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An Active Vibration Control of Beam by Piezoelectric with Fuzzy Approach

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Abstract: Today, the vibration control of structures considers as one of the most important and useful study fields, which requires developing the new ways of control and simulating its effect on flexible structures. Using the piezoelectric sensors and actuators in the purpose of active vibrations control enumerates as one of the most basic devices in designing systems and smart structures. One of the controlling ways, which has been watched an ever-increasing progress, is fuzzy control. This controlling way as one of the active control subdivisions provides some ways to analyze and design the complicated and inaccurate systems. This study presents a study concerning the active vibration control of a smart cantilever beam that is excited by transverse vibrations of the structure that it is clamped to and the clamped end of the beam is experiencing base motion. It has been used fuzzy method in the purpose of designing controller. The electromechanical equations have acquired from energy methods and with Euler-Bernoulli beam theory. In continuation, numerical results have been obtained from an aluminum beam with simulating in MATLAB software. The obtained results show that fuzzy control, in addition to considerable reducing of calculations capacity, has satisfactory function too.

Key words: Piezoelectric, active vibration control, fuzzy control, beam designing system, smart structure

INTRODUCTION

Most structures can model in the form of beam; because of this the vibration control of beam structures is one of the most favored cases in mechanical engineering.

As mentioned before, piezoelectric enumerates as one of the main tools in vibration control of smart structures. The most important function of these elements is the ability of output electric production by the effect of mechanical input and vice versa. Because of mechanical charging, piezoelectric element changes its polarity and produces voltage that this kind of function is called the direct effect of piezoelectric, while these elements are caused of transformation by the effect of electric field that is known as opposite effect of piezoelectric. In smart structures, direct effect is used for measuring the vibrations (sensor) and opposite effect for controlling the vibrations (actuator).

The active vibration control by using piezoelectric has been the case of study for many researchers.

Balamurugan and Narayanan (2003) have examined the active control of smart structures by piezoelectric sensor and actuator; they have used from limited element formulation and have examined the beam, plate and shell structures.

Sodano (2003) examined the application of piezoelectric as a sensor and actuator, in this article has been used a feedback controller and also the comparison between the piezoelectric effects MFC and traditional monolithic PZT has been done.

Vasques and Rodrigues (2006) have presented a numerical study in the field of active vibration control of smart piezoelectric beams and have done a comparison between classical control strategy and optimal control strategy to clarify their effects in controlling the vibrations of beam by piezoelectric layers.

Lin and Zheng (2006) have solved the problem of vibrations control with piezoelectric by making use of a fuzzy controller. They have used a simple model and have not considered the electromechanical effects of piezoelectric (Lin and Zheng, 2006).

In most studies, the finite element formulation and the classical control methods have been utilized, also in some samples of fuzzy control, a simple model for piezoelectric has been used.

In this study, the effects of mass and stiffness and also the coupling effects and piezoelectric capacity have been considered and a more completed model has presented. Besides, the sensor and actuator dynamic equations have been regarded separately. In the following, the vibration control of structure has been preceded by designing an appropriate fuzzy controller.

MODELING

General layout of the system: The examined system includes a smart cantilever beam with 2 piezoelectric layers, above and below of the beam and attached to it that the clamped end of the beam is experiencing base motion (Fig. 1). As a result of the beam vibration,

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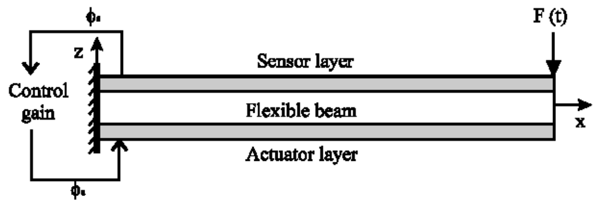

Fig. 1: General layout of the system

piezoelectric sensor produces voltage that sensor voltage and its changes are controller input and applied voltage to the actuator is output of fuzzy controller which finally lead to control and damping of structure vibrations (Jezequel, 1995).

