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Adaptive Controller Design for the Generalized Projective Synchronization of 4-Scroll Systems

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Abstract: This study investigates the adaptive controller design for the Generalized Projective Synchronization (GPS) of identical Liu-Chen 4-scroll chaotic systems, identical Lu-Chen-Cheng 4-scroll chaotic systems and non-identical Liu-Chen and Lu-Chen-Cheng 4-scroll chaotic systems. Lyapunov Stability Theory is the methodology used for establishing the adaptive GPS synchronization results derived in this study. Since, the Lyapunov exponents are not required for these calculations, the proposed Adaptive Control Method is very effective and convenient for achieving the Generalized Projective Synchronization (GPS) of the 4-scroll chaotic systems. Numerical simulations are shown to demonstrate the effectiveness of the adaptive GPS synchronization results derived in this study for the 4-scroll chaotic systems.

Key words: Adaptive control, chaos, chaotic systems, 4-scroll systems, generalized projective synchronization, Liu-Chen system, Lu-Chen-Cheng system

INTRODUCTION

Chaotic systems are nonlinear dynamical systems which are characterized by sensitive dependence on initial conditions. This sensitivity is popularly known as the butterfly effect (Alligood *et al*., 1997).

Chaos is an interesting nonlinear phenomenon and has been studied well in the last 3 decades. Chaos theory has wide applications in several fields like physical systems (Lakshmanan and Murali, 1996), chemical systems (Han *et al*., 1995), ecological systems (Blasius *et al*., 1999), secure communications, etc (Cuomo and Oppenheim, 1993; Kocarev and Parlitz, 1995; Murali and Lakshmanan, 1998).

Chaos synchronization is a phenomenon that may occur when two or more chaotic oscillators are coupled or a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a challenging research problem. In most of the chaos synchronization approaches, the master slave formalism is used. If a particular chaotic system is called the master system and another chaotic system is called the slave system then the idea of the chaos synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically. Since the seminal research by Pecora and Carroll (1990), chaos synchronization problem has been studied well in the chaos literature.

In the last 2 decades, various schemes have been derived for chaos synchronization such as OGY Method (Ott *et al*., 1990), Active Control Method (Ho and Hung, 2002; Huang *et al*., 2004; Chen, 2005; Sundarapandian, 2011d), Adaptive Control Method (Lu *et al*., 2004b; Chen and Lu, 2002; Sundarapandian, 2011a, e), Time Delay Feedback Method (Park and Kwon, 2003), Backstepping Design Method (Wang and Ge, 2001; Xiau-Qun and Jun-An, 2003; Park, 2006; Vincent, 2007), Sampled Data Feedback Synchronization Method (Lee *et al*., 2010), Sliding Mode Control Method (Slotine and Sastry, 1983; Utkin, 1993; Vaidyanathan and Sampath, 2011), etc.

So far, many types of synchronization phenomenon have been presented such as complete synchronization (Pecora and Carroll, 1990), generalized synchronization (Wang and Guan, 2006), anti-synchronization (Zhang and Zhu, 2008; Chiang *et al*., 2008; Sundarapandian, 2011h), hybrid synchronization (Sundarapandian, 2011b, c, f, g), projective synchronization (Jia, 2007), generalized projective synchronization (Li, 2009; Sarasu and Sundarapandian, 2011a, b, 2012), etc.

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Complete Synchronization (CS) is characterized by the equality of state variables evolving in time while Anti-Synchronization (AS) is characterized by the disappearance of the sum of relevant state variables evolving in time. In hybrid synchronization of two chaotic systems, one part of the systems is completely synchronized and the other part is anti-synchronized so that the Complete Synchronization (CS) and Anti-Synchronization (AS) co-exist in the systems. Projective Synchronization (PS) is characterized by the fast that the master and slave systems could be synchronized up to a scaling factor.

In Generalized Projective Synchronization (GPS), the responses of the synchronized dynamical states synchronize up to a constant scaling matrix α. It is easy to see that the complete synchronization and anti-synchronization are special cases of the generalized projective synchronization where the scaling matrix $\alpha = I$ and $\alpha = -I$, respectively.

This study describes the adaptive controller design for the GPS of the identical Liu-Chen 4-scroll systems (Liu and Chen, 2004), the identical Lu-Chen-Cheng 4-scroll systems (Lu *et al*., 2004a) and the non-identical Liu-Chen and Lu-Chen-Cheng 4-scroll systems. The adaptive GPS synchronization results for the 4-scroll systems have been established using the Lyapunov Stability Theory (Hahn, 1967).

SYSTEM DESCRIPTION

In this study, the researchers describe the 4-scroll chaotic systems. The Liu-Chen 4-scroll system (Liu and Chen, 2004) is described by the 3D dynamics:

$$\begin{aligned} \dot{x}_1 &= ax_1 - x_2x_3 \\ \dot{x}_2 &= -bx_2 + x_1x_3 \\ \dot{x}_3 &= -cx_3 + x_1x_2 \end{aligned} \tag{1}$$

where, x_1-x_3 are the states and a-c are constant, positive parameters of the system. The system (1) exhibits a 4-scroll chaotic attractor when the system parameter values are chosen as:

$$a = 0.4,\ b = 12,\ c = 5$$

The 4-scroll strange attractor of the Liu-Chen system (1) is depicted in Fig. 1. The Lu-Chen-Cheng 4-scroll system (Lu *et al*., 2004b) is described by the 3D dynamics:

$$\begin{aligned} \dot{x}_1 &= px_1 - x_2x_3 \\ \dot{x}_2 &= -qx_2 + x_1x_3 + s \\ \dot{x}_3 &= -rx_3 + x_1x_2 \end{aligned} \tag{2}$$

