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A New Analytic Hierarchy Process in Multi-Attribute Group Decision Making

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Abstract: Analytic Hierarchy Process (AHP) is a utility theory based decision making technique, which researches on a premise that the decision making of complex problems can be handled by structuring them into simple and comprehensible hierarchical structure. However, AHP involves human subjective evaluation, which introduces vagueness that necessitates the use of decision making under uncertainty. The concept of Intuitionistic Fuzzy Sets (IFS) is the generalization of the concept of fuzzy set, which germane to uncertainty. The theory of IFS is well suited to dealing with vagueness. In this study, the concept of IFS is applied to AHP and to be called as IF-AHP as a method to handled vagueness in decision making. The aim of this study is to develop a new method for ranking multi-attribute group decision making problem using IFS and to quantify vagueness uncertainties in AHP using IFS for decision making problem. Several linear programming models are constructed to generate optimal weights for attributes. Feasibility and effectiveness of the proposed method are illustrated using a numerical evaluation.

Key words: Fuzzy set, intuitionistic fuzzy set, linear programming model, vagueness, decision making

INTRODUCTION

Multi-Attribute Group Decision Making (MAGDM) approach is often used to solve various decision making and selection problems. This approach often requires the decision makers to provide qualitative and quantitative assessments for determining the performance of each alternative with respect to each criterion and the relative importance of evaluation criteria with respect to the overall objective. TOPSIS, outranking and AHP are the three most frequently used MAGDM techniques.

In the past, numerous studies have used the classical MAGDM analysis method to deal with decision or selection problems. Hwang and Yoon (1981) proposed the TOPSIS method to determine a solution with the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. However, the TOPSIS method does not consider the relative importance of these distances. The outranking decision aid methods are used to compare all couples of actions. Instead of building complex utility function, they determine which actions lead to numerical results that show the concordance and/or the discordance between the actions that can be compared. There are many different fuzzy outranking approaches Roy (1977), Takeda (1982), Siskos *et al*. (1984), Brans *et al*. (1984) and Martel *et al*. (1986). The most well known outranking methods are ELECTRE, ORESTE and PROMETHEE.

In MAGDM, comparative judgements may be used to compare the performance of each evaluation criterion with relative measurement. To facilitate comparative judgements, a pairwise comparison process is commonly used. The concept of pairwise comparisons has been known since the research of (Thurstone, 1927) and has been popularly implemented in the Analytic Hierarchy Process (AHP) of Saaty (1980). AHP has been applied to a wide variety of practical decision problems (Vaidya and Kumar, 2006). Most recently Lazim *et al*. (2009) applied AHP in cancer risk perceptions. Olcer and Odabasi (2004) proposed a new fuzzy multi-attribute decision making method, which is suitable for multiple attribute group decision making problems in a fuzzy environment and this method can deal with the problems of ranking and selection. In their study, fuzzy AHP will be preferred since this method is the only one using a hierarchical structure among goal, attributes, sub attributes and alternatives. Usage of pairwise comparisons is another asset of this method that lets the generation of more precise information about the preferences of decision makers. By using pairwise comparisons, judges are not required to explicitly define a measurement scale for each attribute (Spires, 1991). The pairwise comparisons require qualitative assessment of human beings. Consequently, vagueness dominates the decision making process.

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The vagueness is best described by fuzzy set theory (Zadeh, 1965). Fuzzy based techniques are generalized form of an interval analysis. A fuzzy number described the relationship between an uncertain quality x and a membership function μ_x. In the classical set theory, x is either a member of set A or not, whereas in fuzzy set theory x can be a member of set A with a certain membership function $\mu_x \in [0, 1]$. The non membership is simply a complement of μ_x, which is $v_x = 1 - \mu_x$. A crisp degree of membership μ_x assigned to any given value of x over the universe of discourse may also be subjected to uncertainty. This refers to non specificity, which is associated with the membership μ_x of fuzzy sets. Zadeh (1965) extended the fuzzy set theory to incorporate non specificity through interval valued fuzzy sets, which captures non specificity by an interval $[\mu_L, \mu_U]$ where, μ_L and μ_U represent lower and upper bounds of membership function μ_x, respectively. Atanassov (1986, 1999) defined a non membership function v_x in addition to the membership function μ_x through an Intuitionistic Fuzzy Set (IFS), such that the non specificity is an interval $[\mu_x, 1 - v_x]$. Gau and Buehrer (1993) have explored similar concept, but called it as vague sets. However, Bustince and Burillo (1996) showed that the vague sets are essentially IFS. Further, Cornelis *et al.* (2003) have proved equivalence between intervals valued fuzzy set and IFS. Therefore, both vague sets and interval valued fuzzy sets can be handled using IFS formulation. There are some reported applications of IFS in MCDM, which is Liu and Wang (2007), Li *et al.* (2008), Atanassov *et al.* (2002) and Hong and Choi (2000). Recently, Silavi *et al.* (2006a, b) have demonstrated the possibility of extending AHP using IFS without consider the optimal weight. However, at the best of authors' knowledge, there have been no studies to materialise the extension of AHP in IFS. Thus, this study seeks to address this possibility by proposing a new decision making method. The aim of this study is to propose a model of AHP in IFS decision making environment. This new model is to be called as Intuitionistic Fuzzy Analytic Hierarchy Process (IF-AHP) and to be used throughout the text.