Energy equations (Hamilton's principle): For gaining to the motion equations by using Hamilton's Principle we have (Meirovitch, 1997):

$$\text{V.I.} = \int_{t1}^{t2} [\delta T - \delta U + f\delta x] dt = 0 \tag{1}$$

Which the terms U and T and fδx can be calculated as follows:

$$T = \frac{1}{2}\int_{V_s} \rho_s \underline{\dot{u}}^T \underline{\dot{u}} dV_s + \frac{1}{2}\int_{V_p} \rho_s \underline{\dot{u}}^T \underline{\dot{u}} dV_p \tag{2}$$

$$U = \frac{1}{2}\int_{V_s} \underline{S}^T \underline{T} dV_s + \frac{1}{2}\int_{V_p} \underline{S}^T \underline{T} dV_p - \int_{V_p} \underline{E}^T \underline{D}\, dV_p \tag{3}$$

$$f\delta x = \sum_{i=1}^{nf} \delta\underline{u}(x_i) \cdot \underline{f}_i(x_i) - \sum_{j=1}^{nq} \delta\underline{v} \cdot \underline{q}_j \tag{4}$$

where:

U	=	The potential energy
T	=	The kinetic energy
fδx	=	The external work applied to the system
S	=	The strain
T	=	The stress
E	=	The electric filed
D	=	The electric displacement
V	=	The volume
u	=	The displacement
x	=	The position along the beam
v	=	The applied voltage
q	=	The charge
ρ	=	The density
f	=	The applied force and the subscripts
p and s	=	The piezoelectric material and the substrate, respectively

Constitutive equations: Before continuing with Hamilton's Principle, we examined constitutive equations of piezoelectric elements (Jansson, 2007):

$$\begin{bmatrix} \underline{T} \\ \underline{D} \end{bmatrix} = \begin{bmatrix} c^E & -e^T \\ e & \varepsilon^S \end{bmatrix} \begin{bmatrix} \underline{S} \\ \underline{E} \end{bmatrix} \tag{5}$$

where:

c	=	The modulus of elasticity
ε	=	The dielectric constant and the superscript
S	=	Signifies the parameter was measured at constant strain and the superscript
E	=	Indicates the parameter was measured at constant electric field (short circuit)
e	=	The piezoelectric coupling coefficient and relates the stress to the applied electric field that can be shown as follow:

$$e = d_{ij} c^E \tag{6}$$

where:

c	=	Piezoelectric capacity
d_{ij}	=	The piezoelectric coupling coefficient with the subscript
i and j	=	Referring to the direction of the applied field and the poling, respectively

Now by incorporating piezoelectric properties in Hamilton equations result in Eq. 7:

$$\begin{aligned} \text{V.I.} = \int_{t_1}^{t_2} \Big[& \int_{V_s} \rho_s \delta\underline{\dot{u}}^T \underline{\dot{u}} dV_s + \int_{V_p} \rho_p \delta\underline{\dot{u}}^T \underline{\dot{u}} dV_p \\ & - \int_{V_s} \delta\underline{S}^T c_s \underline{S} dV_s - \int_{V_p} \delta\underline{S}^T c^E \underline{S} dV_p \\ & + \int_{V_p} \delta\underline{S}^T e^T \underline{E} dV_p + \int_{V_p} \delta\underline{E}^T e \underline{S} dV_p \\ & + \int_{V_p} \delta\underline{E}^T \varepsilon^S \underline{E} dV_p + \sum_{i=1}^{nf} \delta\underline{u}(x_i) \cdot \underline{f}(x_i) - \sum_{j=1}^{nq} \delta\underline{v} \cdot \underline{q}_j \Big] \end{aligned} \tag{7}$$

Equations of motion: This equation can now be used to solve for the equations of motion of any mechanical system containing piezoelectric elements. In order to solve Eq. 5 for the cantilever beam with bimorph piezoelectric elements some assumptions must be made.

The 1st assumption follows the Rayleigh-Ritz procedure, which says that the displacement of the beam can be written as the summation of modes in the beam and a temporal coordinate (Luenberger, 1979):

$$u(x,t) = \sum_{i=1}^{N} \phi_i(x) r_i(t) = \underline{\phi}(x) \underline{r}(t) \tag{8}$$

where:

$\Phi_i(x)$ = The assumed mode shapes of the structure, which can be set to satisfy any combination of boundary conditions
r (t) = The temporal coordinate of the displacement
N = The number of modes to be included in the analysis

The 2nd assumption made is to apply the Euler-Bernoulli beam theory. This allows the strain in the beam to be written as the product of the distance from the neutral axis and the second derivative of displacement with respect to the position along the beam. On the basis of this theory, we can write the strain as Eq. 9:

$$\underline{S} = -y\frac{\partial^2 u(u,t)}{\partial x^2} = -y\phi(x)'' r(t) \quad (9)$$