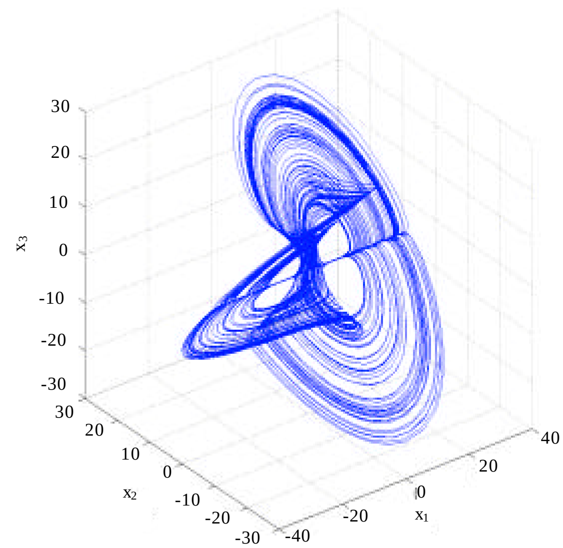

Fig. 1: The 4-scroll attractor of the Liu-Chen system

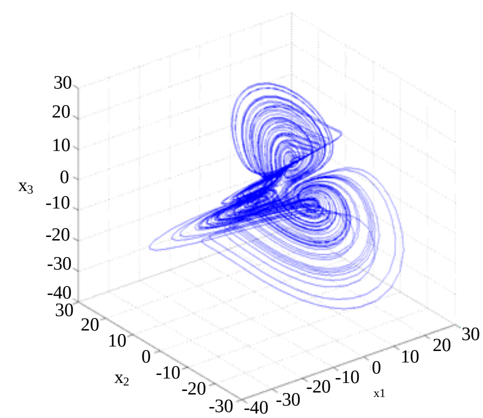

Fig. 2: The 4-scroll attractor of the Lu-Chen-Cheng system

where, x_1-x_3 are the states and p-s are constant, positive parameters of the system. The system (2) exhibits a 4-scroll chaotic attractor when the system parameter values are chosen as:

$$p = 20/7,\ q = 10,\ r = 4,\ s = 5$$

The 4-scroll strange attractor of the Lu-Chen-Cheng system (2) is shown in Fig. 2.

ADAPTIVE GENERALIZED PROJECTIVE SYNCHRONIZATION OF IDENTICAL WANG 3-SCROLL SYSTEMS

Theoretical results: In this study, researchers deploy adaptive control to derive results for the Generalized Projective Synchronization (GPS) of the identical Liu-Chen

4-scroll systems in 2004 when the system parameters are unknown. Thus, the master system is described by the Liu-Chen dynamics:

$$\begin{aligned} \dot{x}_1 &= ax_1 - x_2x_3 \\ \dot{x}_2 &= -bx_2 + x_1x_3 \\ \dot{x}_3 &= -cx_3 + x_1x_2 \end{aligned} \tag{3}$$

where, x_1-x_3 are the states and a-c are unknown parameters of the system. Also, the slave system is described by the controlled Liu-Chen dynamics:

$$\begin{aligned} \dot{y}_1 &= ay_1 - y_2y_3 + u_1 \\ \dot{y}_2 &= -by_2 + y_1y_3 + u_2 \\ \dot{y}_3 &= -cy_3 + y_1y_2 + u_3 \end{aligned} \tag{4}$$

where, y_1-y_3 are the states and u_1-u_3 are the adaptive controls to be designed. The GPS synchronization errors are defined as:

$$e_i = y_i - \alpha_i x_i \, (i = 1,2,3) \tag{5}$$

where, the scales α_1-α_3 are real numbers. The error dynamics is obtained as:

$$\begin{aligned} \dot{e}_1 &= ae_1 - y_2y_3 + \alpha_1 x_2x_3 + u_1 \\ \dot{e}_2 &= -be_2 + y_1y_3 - \alpha_2 x_1x_3 + u_2 \\ \dot{e}_3 &= -ce_3 + y_1y_2 - \alpha_3 x_1x_2 + u_3 \end{aligned} \tag{6}$$

The researchers consider the adaptive controller defined by:

$$\begin{aligned} u_1 &= -\hat{a}e_1 + y_2y_3 - \alpha_1 x_2x_3 - k_1e_1 \\ u_2 &= \hat{b}e_2 - y_1y_3 + \alpha_2 x_1x_3 - k_2e_2 \\ u_3 &= \hat{c}e_3 - y_1y_2 + \alpha_3 x_1x_2 - k_3e_3 \end{aligned} \tag{7}$$

where, $\hat{a}$-$\hat{c}$ are estimates of the parameters a-c, respectively. Substituting Eq. 7 into 6, the researchers obtain the closed-loop error dynamics:

$$\begin{aligned} \dot{e}_1 &= (a - \hat{a})e_1 - k_1e_1 \\ \dot{e}_2 &= -(b - \hat{b})e_2 - k_2e_2 \\ \dot{e}_3 &= -(c - \hat{c})e_3 - k_3e_3 \end{aligned} \tag{8}$$

The researchers define the parameter estimation errors as:

$$e_a = a - \hat{a},\ e_b = b - \hat{b},\ e_c = c - \hat{c} \tag{9}$$

Using Eq. 9, the error dynamics in Eq. 8 is simplified as:

$$\begin{aligned} \dot{e}_1 &= e_a e_1 - k_1e_1 \\ \dot{e}_2 &= -e_b e_2 - k_2e_2 \\ \dot{e}_3 &= -e_c e_3 - k_3e_3 \end{aligned} \tag{10}$$

For the derivation of the update law for adjusting the estimates of parameters, the Lyapunov method is used. The researchers consider the quadratic Lyapunov function defined by:

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2\right) \tag{11}$$

Which is positive definite on R^6. The researchers note that:

$$\dot{e}_a = -\dot{\hat{a}},\ \dot{e}_b = -\dot{\hat{b}},\ \dot{e}_c = -\dot{\hat{c}} \tag{12}$$

Differentiating Eq. 11 along the trajectories of the Eq. 10 and using Eq. 12, the researchers find that:

$$\begin{aligned} \dot{V} = &-k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_a\left[e_1^2 - \dot{\hat{a}}\right] + \\ &e_b\left[-e_2^2 - \dot{\hat{b}}\right] + e_c\left[-e_3^2 - \dot{\hat{c}}\right] \end{aligned} \tag{13}$$

In view of Eq. 13, the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= e_1^2 + k_4e_a \\ \dot{\hat{b}} &= -e_2^2 + k_5e_b \\ \dot{\hat{c}} &= -e_3^2 + k_6e_c \end{aligned} \tag{14}$$

where, the gains k_4-k_6 are positive constants.

Theorem 1: The adaptive control law (7) achieves General Projective Synchronization (GPS) between the identical Liu-Chen 4-scroll Chaotic systems (3) and (4), where the parameter update law is given by (14) and the gains k_i ($i = 1, 2,...,6$) are positive constants. The GPS errors e_i ($i = 1, 2, 3$) and the parameter estimation errors e_a-e_c converge exponentially to zero as $t \to \infty$ for all initial conditions.

Proof: Upon substituting the parameter update law Eq. 14 into the Eq. 13, the researchers obtain the derivative of the quadratic Lyapunov function, V as:

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_a^2 - k_5e_b^2 - k_6e_c^2 \tag{15}$$

Which is a negative definite function on R^6. Hence by Lyapunov stability theory (Hahn, 1967), it follows that the GPS errors e_1-e_3→0 as t→∞ as and the parameter estimator errors e_a-e_c→0 as t→∞ for all initial conditions. This completes the proof.

Numerical results: For the numerical simulations, the 4th order Runge-Kutta Method is used to solve the two systems of differential Eq. 3 and 4 with the adaptive controller (7). The parameter estimates of the identical Liu-Chen systems (3) and (4) are taken so that the systems exhibit 4-scroll chaotic attractors, i.e., a = 0.4, b = 12 and c = 5. The researchers take the state feedback gains as:

$$k_i = 4 \text{ for } i = 1, 2, \ldots, 6$$

The initial values of the parameter estimates are chosen as:

$$\hat{a}(0) = 6,\ \hat{b}(0) = 8,\ \hat{c}(0) = 20$$

The initial values of the master system (3) are chosen as:

$$x_1(0) = 12,\ x_2(0) = 21,\ x_3(0) = -5$$

The initial values of the slave system (4) are chosen as:

$$y_1(0) = -2,\ y_2(0) = 9,\ y_3(0) = 15$$

The GPS scales α_i are chosen as:

$$a_1 = 2.8,\ \ a_2 = 0.6,\ \ a_3 = -1.4$$

Figure 3 shows the GPS between the identical Liu-Chen 4-scroll chaotic systems (3) and (4). Figure 4 shows the time history of the GPS errors e_1-e_3. Figure 5 shows that the parameter estimates $\hat{a}-\hat{c}$ converge to the chosen values of the system a-c, parameters respectively, as t→∞. Figure 6 shows the time history of the parameter estimation errors e_a-e_c.

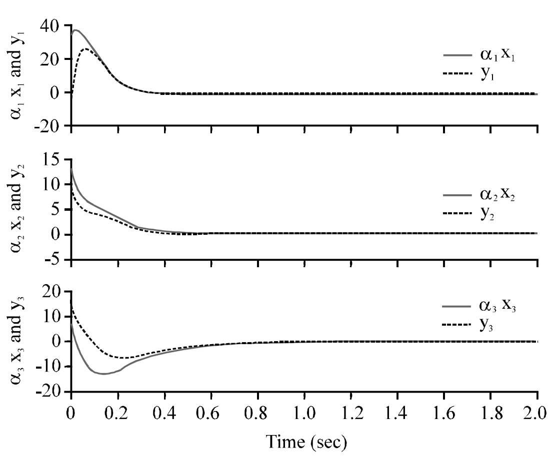

Fig. 3: GPS of the identical Liu-Chen 4-scroll systems (Theorem 1)

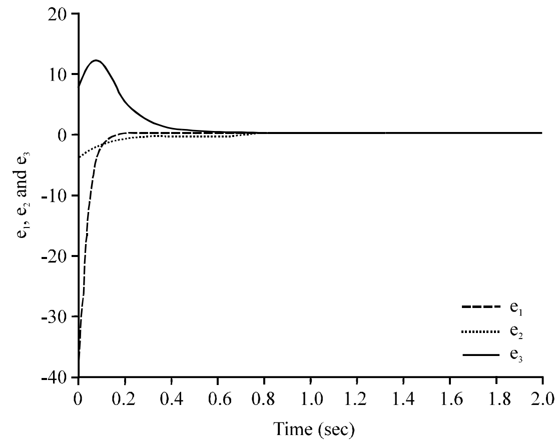

Fig. 4: Time history of the GPS errors (Theorem 1)

