran University of Science and Technology, Tehran, Iran

Page No.: 93-100
Volume: 10, Issue 2, 2017
ISSN: 1997-5422
International Journal of Systems Signal Control and Engineering Application

Abstract: A new optimal method is proposed to tune PI controllers for the First Order Plus Dead Time (FOPDT) models. In this approach, a performance index is employed with specification of the infinity-norm of the sensitivity and complementary sensitivity functions. This criterion determines desire values of PI controller parameters for a bank of FOPDT models. Then an Evolutionary polynomial regression is applied to extract a tuning rule for PI controllers. At the end, the performance of recommended method is evaluated by several case study simulations.

INTRODUCTION

PI and PID controllers have proven to be very useful instruments in industrial automation. Therefore, tuning of PI/PID controllers for achieving a better performance still attracts a great deal of attention. In this study, tuning of PI controllers is studied and investigated. Due to their applicability and simple structure, PI controllers are still the most commonly used controllers in the process control industry (Astrom *et al*., 1998). Therefore, many different approaches have been proposed to tune these controllers, since, 1940’s.

Frequency-domain approaches for tuning linear optimal controllers have been studied since the beginning of the 1980s. Several contributions can be found in a survey paper by Francis and Doyle (Francis and Doyle, 1987). Lee and Yu (1994) presented tuning rules based on frequency-domain analysis of the closed-loop behavior of MPC controllers. Similar work was carried out for designing robust feedback controllers that include mixed time-frequency domain constraints (Schomig *et al*., 1993). Several PID tuning methods were proposed base on the frequency-domain specifications (Ho *et al*., 1995), (Karimi *et al*., 2003). The proposed method to tune PI controllers consists of employing a new set of frequency-domain performance indices which gives desired values of the PI controller's parameters for different models. Then the Evolutionary Polynomial Regression (EPR) is applied in order to extract a tuning rule for the PI controller parameters in terms of model parameters.

MATERIALS AND METHODS

Tuning procedure of the PI controller

Introducing the criterion: Dynamics of many industrial processes can be sufficiently modelled by the stable First-Order Plus Dead Time (FOPDT) transfer function:

$$G(s) = \frac{K}{T_s s + 1} e^{-t_d s} \quad (1)$$

where, K, T_s and t_d are static gain, time constant and time delay of the model, respectively. The PI controller designed for this important category of industrial plants is formulated as:

$$C(s) = K_c(1+\frac{1}{T_I s}) \quad (2)$$

In which K_c and T_I are proportional gain and integral time constant, respectively. This study presents a method to tune PI controllers for the FOPDT models. Accordingly, it leads to extract two formulas for K_c and T_I in terms of the FOPDT model parameters:

$$K_c = f(K, T_s, t_d) \quad (3)$$

$$T_I = g(K, T_s, t_d) \quad (4)$$

Many similar tuning rules have been proposed, since, 70 years ago. They have been designed based on different specifications determined by the designer. In this study, the frequency-domain specifications are employed to tune PI controllers. For this purpose, a criterion is used which considers specifications on the infinity-norm of the sensitivity and complementary sensitivity functions. Such a criterion was used by Garcia *et al*. (2005) to tune PID controllers but it did not provide a tuning formula for the controller.

The mentioned specifications stand for the stability and performances of the closed-loop system. This criterion can be expressed as:

$$CF = (M_m(\rho) - M_m^*)^2 + (M_c(\rho) - M_c^*)^2 \quad (5)$$

In Eq. 5, M_m, known as modulus margin is defined as inverse of the infinity-norm of the sensitivity function. This value ensures a lower bound of $1/(1-M_m)$ for the Gain Margin (GM). Moreover, it is related to the upper bound for the disturbances amplification by the closed-loop system. Typical values of M_m are in the range of 0.5-0.75. The second design parameter M_c, defined as the complementary modulus margin is equal to the inverse of the infinity-norm of the complementary sensitivity function. The value of M_c is related to the resonance peak of the transfer function from set-point to process output and forms a significant performance indicator of the response to set-point changes. A specification on M_c guarantees a lower bound for the Phase Margin (PM):

