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Hybrid Synchronization of Hyperchaotic Wang-Chen and Hyperchaotic Lorenz Systems by Active Non-linear Control

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Abstract: This study investigates the hybrid synchronization of identical hyperchaotic Wang-Chen systems, identical hyperchaotic Lorenz systems and non-identical hyperchaotic Wang-Chen and Lorenz systems. The hyperchaotic Wang-Chen system and hyperchaotic Lorenz system are important models of hyperchaotic systems. Hybrid synchronization of the hyperchaotic systems addressed in this study is achieved through the synchronization of the 1st and 3rd states of the master and slave systems and anti-synchronization of the 2nd and 4th states of the master and slave systems. Active nonlinear control is the method used for the hybrid synchronization of hyperchaotic Wang-Chen and Lorenz systems and the stability results are established using Lyapunov stability theory. Since, the Lyapunov exponents are not required for these calculations, the proposed method is quite effective and convenient to achieve hybrid synchronization of the hyperchaotic systems addressed in this study. Numerical simulations are provided to illustrate the effectiveness of the various synchronization schemes proposed in this study.

Key words: Hybrid synchronization, hyperchaos, hyperchaotic Wang-Chen system, hyperchaotic Lorenz system, non-linear control, India

INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. This sensitivity is popularly known as the butterfly effect (Alligood *et al*., 1997). Chaos is an important non-linear phenomenon and has been widely studied in the last two decades (Alligood *et al*., 1997; Pecora and Carroll, 1990; Lakshmanan and Murali, 1996; Han *et al*., 1995; Blasius *et al*., 1999; Feki, 2003; Murali and Lakshmanan, 1998; Yang and Chua, 1999; Ott *et al*., 1990; Park and Kwon, 2003; Yu and Zhang, 2006; Liao and Tsai, 2000; Konishi *et al*., 1998; Ge and Chen, 2004; Wang and Guan, 2006; Zhang and Zhu, 2008; Chiang *et al*., 2008; Qiang, 2007; Yan and Li, 2006; Li *et al*., 2007; Rui-Hong *et al*., 2010; Wang and Chen, 2008; Wang and Wang, 2007).

Pecora and Carroll (1990) introduced a method to synchronize two identical chaotic systems and showed the possibility of completely synchronizing some chaotic systems. From then on, chaos synchronization has been explored in a wide variety of fields including physical (Lakshmanan and Murali, 1996), chemical (Han *et al*., 1995), ecological (Blasius *et al*., 1999) systems, secure communication (Feki, 2003; Murali and Lakshmanan, 1998), etc.

Since, the seminal research by Pecora and Carroll (1990) a variety of impressive approaches have been proposed for the synchronization of chaotic systems such as PC Method, Sampled-data Feedback Synchronization Method (Yang and Chua, 1999), OGY Method (Ott *at al*., 1990), Time-delay Feedback Method (Park and Kwon, 2003), Backstepping Method (Yu and Zhang, 2006), Adaptive Design Method (Liao and Tsai, 2000), Sliding-mode Control Method (Konishi *at al*., 1998), etc. So far, many types of synchronization phenomenon have been studied such as complete synchronization (Pecora and Carroll, 1990; Lakshmanan and Murali, 1996), phase synchronization (Ge and Chen, 2004), generalized synchronization (Wang and Guan, 2006), anti-synchronization (Zhang and Zhu, 2008; Chiang *et al*., 2008), projective synchronization (Qiang, 2007), etc.

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Complete Synchronization (CS) is characterized by the equality of state variables evolving in time while Anti-Synchronization (AS) is characterized by the disappearance of the sum of relevant state variables evolving in time.

Projective Synchronization (PS) is characterized by the fact that the master and slave systems could by synchronized up to a scaling factor whereas in Generalized Projective Synchronization (GPS), the responses of the synchronized dynamical systems synchronize up to a constant scaling matrix α. It is easy to that the complete synchronization and anti-synchronization are special cases of the generalized projective synchronization where the scaling matrix $\alpha = I$ and $\alpha = -I$, respectively.

In hybrid synchronization of chaotic systems (Rui-Hong *et al.*, 2010), one part of the system is completely synchronized and the other part is anti-synchronized so that Complete Synchronization (CS) and Anti-Synchronization (AS) co-exist in the system. The coexistence of CS and AS is very useful in applications such as secure communication, chaotic encryption schemes, etc.