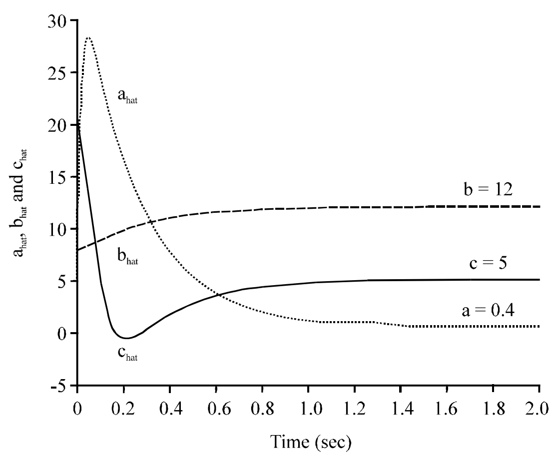

Fig. 5: Time history of the parameter estimates (Theorem 1)

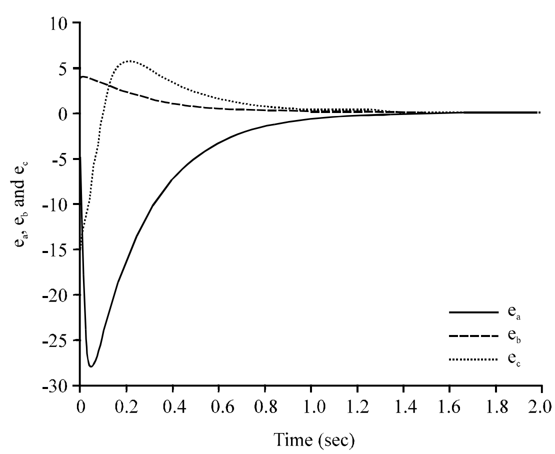

Fig. 6: Time history of the parameter estimation errors (Theorem 1)

ADAPTIVE GENERALIZED PROJECTIVE SYNCHRONIZATION OF IDENTICAL LU-CHEN-CHENG 4-SCROLL CHAOTIC SYSTEMS

Theoretical results: In this study, the researchers deploy adaptive control to derive results for the Generalized Projective Synchronization (GPS) of the identical Lu-Chen-Cheng 4-scroll systems in 2004 when the system parameters are unknown. Thus, the master system is described by the Lu-Chen-Cheng dynamics:

$$\begin{aligned}\dot{x}_1 &= px_1 - x_2x_3\\ \dot{x}_2 &= -qx_2 + x_1x_3 + s\\ \dot{x}_3 &= -rx_3 + x_1x_2\end{aligned} \qquad (16)$$

where, x_1-x_3 are the states and p-s are unknown parameters of the system. Also, the slave system is described by the controlled Lu-Chen-Cheng dynamics:

$$\begin{aligned}\dot{y}_1 &= py_1 - y_2y_3 + u_1\\ \dot{y}_2 &= -qy_2 + y_1y_3 + s + u_2\\ \dot{y}_3 &= -ry_3 + y_1y_2 + u_3\end{aligned} \qquad (17)$$

where, y_1-y_3 are the states and u_1-u_3 are the adaptive controls to be designed. The GPS synchronization errors are defined as:

$$e_i = y_i - a_ix_i (i = 1,2,3) \qquad (18)$$

where, the scales α_1-α_3 are real numbers. The error dynamics is obtained as:

$$\begin{aligned}\dot{e}_1 &= pe_1 - y_2y_3 + a_1x_2x_3 + u_1\\ \dot{e}_2 &= -qe_2 + (y_1y_3 - a_2x_1x_3) + s(1-a_2) + u_2\\ \dot{e}_3 &= -re_3 + y_1y_2 - a_3x_1x_2 + u_3\end{aligned} \qquad (19)$$

The researchers consider the adaptive controller defined by:

$$\begin{aligned}u_1 &= -\hat{p}e_1 + y_2y_3 - a_1x_2x_3 - k_1e_1\\ u_2 &= \hat{q}e_2 - y_1y_3 + a_2x_1x_3 - \hat{s}(1-a_2) - k_2e_2\\ u_3 &= \hat{r}e_3 - y_1y_2 + a_3x_1x_2 - k_3e_3\end{aligned} \qquad (20)$$

where, $\hat{p}-\hat{s}$ are estimates of the parameters p-s, respectively. Substituting Eq. 20 into 19, researchers obtain the closed-loop error dynamics:

$$\begin{aligned}\dot{e}_1 &= (p-\hat{p})e_1 - k_1e_1\\ \dot{e}_2 &= -(q-\hat{q})e_2 + (s-\hat{s})(1-\alpha_2) - k_2e_2\\ \dot{e}_3 &= -(r-\hat{r})e_3 - k_3e_3\end{aligned} \qquad (21)$$

The researchers define the parameter estimation errors as:

$$e_p = p - \hat{p}, e_q = q - \hat{q}, e_r = r - \hat{r}, e_s = s - \hat{s} \qquad (22)$$

Using Eq. 22, the error dynamics in Eq. 21 is simplified as:

$$\begin{aligned}\dot{e}_1 &= e_pe_1 - k_1e_1\\ \dot{e}_2 &= -e_qe_2 + e_s(1-a_2) - k_2e_2\\ \dot{e}_3 &= -e_re_3 - k_3e_3\end{aligned} \qquad (23)$$

For the derivation of the update law for adjusting the estimates of parameters, the Lyapunov method is used. The researchers consider the quadratic Lyapunov function defined by:

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_p^2 + e_q^2 + e_r^2 + e_s^2\right) \qquad (24)$$

Which is positive definite on R^7. The researchers note that:

$$\dot{e}_p = -\dot{p}, \dot{e}_q = -\dot{\hat{q}}, \dot{e}_r = -\dot{\hat{r}}, \dot{e}_s = -\dot{\hat{s}} \qquad (25)$$

Differentiating Eq. 24 along the trajectories of Eq. 23 and using Eq. 25, the researchers find that:

$$\begin{aligned}\dot{V} = &-k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_p\left[e_1^2 - \dot{\hat{p}}\right] +\\ &e_q\left[-e_2^2 - \dot{\hat{q}}\right] + e_r\left[-e_3^2 - \dot{\hat{r}}\right] +\\ &e_s\left[e_2(1-a_2) - \dot{\hat{s}}\right]\end{aligned} \qquad (26)$$

In view of Eq. 26, the estimated parameters are updated by the following law:

$$\begin{aligned}&\dot{\hat{p}} = e_1^2 + k_4e_p,\ \dot{\hat{q}} = -e_2^2 + k_5e_q,\\ &\dot{\hat{r}} = -e_3^2 + k_6e_r,\ \dot{\hat{s}} = e_2(1-a_2) + k_7e_s\end{aligned} \qquad (27)$$

where, the gains k_4-k_7 are positive constants.