$$\Phi_m \geq -\arccos(\frac{M_c^2-2}{2}) + \pi \approx 63M_c - 3 \quad (6)$$

Suggested values for the largest magnitude of the complementary sensitivity function is typically between 1.0 and 1.5 which gives a complementary modulus margin M_c between 0.65 and 1. These values correspond approximately to overshoots of 30 and 0%, respectively. In Eq. 5, M_m* and M_c* are the specified value of the modulus margin. The specifications were set to 1.58 for the infinity-norm of the sensitivity function (M^m* = 0.65) and 1.01 for the infinity-norm of the complementary sensitivity function (M_c*= 0.95) which give GM = 2.87 and PM = 56, respectively.

Defining desired tuning parameters for a set of FOPDT models: In this study, the desired PI controller parameters are defined using the criterion (Eq. 5) for FOPDT models. For this purpose, a range of common FOPDT models is defined using the definition of Normalized Dead Time (NDT):

$$\tau = \frac{t_d}{T_s} \quad (7)$$

This quantity can be used to characterize the difficulty of controlling a process. Roughly speaking, it has been found that processes with small τ are easy to control and the difficulty in controlling the system increases as τ increases (Astrom *et al*., 1992). To create a bank of FOPDT models, for each parameter of the FOPDT model, several levels were allocated which led to 810 models. The experiments arrangement is indicated in Table 1. According to this table, the values of normalized dead time for the created bank of models are set between 0.011 and 9 which include a wide range of FOPDT industrial models. The similar empirical rules have been developed for other tuning methods based on the simulation of a large number of systems. For example, Ziegler Nichols method works well for the models with normalized dead time under 0.4. For each 810 produced model, desired K_c and T_I are needed to be defined. Here, desired values are defined as values of K_c and T_I which minimize the cost function (Eq. 5). After performing the simulations, a bank of data consisting FOPDT model parameters and their corresponding desired PI parameters is constructed. These data are then employed to tune lambda by means of a method discussed later.

Evolutionary polynomial regression: Evolutionary Polynomial Regression (EPR) is a data-driven method based on evolutionary computing, aimed at searching polynomial structures representing a system. This method is a two-stage technique for constructing symbolic models which consists of: structure identification and parameter estimation. In the first stage, EPR searches for symbolic

Table 1: Experiments arrangements for FOPDT models

FOPDT parameters	Level 1	Level 2	Level 3	…	Level 9	Level 10
K	1	2	3	…	9	-
T_s	1	2	3	…	9	-
t_d	0.1	1	2	...	8	9

structures by Genetic Algorithm (GA) and it estimates constant values by solving a Least Squares (LS) linear problem in the second stage.

A general EPR expression can be presented as follows (Giustolisi and Savic, 2006):

$$y = \sum_{j=1}^{n} F(X, f(X), a_j) + a_0 \tag{8}$$

Where:

y = The estimated vector of the output of process
a_j = An adjustable parameter for the jth term
F = A function in the space dimensionally equal to the number of inputs
X = The input variables matrix
f = A user defined function
n = The number of terms of the target expression

The general functional structure represented by F(X, f(x), a_j) is constructed from elementary functions using a Genetic Algorithm (GA) strategy. The GA is employed to select the useful input vectors from X to be combined.

In development of EPR models, a number of restrictions can be implemented to control the constructed models in terms of the length of the equations, type of the functions used, number of terms, range of exponents, number of generations, etc. As a result, it is potential to obtain different models for a special problem which allows the user to acquire additional insight about the problem. By implementing the EPR procedure, the evolutionary process starts with a constant mean of output values. By increasing the number of evolutions, it progressively picks up the different participating parameters in order to form equations representing the constitutive relationship. The level of accuracy at each stage is evaluated based on the Coefficient of Determination (CoD) as the fitness function that can be computed as:

$$\text{CoD} = 1 - \frac{N-1}{N} \frac{\sum_N (\hat{y} - y_p)^2}{\sum_N (y_p - \text{avg}(y_p))^2} \tag{9}$$

Where:

$\hat{y}$ = The actual desired output
y_p = The output predicted by EPR
N = The number of data points on which the CoD is computed

If the model fitness is not satisfactory or the other termination criteria are not fulfilled, another evolution is needed in order to achieve a new model.