In this study, researchers use the active non-linear control method for the hybrid synchronization of identical and different hyperchaotic Wang-Chem system and hyperchaotic Lorenz system (2007). For the hybrid synchronization of the 4-D hyperchaotic systems considered in this study, the 1st and 3rd states of the master and slave systems are Completely Synchronized (CS) and the 2nd and 4th states are Anti-Synchronized (AS). The active nonlinear control method is a simple and effective method for the hybrid synchronization of hyperchaotic systems.

MATERIALS AND METHODS

Hybrid synchronization of identical hyperchaotic Wang-Chen systems: In this study, we consider the hybrid synchronization of identical hyperchaotic Wang-Chen systems (Wang and Chen, 2008). Thus, the master system is described by the Wang-Chen dynamics:

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2x_3 \\
\dot{x}_2 &= cx_1 - x_1x_3 - x_2 - 0.5x_4 \\
\dot{x}_3 &= x_1x_2 - 3x_3 \\
\dot{x}_4 &= 0.5x_1x_3 - bx_4
\end{aligned} \tag{1}$$

Where, x_1 (i = 1, 2, 3, 4) are the state variables and a-c are real parameters with a>0, b<0 and c>0. When a = 40,

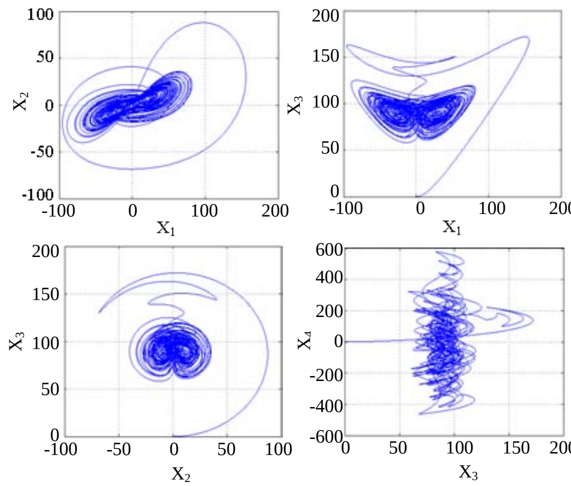

Fig. 1: State portrait of hyperchaotic Wang-Chen system

b = 1.7 and c = 88, the Wang-Chen system (1) is hyperchaotic as shown in Fig. 1. The hyperchaotic Wang-Chen dynamics is also taken as the slave system which is described by the dynamics:

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + y_2y_3 + u_1 \\
\dot{y}_2 &= cy_1 - y_1y_3 - y_2 - 0.5y_4 + u_2 \\
\dot{y}_3 &= y_1y_2 - 3y_3 + u_3 \\
\dot{y}_4 &= 0.5y_1y_3 - by_4 + u_4
\end{aligned} \tag{2}$$

Where, y_i (i = 1, 2, 3) are the state variables and u_i (i = 1, 2, 3) are the active controls. For the hybrid synchronization of the identical hyperchaotic Wang-Chen systems (1 and 2), the errors are defined as:

$$\begin{aligned}
e_1 &= y_1 - x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 - x_3 \\
e_4 &= y_4 + x_4
\end{aligned} \tag{3}$$

From the error Eq. 3, it is obvious that one part of the hyperchaotic systems (1 and 2) is completely synchronized (1st and 3rd states) while the other part is anti-synchronized (2nd and 4th states) so that Complete Synchronization (CS) and Anti-synchronization (AS) co-exist in the synchronization process. A simple calculation yields, the error dynamics as:

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) - 2ax_2 + y_2y_3 - x_2x_3 + u_1 \\
\dot{e}_2 &= ce_1 - e_2 - 0.5e_4 + 2cx_1 - y_1y_3 - x_1x_3 + u_2 \\
\dot{e}_3 &= -3e_3 + y_1y_2 - x_1x_2 + u_3 \\
\dot{e}_4 &= -be_4 + 0.5(y_1y_3 + x_1x_3) + u_4
\end{aligned} \tag{4}$$

Researchers consider the nonlinear controller defined by:

$$\begin{aligned} u_1 &= -ae_2 + 2ax_2 - y_2y_3 + x_2x_3 \\ u_2 &= -ce_1 + 0.5e_4 - 2cx_1 + y_1y_3 + x_1x_3 \\ u_3 &= -y_1y_2 + x_1x_2 \\ u_4 &= (b-1)e_4 - 0.5(y_1y_3 + x_1x_3) \end{aligned} \quad (5)$$