Theorem 2: The adaptive control law (20) achieves General Projective Synchronization (GPS) between the identical Lu-Chen-Cheng 4-scroll chaotic systems (16) and (17) where the parameter update law is given by (27) and the gains k_i (i = 1, 2,....,7) are positive constants. The GPS errors e_i (I = 1, 2, 3) and the parameter estimation errors e_p-e_s converge exponentially to zero as $t\to\infty$ for all initial conditions.

Proof: Upon substituting the parameter update law (27) into the Eq. 26, the researchers obtain the derivative of the quadratic Lyapunov function V as:

$$\begin{aligned}\dot{V} = &-k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_p^2 -\\ &k_5e_q^2 - k_6e_r^2 - k_7e_s^2\end{aligned} \qquad (28)$$

Which is a negative definite function on R^7. Hence by Lyapunov stability theory (Hahn, 1967), it follows that the GPS errors e_1-$e_3 \to 0$ as $t \to \infty$ and the parameter estimator errors e_p-$e_s \to 0$ as $t \to \infty$ for all initial conditions.

Numerical results: For the numerical simulations, the fourth order Runge-Kutta Method is used to solve the two systems of differential Eq. 16 and 17 with the adaptive controller (20). The parameter estimates of the identical Lu-Chen-Cheng systems (16) and (17) are taken so that the systems exhibit 4-scroll chaotic attractors, i.e., $p = 20/7$, $q = 10$, $r = 4$, $s = 5$.

The researchers take the state feedback gains as $k_i = 4$ for $i = 1, 2, \ldots, 7$. The initial values of the parameter estimates are chosen as:

$$\hat{p}(0) = 5, \hat{q}(0) = 2, \hat{r}(0) = 6, \hat{s}(0) = 4$$

The initial values of the master system (16) are chosen as:

$$x_1(0) = 12, \; x_2(0) = 8, \; x_3(0) = -6$$

The initial values of the slave system (17) are chosen as:

$$y_1(0) = 9, \; y_2(0) = -10, \; y_3(0) = 2$$

The GPS scales α_i are chosen as:

$$a_1 = -2.9, \; a_2 = 1.7, \; a_3 = 1.5$$

Figure 7 shows the GPS between the identical Lu-Chen-Cheng 4-scroll chaotic systems (16) and (17). Figure 8 shows the time history of the GPS errors e_1-e_3. Figure 9 shows that the parameter estimates $\hat{p} - \hat{s}$

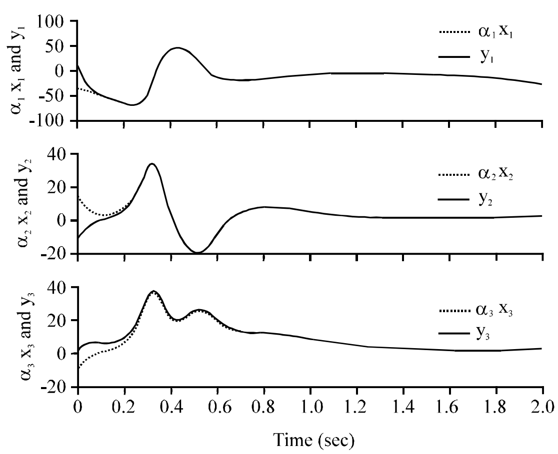

Fig. 7: GPS of the Identical Lu-Chen-Cheng 4-scroll system (Theorem 2)

converge to the chosen values of the system parameters p-s, respectively as $t \to \infty$. Figure 10 shows the time history of the parameter estimation errors e_p-e_s.

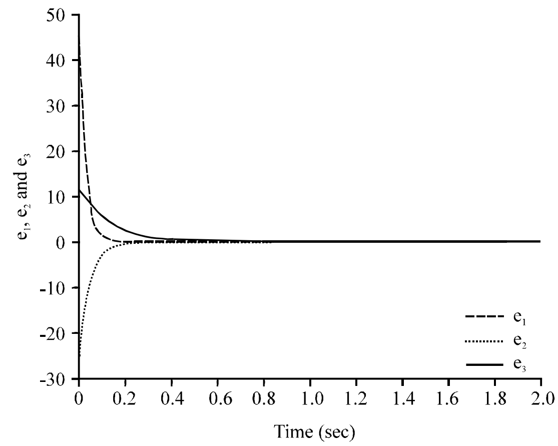

Fig. 8: Time history of GPS errors (Theorem 2)

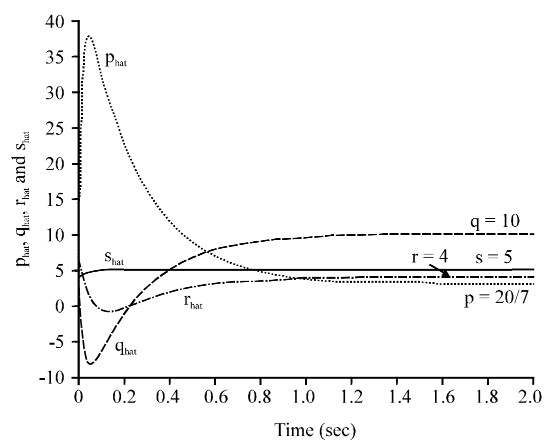

Fig. 9: Time history of parameters estimates (Theorem 1)