The 3rd is that the electric potential across the piezoelectric element is constant. This assumption also indicates that no field is applied to the beam, which in latter equations designates the beam to be inactive material:

$$\underline{E} = \psi(y)v(t) = \begin{cases} -v/t_p & t/2 < y < t/2 + t_p \\ 0 & -t/2 < y < t/2 \\ v/t_p & -t/2 - t_p < y < -t/2 \end{cases} \quad (10)$$

Using the previous assumptions, we can simplify the variational indicator to include terms that represent physical parameters. By doing this the equations describing the system become more recognizable when compared to those of a typical system and help give physical meaning to the parameters in the equations of motion. The mass matrices for the system can be written as:

$$M_s = \int_{V_s} \rho_s \underline{\phi}^T(x)\underline{\phi}(x)dV_s \quad (11)$$

$$M_p = \int_{V_p} \rho_p \underline{\phi}^T(x)\underline{\phi}(x)dV_p \quad (12)$$

The stiffness matrices can be written as:

$$K_s = \int_{V_s} y^2 \underline{\phi}^T(x)'' c_s \underline{\phi}(x)'' dV_s \quad (13)$$

$$K_p = \int_{V_p} y^2 \underline{\phi}^T(x)'' c^E \underline{\phi}(x)'' dV_p \quad (14)$$

The electromechanical coupling matrix, Θ and the capacitance matrix, C_p, are defined by:

$$\Theta = -\int_{V_p} y\underline{\phi}^T(x)'' e^T \psi(y)dV_p \quad (15)$$

$$C_p = \int_{V_p} \psi^T(y)\varepsilon^S \psi(y)dV_p \quad (16)$$

By substituting, the obtained parameters and with proportional damping assumption for the system, we can calculate the Eq. 17 and 18 of motion as:

$$\begin{aligned}(M_s + M_p)\ddot{\underline{r}}(t) + C\dot{\underline{r}}(t) + (K_s + K_p)\\ \underline{r}(t) - \Theta v(t) = \sum_{i=1}^{nm}\phi(x_i)^T f_i(t)\end{aligned} \quad (17)$$

$$\Theta^T r(t) + C_p v(t) = q(t) \quad (18)$$

By assuming q (t) = 0 and considering that v (t) includes sensor and actuator voltage, then Eq. 19 and 20 can be like this:

$$\begin{aligned}(M_s + M_p)\ddot{\underline{r}}(t) + C\dot{\underline{r}}(t) + (K_s + K_p)\underline{r}(t) + \\ \Theta_S C_{PS}^{-1}\Theta_S^T r(t) = -\Theta_a v_a(t) + \sum_{i=1}^{nm}\phi(x_i)^T f_i(t)\end{aligned} \quad (19)$$

$$v_S(t) = -C_{PS}^{-1}\Theta_S^T r(t) \quad (20)$$

ACTIVE CONTROL OF VIBRATIONS

State space design: In order to design controller and analysis of the system behavior, by choosing the below state variables, equations in state area will obtain:

$$X(t) = [r(t)\dot{r}(t)] \quad (21)$$

$$\dot{X}(t) = AX(t) + B_v U_v(t) + B_r U_r(t) \quad (22)$$

$$\text{Sensor voltage: } Y(t) = C_n X(t) \quad (23)$$

$$\text{Displacement: } Y(t) = C_r X(t) \quad (24)$$

where:

A = The system matrix
B_v and B_r = Are the mechanical and electrical input matrices
C_v and C_r = The output matrices
$U_r(t)$ and $U_v(t)$ = The mechanical and electrical input vectors
Y(t) = The output vector, given by

$$A = \begin{bmatrix} 0 & I \\ -\left((M_S + M_P)^{-1}(K_S + K_P)\right) & \\ -(M_S + M_P)^{-1}\Theta_S C_{PS}{}^{-1}\Theta_S{}^T & -(M_S + M_P)^{-1} C \end{bmatrix} \quad (25)$$

$$B_v = \begin{bmatrix} 0 \\ -(M_S + M_P)^{-1}\Theta_P \end{bmatrix} \quad (26)$$

$$B_r = \begin{bmatrix} 0 \\ (M_S + M_P)^{-1} F \end{bmatrix} \quad (27)$$

$$C_v = \begin{bmatrix} -C_P{}^{-1}\Theta^T & 0 \end{bmatrix} \quad (28)$$

$$C_r = \begin{bmatrix} \Phi(x_i) & 0 \end{bmatrix} \quad (29)$$

Controller design: As we have seen in Fig. 2 , there has been used a fuzzy controller in order to control structure vibrations, which the sensor voltage and its changes are as controller input and applied voltage in actuator is output of fuzzy controller (Lin, 2005).