Substitution of Eq. 5 into 4 yields, the linear system:

$$\dot{e}_1 = -ae_1,\ \dot{e}_2 = -e_2,\ \dot{e}_3 = -3e_3,\ \dot{e}_4 = -e_4 \quad (6)$$

The candidate Lyapunov function is taken as:

$$V(e) = \frac{1}{2}e^Te = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right)$$

Which is a positive definite function on R^4. A simple calculation gives:

$$\dot{V}(e) = -ae_1^2 - e_2^2 - 3e_3^2 - e_4^2$$

Which is a negative definite function on R^4 since, a>0. Thus, by Lyapunov stability theory (Hahn, 1967), the error dynamics (6) is globally exponentially stable.

RESULTS AND DISCUSSION

Theorem 1: The identical hyperchaotic Wang-Chen systems (1 and 2) are globally and exponentially hybrid synchronized with the active non-linear controller (5).

Numerical simulations: For the numerical simulations, the 4th order Runge-Kutta Method with step-size $h = 10^{-6}$ is used to solve the differential Eq. 1 and 2 with the active non-linear controller (5). The parameters of the identical Wang-Chen systems (1 and 2) are selected as:

$$a = 40,\ b = -1.7,\ c = 88$$

So that, the systems (1 and 2) are hyperchaotic. The initial values of the master system (1) are chosen as:

$$x_1(0) = 15,\ x_2(0) = 7,\ x_3(0) = 12,\ x_4(0) = 9$$

and the initial values of the slave system (2) are chosen as:

$$y_1(0) = 4,\ y_2(0) = 14,\ y_3(0) = 6,\ y_4(0) = 2$$

Figure 2 shows the hybrid synchronization of the identical hyperchaotic Wang-Chen systems (1 and 2).

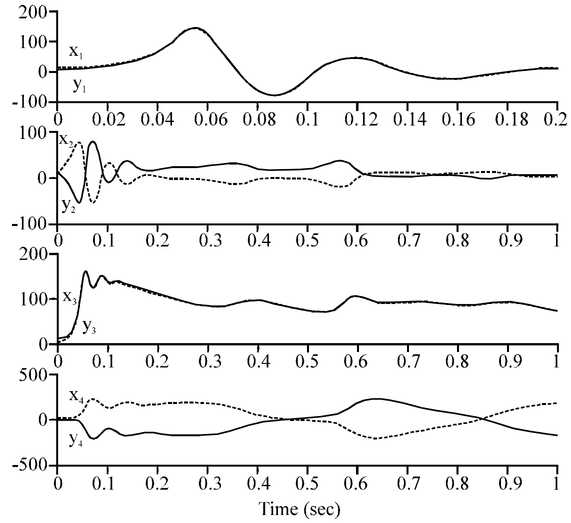

Fig. 2: Hybrid synchronization of the hyperchaotic Wang-Chen systems

Hybrid synchronization of identical hyperchaotic Lorenz systems: In this study, we onsider the hybrid synchronization of identical hyperchaotic Lorenz systems (Wang and Wang, 2007). Thus, the master system is described by the Lorenz dynamics as:

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) + x_4 \\ \dot{x}_2 &= \gamma x_1 - x_2 - x_1x_3 \\ \dot{x}_3 &= x_1x_2 - \beta x_3 \\ \dot{x}_4 &= -x_1x_3 - rx_4 \end{aligned} \quad (7)$$

Where, x_i (i = 1, 2, 3) are the state variables and α, β, γ, r are positive real parameters. When α and 10, β = 8/3, γ = 28 and r = 1, the system (7) is hyperchaotic as shown in Fig. 3. The hyperchaotic Lorenz dynamics is also taken as the slave system which is described by the dynamics:

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + y_4 + u_1 \\ \dot{y}_2 &= \gamma y_1 - y_2 - y_1y_3 + u_2 \\ \dot{y}_3 &= y_1y_2 - \beta y_3 + u_3 \\ \dot{y}_4 &= -y_1y_3 - ry_4 + u_4 \end{aligned} \quad (8)$$