Application of EPR for tuning lambda: In this section, EPR is applied to the data obtained in section 2.2. The least square method was used to estimate parameters using a singular value decomposition solver (Giustolisi and Savic, 2006). The range of exponents has been set to [-2 -1.9 … 0 … 1.9 2]. The value 0 has been set among the exponent, so that, EPR can deselect those inputs which are not essential for the model. The number of generation, according to the previous experiences of EPR, is set to 10. This parameter is related to number of inputs and outputs and to the length of the expression. Finally, this analysis leads to the following tuning rule for K_c and T_I:

$$K_c = \frac{1}{K}\left(0.7511\left(\frac{T_s}{t_d}\right)^{0.8} + 0.032\right) \tag{10}$$

$$T_I = 2.0389 T_s^{0.6} t_d^{0.2} - 0.22017 \tag{11}$$

In the next sections, these formulas are investigated in details and their performances are compared to the other tuning rules.

Comparison: In this section, a thorough comparison is done between the proposed method and some well-known PI tuning formulas. For the comparison, several criteria are considered to be compared such as:

- Settling time
- Overshoot (%)
- The Integrated Absolute Error (IAE) defined as:

$$\text{IAE} = \int_0^{\infty} |r(t) - y(t)| dt$$

- The Total Variation (TV) of the manipulated input defined as:

$$\text{TV} = \sum_{k=1}^{\infty} |u(k+1) - u(k)|$$

- Gain Margin (GM)
- Phase Margin (PM)

The following PI tuning approaches are considered for the comparison:

- Process reaction curve method of Ziegler–Nichols (Z-N) (Ziegler *et al*., 1942)
- Murrill tuning rules based on minimizing IAE, ITAE, and ISE (Murril, 1967)
- Zhuang and Atherton (Z&A) tuning rules based on minimizing of ISE, ISTE and IST2E (Zhuang and Atherton, 1993)
- Chien, Hrones and Reswick Method (CHR) which is a modification of the Ziegler-Nichols methods (Chien *et al*., 1952)

- Smith tuning rules that gives a specified closed loop response (Smith and Corripio, 2006)
- Astrom and Hagglund tuning rules that maximize performance subject to a constraint on the degree of robustness (Juneja *et al*., 2010)

The tuning formulas of the mentioned methods are given in Table 2. Many of these tuning formulas seem quite out of date. However, these methods are representative and they are generally bases comparison for more recent tuning methods.

Due to existence of the factor (1/K) in Kc formula of all tuning rules, without loss of generality, we set K = 1 in simulations. Accordingly, 90 FOPDT models are considered for comparison. As it can be seen from Table 2, many of tuning rules are valid only for small values of NDT. Accordingly, the comparison is carried out in two cases:

First, the FOPDT models whose NDT values are between 0.11 and 1 are discussed. These models include 36 models of totally 90 considered FOPDT models. In this case, the proposed tuning method is compared to the methods in the first part of Table 2. For each model, the criteria defined in section 2.1 are calculated and the average values of different criteria were considered as new performance indices. These comparisons can be observed in Fig. 1. Form this Fig. 1, the following observations can be seen:

Figure 1a shows the average values of settling time for different methods. It can be seen that the proposed method gives an average settling time equal to 22.3 which is the smallest value after the value provided by Z&A (IST2E) which is about 15.