The designed fuzzy system is consists of two inputs, an output and 25 controller rules that Fig. 3 shows a summarized graphical representation of that.

The applied Membership functions for changing the input and output variables to linguistic control variables are triangular that can be calculated as follow Foully and Galichet (1995) (Fig. 4):

$$u(x) = \begin{cases} 0, & x < a_1 \\ (x - a_1)/(a_2 - a_1) & a_1 \le x \le a_2 \\ (a_3 - x)/(a_3 - a_2) & a_2 \le x \le a_3 \\ 0, & x > a_3 \end{cases} \quad (30)$$

The rule base design of fuzzy subsystems is based on presimulation investigations and also by using some general rules. The statements employ fuzzy quantities, such as Negative Big (NB), Negative Small (NS), Zero (ZE), Positive Small (PS) and Positive Big (PB), which require corresponding membership functions (Reznik, 1997) (Fig. 5).

This study applies fuzzy control rules of state evaluation, which are similar to the institutional thinking of humans:

$$R^i : \text{if e is } A_{1i}, \Delta e \text{ is } B_{1i}, \text{ then u is } C_{1i} \quad (31)$$

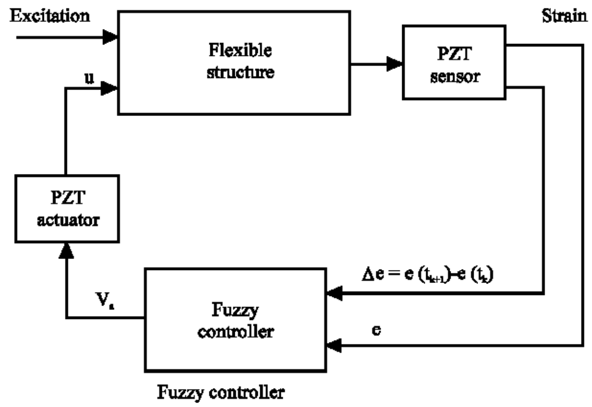

Fig. 2: Block diagram for the controller

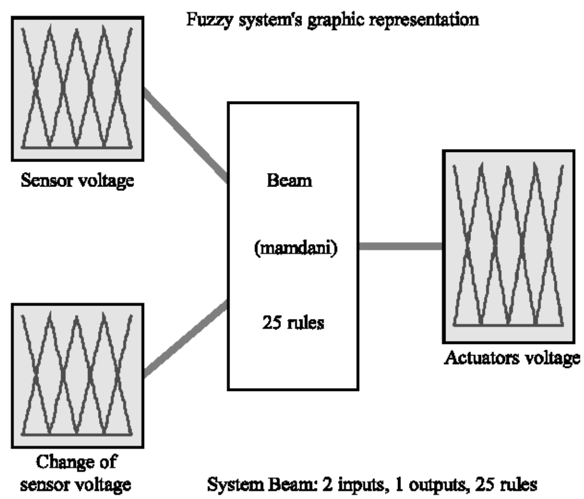

Fig. 3: Fuzzy controller's graphical representation

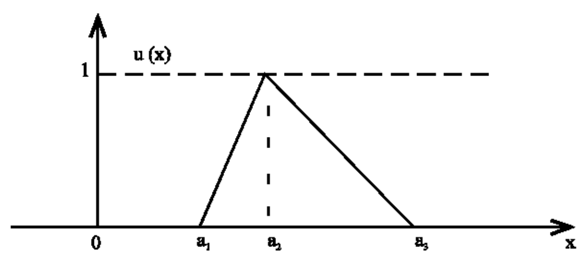

Fig. 4: Triangular fuzzy number

where:

e, Δe and u	=	Denote the system variables (error, error change and output voltage)
A_{1i}, B_{1i} and C_{1i}	=	The linguistic values of the fuzzy variables to express the universe of discourse of the fuzzy sets (Lie-Xin, 2000)