Where y_i (i = 1, 2, 3, 4) are the state variables and u_i (i = 1, 2, 3, 4) are the active controls. For the hybrid synchronization of the identical hyperchaotic Lorenz systems (7 and 8), the errors are defined as:

$$\begin{aligned} e_1 &= y_1 - x_1,\ \ e_2 = y_2 + x_2 \\ e_3 &= y_3 - x_3,\ \ e_4 = y_4 + x_4 \end{aligned} \quad (9)$$

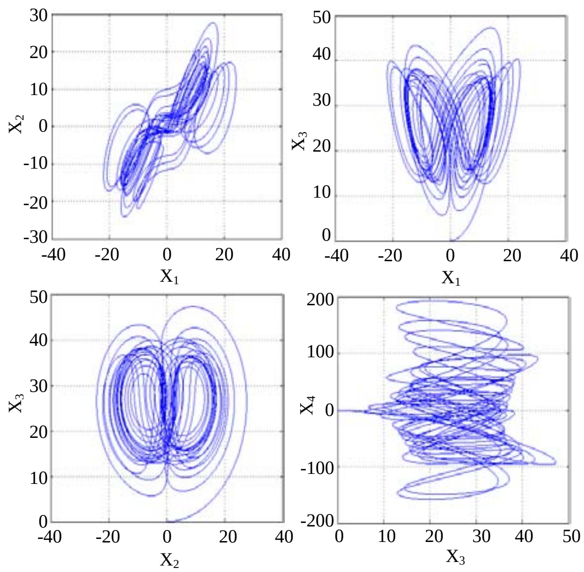

Fig. 3: State portrait of hyperchaotic Lorenz system

From the error Eq. 9, it is obvious that one part of the hyperchaotic systems (7 and 8) is completely synchronized (1st and 3rd states) while the other part is anti-synchronized (2nd and 4th states) so that, Complete Synchronization (CS) and Anti-synchronization (AS) co-exist in the synchronization process. A simple calculation yields the error dynamics as:

$$\begin{aligned}
\dot{e}_1 &= \alpha(e_2 - e_1) + e_4 - 2\alpha x_2 - 2x_4 + u_1 \\
\dot{e}_2 &= \gamma e_1 - e_2 + 2\gamma x_1 - y_1 y_3 - x_1 x_3 + u_2 \\
\dot{e}_3 &= -\beta e_3 + y_1 y_2 - x_1 x_2 + u_3 \\
\dot{e}_4 &= -r e_4 - y_1 y_3 - x_1 x_3 + u_4
\end{aligned} \quad (10)$$

Researchers consider the non-linear controller defined by:

$$\begin{aligned}
u_1 &= -\alpha e_2 - e_4 + 2\alpha x_2 + 2x_4 \\
u_2 &= -\gamma e_1 - 2\gamma x_1 + y_1 y_3 + x_1 x_3 \\
u_3 &= -y_1 y_2 + x_1 x_2 \\
u_4 &= y_1 y_3 + x_1 x_3
\end{aligned} \quad (11)$$

Substitution of Eq. 11 and 10 yields, the linear system:

$$\begin{aligned}
\dot{e}_1 &= -\alpha e_1, \quad \dot{e}_2 = -e_2 \\
\dot{e}_3 &= -\beta e_3, \quad \dot{e}_4 = -r e_4
\end{aligned} \quad (12)$$

The candidate Lyapunov function is taken as:

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right)$$

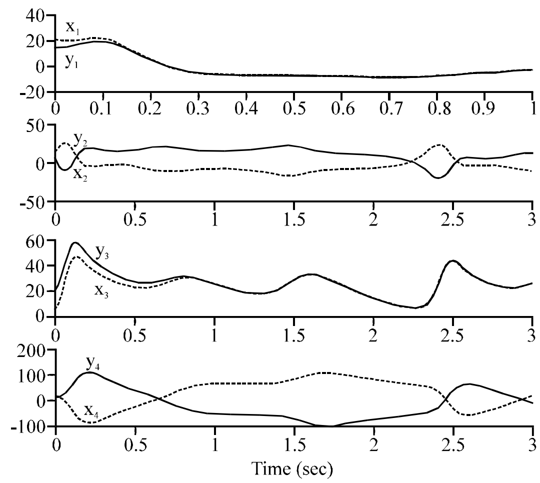

Fig. 4: Hybrid synchronization of the hyperchaotic Lorenz systems

Which is a positive definite function on R^4. A simple calculation gives:

$$\dot{V}(e) = -\alpha e_1^2 - e_2^2 - \beta e_3^2 - r e_4^2$$

Which is a negative definite function on R^4 since, α, β and r are positive real constants. Thus, by Lyapunov Stability Theory (Hahn, 1967), the error dynamics (12) is globally exponentially stable. Hence, we obtain the following result.