The proposed method provides small values of overshoot with an average of 4.3%. Surprisingly, Z&A (IST2E) leads to a smaller overshoot compared to the

Table 2: PI tuning formulas

Category	Methods	K_c	T_I	Comment
1	Murrill (ITAE)	$[0.859(T_s/t_d)^{0.977}]/K$	$[T_s(t_d/T_s)^{0.438}]/0.552$	$0.1<\tau<1$
	Murrill(ISE)	$[1.035(T_s/t_d)^{0.959}]/K$	$[T_s(t_d/T_s)^{0.739}]/0.492$	$0.1<\tau<1$
	Murrill (IAE)	$[0.984(T_s/t_d)^{0.986}]/K$	$[T_s(t_d/T_s)^{0.707}]/0.608$	$0.1<\tau<1$
	Z & A (ISE)	$[0.98(T_s/t_d)^{0.892}]/K$	$T_s/[0.69-0.155(t_d/T_s)]$	$0.1<\tau<1$
	Z & A (ISTE)	$[0.712(T_s/t_d)^{0.921}]/K$	$T_s/[0.968-0.247(t_d/T_s)]$	$0.1<\tau<1$
	Z & A (IST2E)	$[0.569(T_s/t_d)^{0.951}]/K$	$T_s/[1.023-0.179(t_d/T_s)]$	$0.1<\tau<1$
	Ziegler-Nichols	$[0.9(T_s/t_d)]/K$	$3.33*T_d$	$0.1<\tau<1$
2	Smith	$[0.5(T_s/t_d)]/K$	T_s	-
	CHR (os.0%)	$[0.35(T_s/t_d)]/K$	$1.2T_s$	-
	CHR (os.20%)	$[0.6(T_s/t_d)]/K$	T_s	-
	As. & Hag.	$[0.14+0.28(T_s/t_d)]/K$	$0.33t_d+6.8T_st_d/(10t_d+T_s)$	-
	Klein	$0.28T_s/K(t_d+0.1T_s)$	$0.53T_s$	-
	Stogestad	$[0.5(T_s/t_d)]/K$	$\min(T_s, 3t_d)$	-

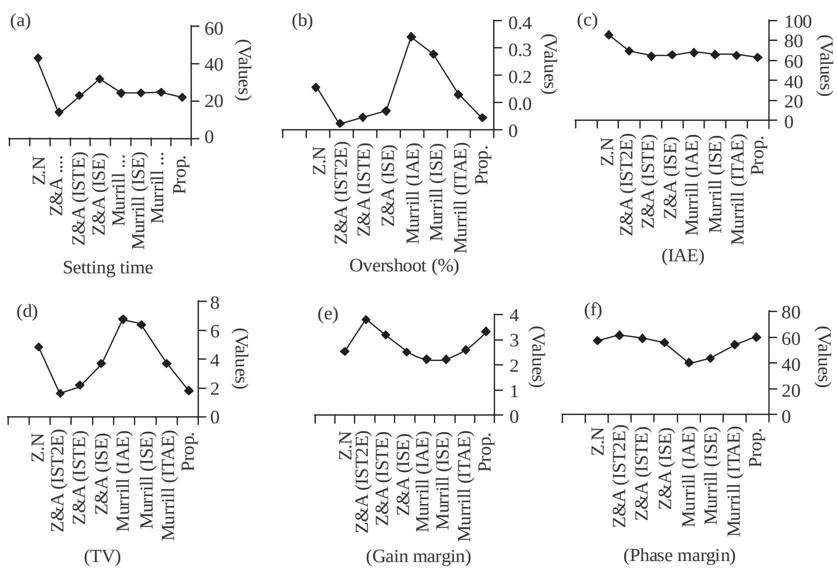

Fig. 1(a-f): Comparison between the proposed method and the methods designed for the processes with small τ