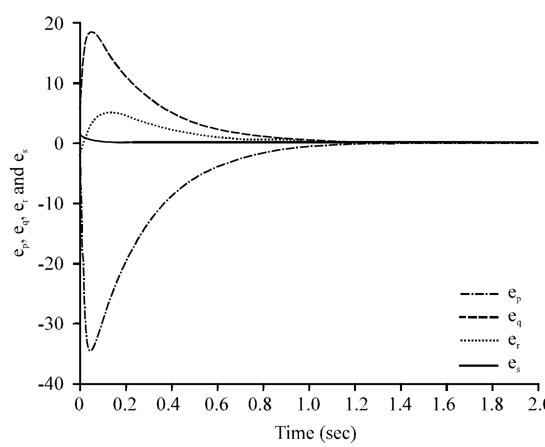

Fig. 10: Time history of the parameter estimation errors (Theorem 2)

ADAPTIVE GENERALIZED PROJECTIVE SYNCHRONIZATION OF LIU-CHEN AND LU-CHEN-CHENG 4-SCROLL SYSTEMS

Theoretical results: In this study, the researchers deploy adaptive control to derive results for the Generalized Projective Synchronization (GPS) of the non-identical Liu-Chen and Lu-Chen-Cheng 4-scroll systems in 2004 when the system parameters are unknown. Thus, the master system is described by the Liu-Chen dynamics:

$$\begin{aligned} \dot{x}_1 &= ax_1 - x_2x_3 \\ \dot{x}_2 &= -bx_2 + x_1x_3 \\ \dot{x}_3 &= -cx_3 + x_1x_2 \end{aligned} \quad (29)$$

where, x_1-x_3 are the states and a-c are unknown parameters of the system. Also, the slave system is described by the controlled Lu-Chen-Cheng dynamics:

$$\begin{aligned} \dot{y}_1 &= py_1 - y_2y_3 + u_1 \\ \dot{y}_2 &= -qy_2 + y_1y_3 + s + u_2 \\ \dot{y}_3 &= -ry_3 + y_1y_2 + u_3 \end{aligned} \quad (30)$$

where, y_1-y_3 are the states, p-s are unknown parameters of the system and u_1-u_3 are the adaptive controls to be designed. The GPS synchronization errors are defined as:

$$e_i = y_i - a_i x_i (i = 1,2,3) \quad (31)$$

where, the scales α_1-α_3 are real numbers. The error dynamics is obtained as:

$$\begin{aligned} \dot{e}_1 &= py_1 - y_2y_3 - \alpha_1(ax_1 - x_2x_3) + u_1 \\ \dot{e}_2 &= -qy_2 + y_1y_3 + s - \alpha_2(-bx_2 + x_1x_3) + u_2 \\ \dot{e}_3 &= -ry_3 + y_1y_2 - \alpha_3(-cx_3 + x_1x_2) + u_3 \end{aligned} \quad (32)$$

The researchers consider the adaptive controller as:

$$\begin{aligned} u_1 &= -\hat{p}y_1 + y_2y_3 + \alpha_1(\hat{a}x_1 - x_2x_3) - k_1e_1 \\ u_2 &= \hat{q}y_2 - y_1y_3 - \hat{s} + \alpha_2(-\hat{b}x_2 + x_1x_3) - k_2e_2 \\ u_3 &= \hat{r}y_3 - y_1y_2 + \alpha_3(-\hat{c}x_3 + x_1x_2) - k_3e_3 \end{aligned} \quad (33)$$

where, $\hat{a},\hat{b},\hat{c},\hat{p},\hat{q},\hat{r},\hat{s}$ are estimates of the parameters, $a,b,c,p,q,r,s,$ respectively. Substituting Eq. 33 into 32, the researchers obtain the closed loop error dynamics:

$$\begin{aligned} \dot{e}_1 &= (p - \hat{p})y_1 - a_1x_1(a - \hat{a}) - k_1e_1 \\ \dot{e}_2 &= -(q - \hat{q})y_2 + (s - \hat{s}) + a_2x_2(b - \hat{b}) - k_2e_2 \\ \dot{e}_3 &= -(r - \hat{r})y_3 + a_3x_3(c - \hat{c}) - k_3e_3 \end{aligned} \quad (34)$$

The researchers define the parameter estimation errors as:

$$\begin{aligned} &e_a = a - \hat{a}, e_b = b - \hat{b}, e_c = c - \hat{c}, e_p = p - \hat{p} \\ &e_q = q - \hat{q}, e_r = r - \hat{r}, e_s = s - \hat{s} \end{aligned} \quad (35)$$

Using Eq. 35, the error dynamics in Eq. 34 is simplified as:

$$\begin{aligned} \dot{e}_1 &= e_py_1 - \alpha_1x_1e_a - k_1e_1 \\ \dot{e}_2 &= -e_qy_2 + e_s + \alpha_2x_2e_b - k_2e_2 \\ \dot{e}_3 &= -e_ry_3 + \alpha_3x_3e_c - k_3e_3 \end{aligned} \quad (36)$$