Theorem 2: The identical hyperchaotic Lorenz systems (7 and 8) are globally and exponentially hybrid synchronized with the active nonlinear controller (11).

Numerical simulations: For the numerical simulations, the fourth order Runge-Kutta Method with step-size $h = 10^{-6}$ is used to solve the differential Eq. 7 and 8 with the active nonlinear controller (11). The parameters of the identical Lorenz systems (7 and 8) are selected as:

$$\alpha = 10, \ \beta = \frac{8}{3}, \ \gamma = 28, \ r = 1$$

So that, the systems (7 and 8) are hyperchaotic. The initial values of the master system (7) are chosen as:

$$x_1(0) = 20, \ x_2(0) = 11, \ x_3(0) = 5, \ x_4(0) = 12$$

and the initial values of the slave system (8) are chosen as:

$$y_1(0) = 14, \ y_2(0) = 5, \ y_3(0) = 20, \ y_4(0) = 8$$

Figure 4 shows the hybrid synchronization of the identical hyperchaotic Lorenz systems (7 and 8).

Hybrid synchronization of hyperchaotic Wang-Chen and hyperchaotic Lorenz systems: In this study, we consider the hyperchaotic Wang-Chen system (Wang and Chen, 2008) as the master system which is described by the dynamics:

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_2x_3\\ \dot{x}_2 &= cx_1 - x_1x_3 - x_2 - 0.5x_4\\ \dot{x}_3 &= x_1x_2 - 3x_3\\ \dot{x}_4 &= 0.5x_1x_3 - bx\end{aligned} \tag{13}$$

Where, x_i (i = 1, 2, 3, 4) are state variables and a-c are real constants with a>0, b<0 and c>0. When a = 40, b = -1.7 and c = 88, the Wang-Chen system (1) is hyperchaotic. We consider the Lorenz system (Wang and Wang, 2007) as the slave system which is described by the dynamics:

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + y_4 + u_1\\ \dot{y}_2 &= \gamma y_1 - y_2 - y_1y_3 + u_2\\ \dot{y}_3 &= y_1y_2 - \beta y_3 + u_3\\ \dot{y}_4 &= -y_1y_3 - ry_4 + u_4\end{aligned} \tag{14}$$

Where, y_i (i = 1, 2, 3, 4) are the state variables and u_i (i = 1, 2, 3, 4) are the active controls. When α = 10, β = 8/3, γ = 28 and r = 1, the system (14) is hyperchaotic. For the hybrid synchronization of the hyperchaotic systems (13 and 14), the errors are defined as:

$$\begin{aligned}e_1 &= y_1 - x_1, \quad e_2 = y_2 + x_2\\ e_3 &= y_3 - x_3, \quad e_4 = y_4 + x_4\end{aligned} \tag{15}$$

From the error Eq. 15, it is obvious that one part of the hyperchaotic systems (13 and 14) is completely synchronized (1st and 3rd states) while the other part is anti-synchronized (2nd and 4th states) so that, Complete Synchronization (CS) and Anti-synchronization (AS) co-exist in the synchronization process. A simple calculation yields the error dynamics as:

$$\begin{aligned}\dot{e}_1 &= \alpha(e_2 - e_1) + e_4 + (a-\alpha)x_1 - (a+\alpha)x_2\\ &\quad - x_4 - x_2x_3 + u_1\\ \dot{e}_2 &= \gamma e_1 - e_2 + (c+\gamma)x_1 - y_1y_3 - x_1x_3\\ &\quad - 0.5x_4 + u_2\\ \dot{e}_3 &= -\beta e_3 + (3-\beta)x_3 + y_1y_2 - x_1x_2 + u_3\\ \dot{e}_4 &= -re_4 + (r-b)x_4 - y_1y_3 + 0.5x_1x_3 + u_4\end{aligned} \tag{16}$$