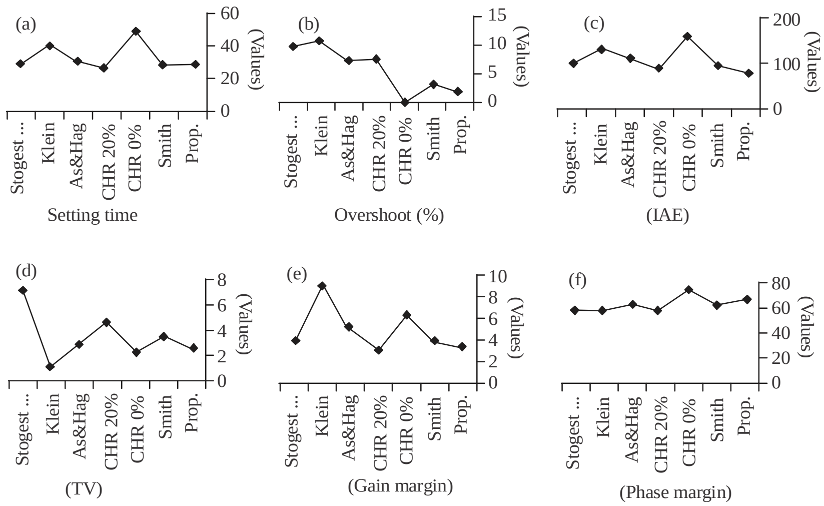

Fig. 2(a-f): Comparison between the proposed method and the methods designed for the processes with different τ

others. The mean values of IAE for different methods are closed together. The proposed method gives a small TV compared to other methods.

Gain margin and phase margin of the proposed method have average values of 3.24 and 60.3, respectively. As it was expected from the predefined gain margin and phase margin specifications in section 2.1, the minimum values of GM and PM for all 36 models are respectively 2.91 and 56.83 which provide a good performance for control system.

Although, the proposed tuning rules have a suitable performance for FOPDT models with $0.1<\tau<1$, an exact comparison shows that the A&Z (IST2E) method gives a better performance compared to the proposed method. However, this superiority is reliable only for the FOPDT models with small NDT while the proposed method can be applied to dominant time delay systems. Therefore, the proposed method is preferred to be used in spite of the superiority of A&Z (IST2E) for models with small normalized dead time.

Here, the proposed method is compared to the methods in second part of Table 2. In this case, the PI controller is applied to the FOPDT models with different values of τ including dominant time delay systems. The recommended tuning rules and the others are applied to 90 FOPDT models and the average values of different criteria were calculated as indicators of the comparison. This comparison is depicted in Fig. 2.

The average value of settling time for the proposed method is 27.9, while the smallest value of settling time is belong to CHR (20% o.s.) which is 26.5. Therefore, the proposed method results in a good settling time for the response.

The mean value of overshoot for the proposed method is 2.1% which is the lowest value of overshoot after CHR (0% o.s.) methods. CHR (0% o.s.) has no overshoot of response but it leads to a settling time twice the proposed method. From Fig 2b, it can be seen that CHR (20% o.s.) leads to an overshoot equal to 8.1% which is a large overshoot value compared to the proposed method. So, it seems that the slight superiority of CHR (20% o.s.) in settling time cannot justify its large overshoot compared to the proposed method.

The average values of IAE for different tuning rules are indicated in Fig. 2c. Obviously, the proposed method leads to a smaller mean value of IAE compare to the others. The mean values of TV for different methods are compared in Fig. 2d. The proposed method has a small value of TV compared to the most other methods.

The averages values of gain margin and phase margin achieved by the proposed method are 3.54 and 65.12, respectively. The minimum values of GM and PM for all 90 models are respectively 2.89 and 56.83 which fulfil the predefined specifications in section 2.1. Figure 2e shows that the other methods provide larger values of GM compared to the proposed tuning rules. For example, the Klein tuning rules provides an average GM more than 8.8 which is a large value of GM. However, overweighting on gain margin criterion can cause a poor performance of the method.

The above observations of the simulations on 90 FOPDT models show that the proposed tuning rules provide a good efficiency for the control system.

RESULTS AND DISCUSSION

Simulation study: The proposed formulas for tuning PI controllers are tested via. several simulations on FOPDT models with different values of NDT. To more evaluate the proposed method, the closed-loop responses of other tuning methods are compared to the proposed method's response:

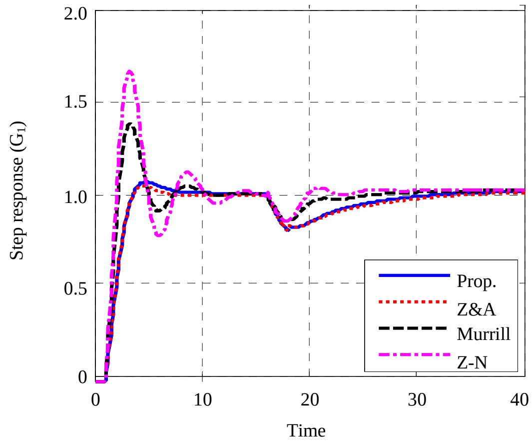

Fig. 3: Output responses of different tuning methods of G_1

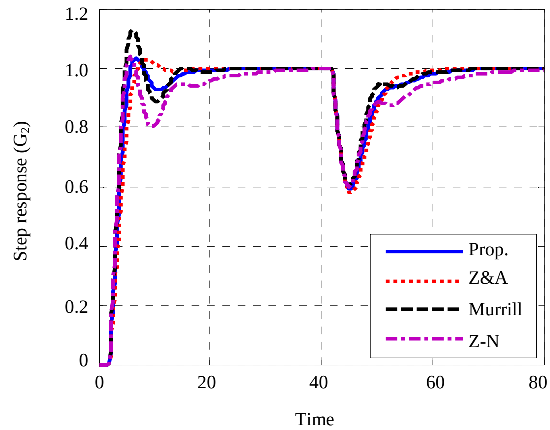

Fig. 4: Output responses of different tuning methods of G_2

$$G_1(s) = \frac{e^{-s}}{8s+1} \quad (12)$$

This model has a small NDT = 0.125. For comparison, the Zhuang and Atherton (IST2E) and the Murrill (IAE) tuning rules were chosen for their high performance shown in section 3 and the Ziegler-Nichols method was considered as the most common tuning rule. Fig. 3 represents the output response in presence of a step input disturbance of magnitude 1 (100% of set point). It can be seen that the Murrill and Z-N lead to an oscillatory response with a large overshoot while Z&A method and the proposed tuning rules give a suitable response with a small overshoot. However, the disturbance damping in Z-A and Murrill is swifter than the others:

$$G_2(s) = \frac{e^{-2s}}{3s+1} \quad (13)$$

For this model, the NDT value is 0.66. the output response in presence of disturbance (70% of setpoint) are shown in Fig. 4. The proposed method and Z&A (IST2E) methods result in better responses (low value of overshoot and settling time). The Ziegler-Nichols tuning approach shows a slow response both in setpoint tracking and disturbance rejection.

In above two examples, the FOPDT models with small NDT were considered and the proposed method was compared to those methods which were designed for small τ (category 1 of Table 2). In the next examples, the second part of tuning rules given in Table 2 is compared to the proposed method which are applicable for the FOPDT models with different NDT. For the comparison, the smith method and CHR (20% o.s.) are chosen which have shown good performances in section 3 compared the others:

$$G_3(s) = \frac{e^{-3s}}{2s+1} : \tau = 1.5 \quad (14)$$

$$G_4(s) = \frac{e^{-4s}}{s+1} ; \tau = 4 \quad (15)$$

$$G_5(s) = \frac{e^{-12s}}{2s+1} ; \tau = 6 \quad (16)$$

$$G_6(s) = \frac{e^{-20s}}{2s+1} ; \tau = 10 \quad (17)$$

Figure 5 shows the closed loop responses of all PI controllers tuned by three mentioned methods for the above FOPDT models. The simulations were performed in presence of step input disturbance of magnitude 0.5 (50% of setpoint). It can be observed that the proposed method gives a smooth response both in setpoint tracking and disturbance rejection without any overshoot while the other methods lead to responses with overshoot and same settling point as the propose method.

The above examples show the good performance of the proposed tuning rules for large values of NDT. The similar simulations are performed for the FOPDT models with small values of NDT in next examples:

$$G_7(s) = \frac{e^{-0.5s}}{5s+1} ; \tau = 0.1 \quad (18)$$

$$G_8(s) = \frac{e^{-0.1s}}{20s+1} ; \tau = 0.005 \quad (19)$$

In Fig. 6a the closed loop response for G7 is presented in presence of input disturbance of magnitude 1. The proposed method still gives a convenient response with a small overshoot. However, for the FOPDT models which have very small values