For the derivation of the update law for adjusting the estimates of parameters, the Lyapunov method is used. The researchers consider the quadratic Lyapunov function defined by:

$$V = \frac{1}{2}\begin{bmatrix} e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2 + \\ e_p^2 + e_q^2 + e_r^2 + e_s^2 \end{bmatrix} \quad (37)$$

which is positive definite on R^7. The researchers note that:

$$\begin{aligned} &\dot{e}_a = -\dot{\hat{a}}, \dot{e}_b = -\dot{\hat{b}}, \dot{e}_c = -\dot{\hat{c}}, \dot{e}_p = -\dot{\hat{p}}, \\ &\dot{e}_q = -\dot{\hat{q}}, \dot{e}_r = -\dot{\hat{r}}, \dot{e}_s = -\dot{\hat{s}} \end{aligned} \quad (38)$$

Differentiating Eq. 37 along the trajectories of the system (36) and using Eq. 38, the researchers find that:

$$\begin{aligned} \dot{V} = &-k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_a\left[-\alpha_1x_1e_1 - \dot{\hat{a}}\right] + \\ &e_b\left[\alpha_2e_2x_2 - \dot{\hat{b}}\right] + e_c\left[\alpha_3x_3e_3 - \dot{\hat{c}}\right] + \\ &e_p\left[y_1e_1 - \dot{\hat{p}}\right] + e_q\left[-y_2e_2 - \dot{\hat{q}}\right] + \\ &e_r\left[-y_3e_3 - \dot{\hat{r}}\right] + e_s\left[e_2 - \dot{\hat{s}}\right] \end{aligned} \quad (39)$$

In view of Eq. 39, the estimated parameters are updated by the following law:

$$\begin{aligned} &\dot{\hat{a}} = -\alpha_1x_1e_1 + k_4e_a, \dot{\hat{b}} = \alpha_2x_2e_2 + k_5e_b, \\ &\dot{\hat{c}} = \alpha_3x_3e_3 + k_6e_c, \dot{\hat{p}} = y_1e_1 + k_7e_p, \\ &\dot{\hat{q}} = -y_2e_2 + k_8e_q, \dot{\hat{r}} = -y_3e_3 + k_9e_r, \\ &\dot{\hat{s}} = e_2 + k_{10}e_s \end{aligned} \quad (40)$$

where, the gains k_i ($i = 4,...,10$) are positive constants.

Theorem 3: The adaptive control law (33) achieves General Projective Synchronization (GPS) between the non-identical Liu-Chen 4-scroll system (29) and the Lu-Chen-Cheng 4-scroll chaotic system (30) where the parameter update law is given by (40) and the gains k_i (i = 1, 2,...,10) are positive constants. The GPS errors e_i (i = 1, 2, 3) and the parameter estimation errors e_a, e_b, e_c, e_p, e_q, e_r, e_s converge exponentially to zero as $t\to\infty$ for all initial conditions.

Proof: Upon substituting the parameter update law (40) into Eq. 39, researchers obtain the derivative of the quadratic Lyapunov function V as:

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_a^2 - k_5e_b^2 - k_6e_c^2 - k_7e_p^2 - k_8e_q^2 - k_9e_r^2 - k_{10}e_s^2 \quad (41)$$

which is a negative definite function on R^{10}. Hence by Lyapunov stability theory (42), it follows that the GPS errors e_1-$e_3\to 0$ as $t\to\infty$ and the parameter estimator errors $e_a\to 0$, $e_b\to 0$, $e_c\to 0$, $e_p\to 0$, $e_q\to 0$, $e_r\to 0$, $e_s\to 0$ as $t\to\infty$ for all initial conditions.

Numerical results: For the numerical simulations, the 4th order Runge-Kutta Method is used to solve the two systems of differential Eq. 29 and 30 with the adaptive controller (33).

The parameter estimates of the non-identical Liu-Chen system (29) and the Lu-Chen-Cheng system (30) are chosen so that the systems exhibit 4-scroll chaotic attractors, i.e.,

$$a = 0.4,\ b = 12,\ c = 5,\ p = 20/7,\ q = 10,\ r = 4,\ s = 5$$

The researchers take the state feedback gains as:

$$k_i = 4 \text{ for } i = 1, 2, 3, \ldots,10$$

The initial values of the parameter estimates are chosen as:

$$\hat{a}(0) = 3,\ \hat{b}(0) = 4,\ \hat{c}(0) = 19,$$
$$\hat{p}(0) = 7,\ \hat{q}(0) = 8,\ \hat{r}(0) = 12,\ \hat{s}(0) = 10$$

The initial values of the master system (16) are chosen as:

$$x_1(0) = 16,\ x_2(0) = -7,\ x_3(0) = 19$$

The initial values of the slave system (17) are chosen as:

$$y_1(0) = -6,\ y_2(0) = 14,\ y_3(0) = 5$$

The GPS scales are chosen as:

$$a_1 = 1.9, a_2 = -2.5, a_3 = 3.7$$

Figure 11 shows the GPS between the Liu-Chen and Lu-Chen-Cheng 4-scroll chaotic systems. Figure 12 shows the time history of the GPS errors e_1-e_3. Figure 13