We consider the non-linear controller defined by:

$$\begin{aligned}u_1 &= -\alpha e_2 - e_4 - (a-\alpha)x_1 + (a+\alpha)x_2 + x_4 + x_2x_3\\ u_2 &= -\gamma e_1 - (c+\gamma)x_1 + 0.5x_4 + y_1y_3 + x_1x_3\\ u_3 &= (\beta - 3)x_3 - y_1y_2 + x_1x_2\\ u_4 &= (b-r)x_4 + y_1y_3 - 0.5x_1x_3\end{aligned} \tag{17}$$

Substitution of Eq. 17 into 16 yields, the linear system:

$$\begin{aligned}\dot{e}_1 &= -\alpha e_1, \ \dot{e}_2 = -e_2\\ \dot{e}_3 &= -\beta e_3, \ \dot{e}_4 = -re_4\end{aligned} \tag{18}$$

The candidate Lyapunov function is taken as:

$$V(e) = \frac{1}{2}e^Te = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right)$$

Which is a positive definite function on R^4A simple calculation gives:

$$\dot{V}(e) = -\alpha e_1^2 - e_2^2 - \beta e_3^2 - re_4^2$$

Which is a negative definite function on R^4 since, α, β and r are positive real constants. Thus by Lyapunov Stability Theory (Hahn, 1967), the error dynamics (18) is globally exponentially stable. Hence, we obtain the following result.

Theorem 3: The hyperchaotic Wang-Chen system (13) and hyperchaotic Lorenz system (14) are globally and exponentially hybrid synchronized with the active non-linear controller (17).

Numerical simulations: For the numerical simulations, the 4th order Runge-Kutta Method with step-size $h = 10^{-6}$ is used to solve the differential Eq. 13 and 14 with the active nonlinear controller (11). The parameters of the identical Wang-Chen system (13) are selected as:

$$a = 40, \ b = -1.7, \ c = 88$$

So that, the Wang-Chen system (13) is hyperchaotic. The parameters of the Lorenz system (14) are selected as:

$$\alpha = 10, \ \beta = \frac{8}{3}, \ \gamma = 28, \ r = 1$$

So that, the Lorenz system (14) is hyperchaotic. The initial values of the master system (13) are chosen as:

$$x_1(0) = 30, \ x_2(0) = 24, \ x_3(0) = 15, \ x_4(0) = 6$$

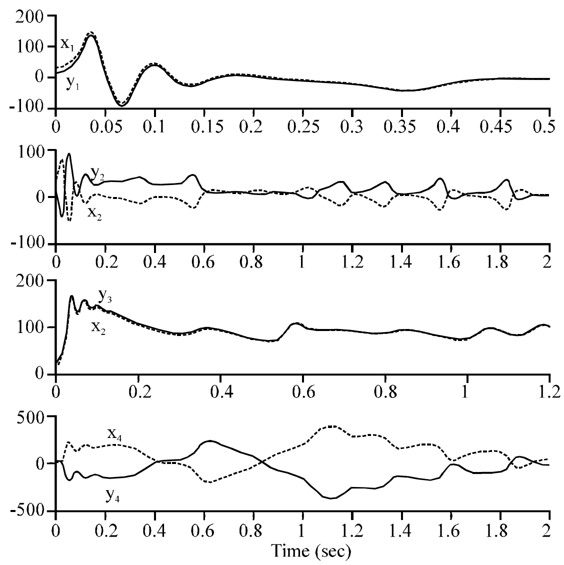

Fig. 5: Hybrid synchronization of the hyperchaotic Wang-Chen and hyperchaotic Lorenz systems

The initial values of the slave system (14) are chosen as:

$$y_1(0) = 12,\ y_2(0) = 10,\ y_3(0) = 23,\ y_4(0) = 17$$

Figure 5 shows the hybrid synchronization of the different hyperchaotic systems (13 and 14).

CONCLUSION

In this study, researchers deployed active nonlinear control method for the hybrid synchronization of the following three types of hyperchaotic systems:

- Identical hyperchaotic Wang-Chen systems
- Identical hyperchaotic Lorenz systems (2007)
- Non-identical hyperchaotic Wang-Chen system and hyperchaotic Lorenz system (2007)

The global hybrid synchronization results for the cases A-C are established using Lyapunov Stability Theory. Numerical simulations are shown to illustrate the effectiveness of the proposed hybrid synchronization schemes for the cases A-C. Since, Lyapunov exponents are not required for these calculations, the proposed active nonlinear control method is effective and convenient to achieve the hybrid synchronization of the hyperchaotic systems addressed in this study.

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