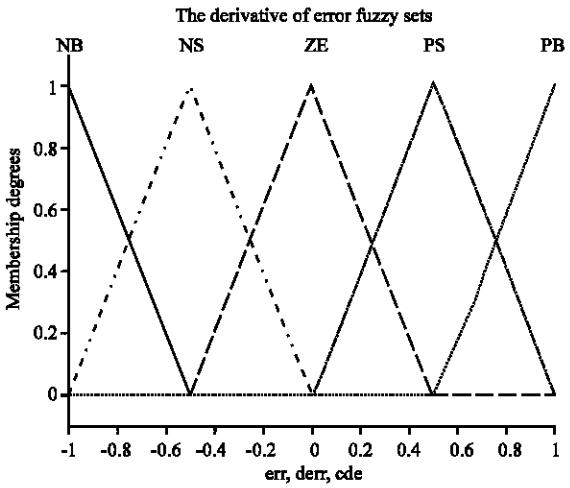

Fig. 5: Membership functions for input and output

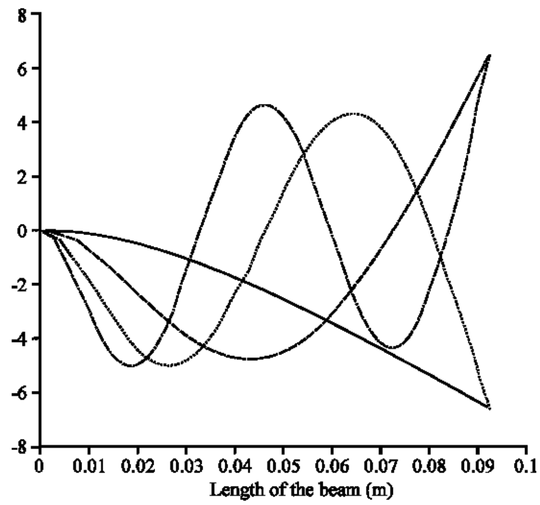

Fig. 6: The first four mode shapes of structure vibration

The proposed fuzzy inference method is Mamdani implication. The fuzzy sets must be defuzzified to obtain the appropriate control output for this control system. The MoM (Middle-of-Maxima) method was adopted to defuzzify the output variable.

SIMULATIONS

The numerical results have obtained for an aluminum beam with rectangular cross section that the common properties of the system parameters are:

Beam properties:

- Width = 25.4e-3 (m)
- Depth = 0.254e-3 (m)
- Length of the beam = 92.6e-3 (m)
- Poisson's Ration = 0.33
- Beam density = 4889 (kg/m^3)
- Modulus of Beam = 3.4×10^10 (N/m^2)

Piezoelectric properties:

- Thickness = 0.254e-3 (m)
- Width = 20.574e-3 (m)
- Dielectric Constant = 1800
- Dielectric Permittivity = K3×8.85e-12
- Piezoelectric Density = 4889 (kg/m^3)
- Modulus of PZT = 3.4×10^10 (N/m^2)
- Piezoelectric coefficient = -179e-12 (m/v)
- Piezoelectric Voltage Coefficient = -11.0e-3
- Poisson's Ration = 0.31

Figure 6 shows the first 4 mode shapes of structure vibration that has obtained from analyzing equations of motion.

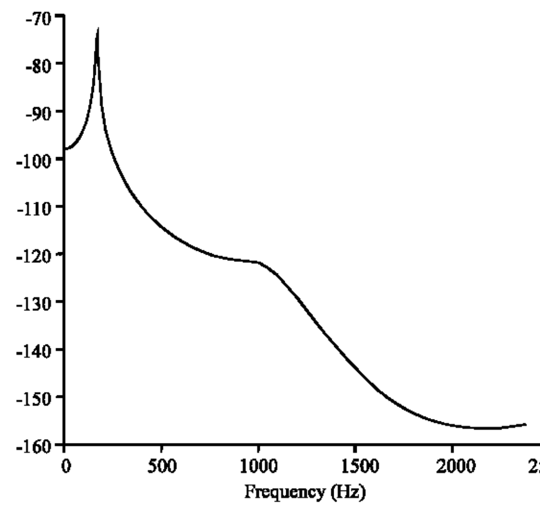

Fig. 7: Frequency response of the model

The beam was excited by a sinusoidal input and the frequency response of the model is shown in Fig. 7.