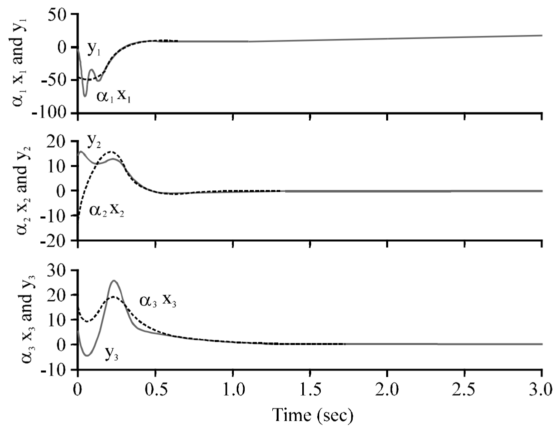

Fig. 11: GPS of the Liu-Chen and Lu-Chen-Cheng 4-Scroll chaotic systems

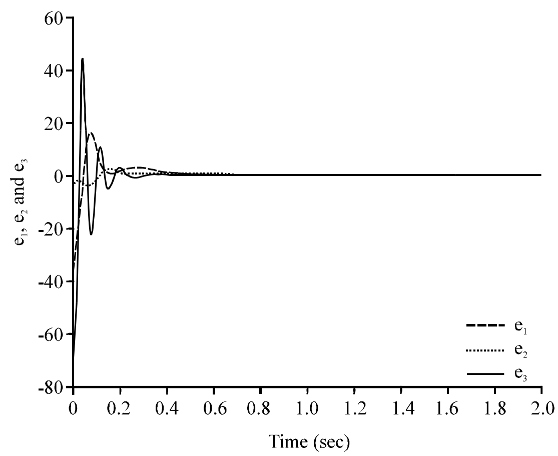

Fig. 12: Time history of the GPS errors (Theorem 3)

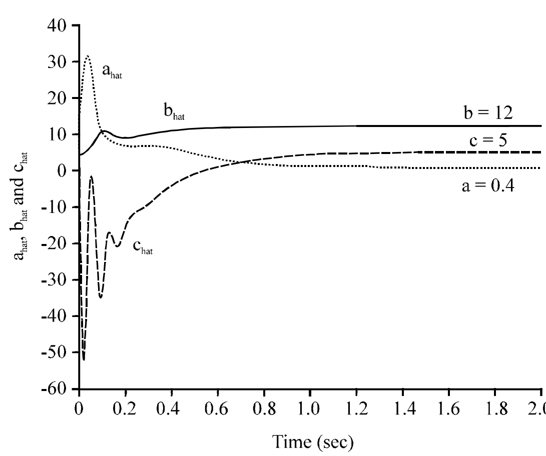

Fig. 13: Time history of the parameter estimates (Theorem 3)

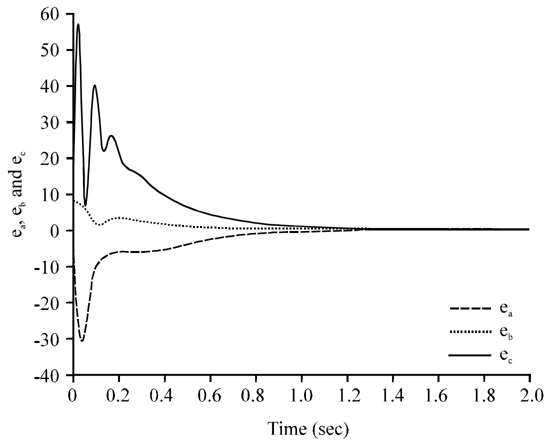

Fig. 14: Time history of the parameter estimation errors e_a-e_c (Theorem 3)

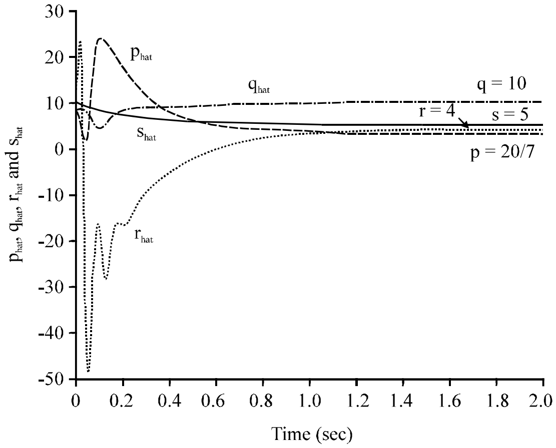

Fig. 15: Time history of the parameter estimates $\hat{p}, \hat{q}, \hat{r}, \hat{s}$ (Theorem 3)

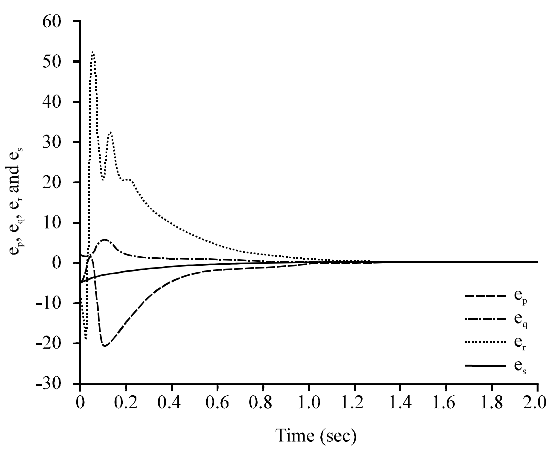

Fig. 16: Time history of the parameter estimation errors e_p-e_s (Theorem 3)

shows that the parameter estimates $\hat{a}-\hat{c}$ converge to the chosen values of the system parameters a-c, respectively as $t\to\infty$. Figure 14 shows the time history of the parameter estimation errors e_a-e_c. Figure 15 shows that the parameter estimates $\hat{p}-\hat{s}$ converge to the chosen values of the system parameters p-s, respectively as $t\to\infty$. Figure 16 shows the time history of the parameter estimation errors e_p-e_s.

CONCLUSION

In this study, researchers have designed adaptive controllers for achieving Generalized Projective Synchronization (GPS) of 4-scroll chaotic attractors, viz., the identical Liu-Chen 4-scroll chaotic systems in 2004, the identical Lu-Chen-Cheng 4-scroll chaotic systems and the non-identical Liu-Chen and Lu-Chen-Cheng 4-scroll chaotic systems when the system parameters are unknown.

The adaptive GPS synchronization results for the 4-scroll chaotic systems have been proved using the Lyapunov stability theory. Numerical simulations have been presented to validate and demonstrate the effectiveness of the GPS synchronization results derived in this study.

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