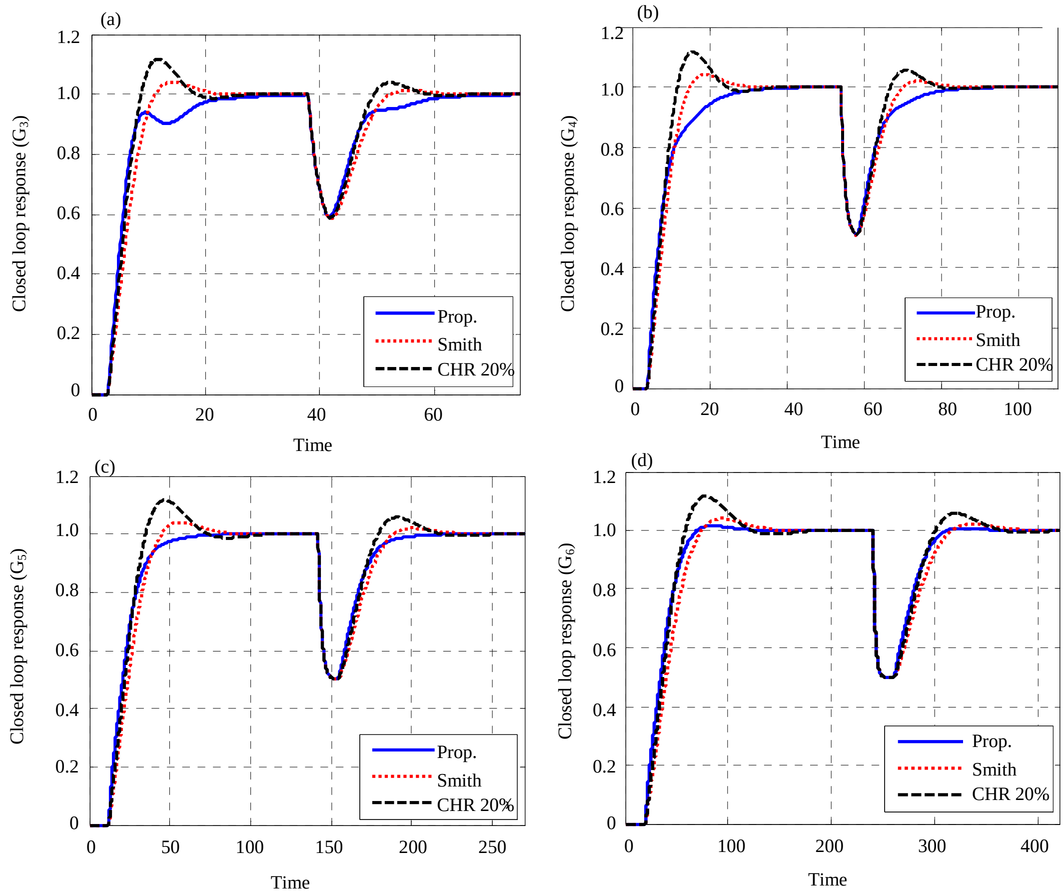

Fig. 5(a-d): Output responses of different tuning methods of G_3, G_4, G_5 and G_6