In order to examine, the efficiency and operation of the designed controller, the results have been presented in the form of a diagram with simulating in Matlab software (Mokhtari and Michel, 2006).

Figure 8 and 9 compare the system's response to the impulse excitation in 2 controlled and uncontrolled states, the Fig. 8 shows sensor voltage and the Fig. 9 shows tip displacement of the beam.

As we can see here the designed controller could have reduced the system's vibrations in an efficient amount so that the system can follow the input with closer approximation.

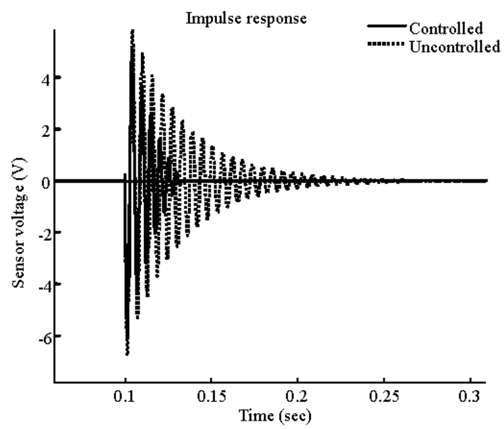

Fig. 8: Comparison of system's response to impulse excitation in controlled and uncontrolled states

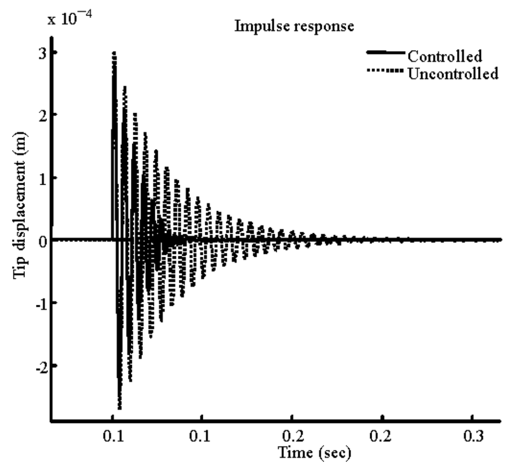

Fig. 9: Comparison of system's response to impulse excitation in controlled and uncontrolled states

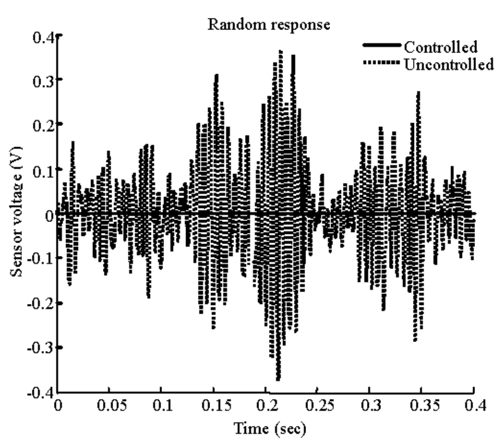

Fig. 10: The comparison of system's response to the random excitation in controlled and uncontrolled states

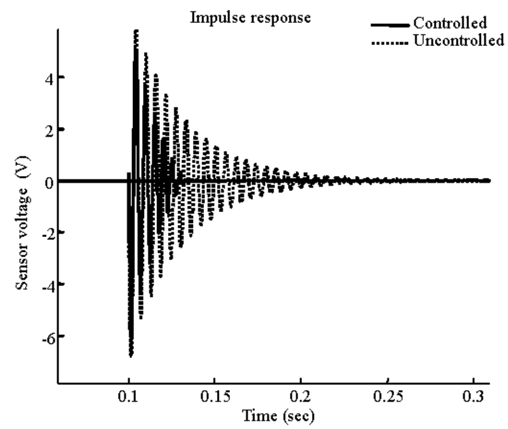

Fig. 11: The comparison of system's response to the white noise excitation in controlled and uncontrolled states

Figure 10 and 11 compare the system's response to random and white noise excitations in both controlled and uncontrolled states, which means the appropriate operation of designed controller.

CONCLUSION

This study has been proceeded to examine the active control of vibrations of a smart cantilever beam by piezoelectric and has presented a complete model from electromechanical properties of piezoelectric. Also by designing a logic fuzzy controller in addition to considerable reduction of computations capacity as we can see from obtained results, an appropriate operation has gained.

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