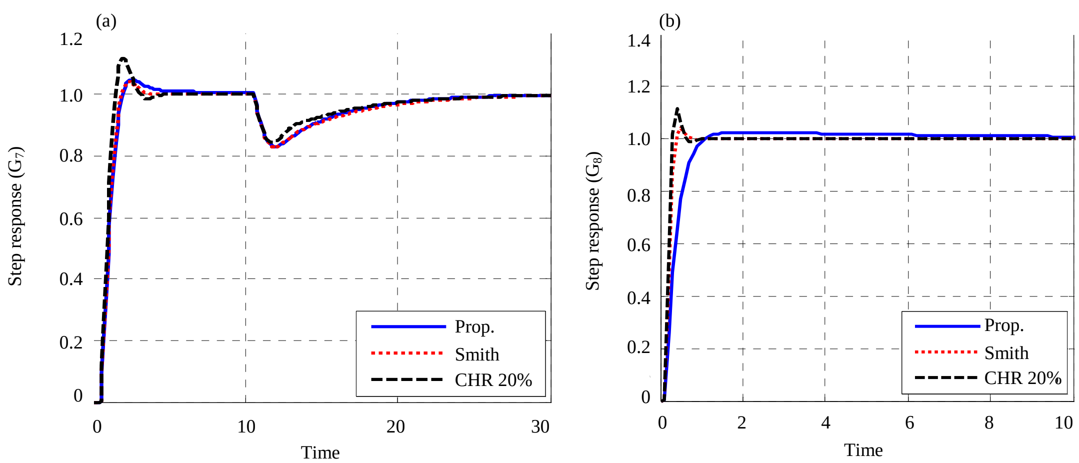

Fig. 6(a, b): Output responses of different tuning methods of G_7 and G_8

of τ, like G_8, the propose method leads to a response with smooth behavior but a large settling time. Thus, for FOPDT models with very small values of NDT, the proposed method gives a slow response.

CONCLUSION

In this study, an optimal data-based method was proposed to tune the PI controller's parameters for FOPDT models. The procedure of the proposed approach was based on minimizing a frequency-domain criterion and employing EPR to extract the tuning rules. A thorough comparison was performed to evaluate the performance of the proposed method on a bank of different FOPDT models. It was observed that the proposed method works well for a wide range of FOPDT models. It gives smooth responses with small values of overshoot for delay dominant systems. However, for the models with very small normalize dead time the recommended tuning rules give a slow response. A further study with focus on SOPDT models and PID controllers is suggested.

REFERENCES

Astrom, K.J., C.C. Hang, P. Persson and W.K. Ho, 1992. Towards intelligent PID control. Automatica, 28: 1-9.

Astrom, K.J., H. Panagopoulos and T. Hagglund, 1998. Design of PI controllers based on non-convex optimization. Automatica, 34: 585-601.

Chien, K.L., J.A. Hrones and J.B. Reswick, 1952. On the automatic control of generalized passive systems. Trans. ASME., 74: 175-185.

Francis, B.A. and J.C. Doyle, 1987. Linear control theory with an \bfH∞ optimality criterion. SIAM. J. Control Optim., 25: 815-844.

Garcia, D., A. Karimi and R. Longchamp, 2005. PID controller design with specifications on the infinity-norm of sensitivity functions. IFAC Proc. Vol., 38: 177-182.

Giustolisi, O. and D.A. Savic, 2006. A symbolic data-driven technique based on evolutionary polynomial regression. J. Hydroinf., 8: 207-222.

Ho, W.K., C.C. Hang and L.S. Cao, 1995. Tuning of PID controllers based on gain and phase margin specifications. Automatica, 31: 497-502.

Juneja, P.K., A.K. Ray and R. Mitra, 2010. Various controller design and tuning methods for a first order plus dead time process. Int. J. Comput. Sci. Commun., 1: 161-165.

Karimi, A., D. Garcia and R. Longchamp, 2003. PID controller tuning using Bode’s integrals. IEEE. Trans. Control Syst. Technol., 11: 812-821.

Lee, J. and Z.H. Yu, 1994. Tuning of model predictive controllers for robust performance. Comput. Chem. Eng., 18: 15-37.

Murrill, P.W., 1967. Automatic Control of Processes. International Textbook Company, Scranton, Pennsylvania,.

Schomig, E., M. Sznaier and Z.Q. Wang, 1993. Robust state-feedback controllers for systems under mixed time/frequency domain constraints. Proceedings of 32nd IEEE Conference on Decision and Control, December 15-17, 1993, IEEE, San Antonio, Texas, USA., pp: 2584-2589.

Smith, C.A. and A.B. Corripio, 2006. Principles and Practice of Automatic Process Control. 3rd Edn., John Wiley and Sons, New York, ISBN: 978-0-471-43190-9, Pages: 563.

Zhuang, M. and D.P. Atherton, 1993. Automatic tuning of optimum PID controllers. IEE Proc. D: Control Theory Applic., 140: 216-224.

Ziegler, J.G., N.B. Nichols and N.Y. Rochester, 1942. Optimum settings for automatic controllers. Trans. Am. Soc. Mech. Eng., 64: 759-765.