MATERIALS AND METHODS

Definition of Intuitionistic Fuzzy Set (IFS) from Liu and Wang (2007) is reproduced to make the study self-contained.

An Intuitionistic Fuzzy Set (IFS) A in U is an object having the following form:

$$A = \{(u, \mu_A(u), v_A(u)) \mid u \in U\} \quad (1)$$

where, the functions μ_A: U→[0, 1] and v_A: U→[0, 1] define the degree of membership and the degree of non-membership of the element u∈U in A, respectively and for every u∈U:

$$0 \le \mu_A(u) + v_A(u) \le 1 \quad (2)$$

Obviously, each ordinary fuzzy set may be written as:

$$\{(u, \mu_A(u), 1 - \mu_A(u)) \mid u \in U\} \quad (3)$$

For IFS, the degree of non determinacy π_x (or non specificity) of the element A in u∈U is defined as follows:

$$\pi_A(u) = 1 - \mu_A(u) - v_A(u) \quad (4)$$

Clearly in IFS, a degree of membership and a degree of non membership are independent and the sum of these grades is not >1.

There are many fuzzy AHP methods proposed by various authors. These methods are systematic approaches to the alternative selection and justification problem by using the concepts of fuzzy set theory and hierarchical structure analysis. Now, we combine intuitionistic fuzzy set with the Analytic Hierarchy Process (AHP) to propose the Intuitionistic Fuzzy Analytic Hierarchy Process (IF-AHP). Instead of using one-sided evaluation in AHP, the IF-AHP is using two-sided complement evaluation of IFS. The approach can be written in the following steps.

Step 1: Develop a hierarchical structure.

Assume that there exist an alternative set $A = \{A_1, A_2..., A_m\}$, which consist of m alternatives $A_i = \{i = 1, 2,..., m\}$ from which the most preferred alternatives is to be selected by a group of K decision makers $P_k(k = 1, 2,..., K)$. Denote the set of all criteria $C = \{C_1, C_2,..., C_m\}$. Assume that the decision maker $P_k(k = 1, 2,..., K)$ construct an IFS $X^k_{ij} = \{\langle A_i, \mu^k_{ij}, v^k_{ij}\rangle\}$ where, μ^k_{ij} and v^k_{ij} are the degree of membership (or satisfaction) and the degree of non-membership (or non-satisfaction) of the alternative $A_i \in A$ with respect to the criteria $C_j \in C$ given by P_k, respectively and $0 \le \mu^k_{ij} \le 1$, $0 \le v^k_{ij} \le 1$ and $0 \le \mu^k_{ij} + v^k_{ij} \le 1$. Thus, IF-AHP can be expressed concisely in the matrix as follows:

$$F^k = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} \langle \mu^k_{11}, v^k_{11} \rangle & \langle \mu^k_{12}, v^k_{12} \rangle & \cdots & \langle \mu^k_{1n}, v^k_{1n} \rangle \\ \langle \mu^k_{21}, v^k_{21} \rangle & \langle \mu^k_{22}, v^k_{22} \rangle & \cdots & \langle \mu^k_{2n}, v^k_{2n} \rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle \mu^k_{m1}, v^k_{m1} \rangle & \langle \mu^k_{m2}, v^k_{m2} \rangle & \cdots & \langle \mu^k_{mn}, v^k_{mn} \rangle \end{pmatrix} \end{array} \quad (5)$$

$$k = 1, 2, ..., K$$

The decision maker P_k can change their evaluations by adjusting the value of the intuitionistic index. The intuitionistic indices is $\pi^k_{ij} = 1- \mu^k_{ij} - v^k_{ij}$. So, in fact their evaluation lies in the closed interval $[\mu^{kl}_{ij}, \mu^{ku}_{ij}] = [\mu^k_{ij}, \mu^k_{ij} + \pi^k_{ij}]$ where, $\mu^{kl}_{ij} = \mu^k_{ij}$ and $\mu^{ku}_{ij} = \mu^k_{ij} + \pi^k_{ij} = 1 - v^k_{ij}$. Obviously, $0 \le \mu^{kl}_{ij} \le \mu^{ku}_{ij} \le 1$ for all $A_i \in A$ and $C_j \in C$. Thus, IF-AHP can be expressed concisely in the matrix as follows:

$$F^k = \begin{matrix} & A_1 & A_2 & \cdots & A_n \\ X_1 & \langle \mu^{kl}_{11}, \mu^{ku}_{11} \rangle & \langle \mu^{kl}_{12}, \mu^{ku}_{12} \rangle & \cdots & \langle \mu^{kl}_{1n}, \mu^{ku}_{1n} \rangle \\ X_2 & \langle \mu^{kl}_{21}, \mu^{ku}_{21} \rangle & \langle \mu^{kl}_{22}, \mu^{ku}_{22} \rangle & \cdots & \langle \mu^{kl}_{2n}, \mu^{ku}_{2n} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ X_m & \langle \mu^{kl}_{m1}, \mu^{ku}_{m1} \rangle & \langle \mu^{kl}_{m2}, \mu^{ku}_{m2} \rangle & \cdots & \langle \mu^{kl}_{mn}, \mu^{ku}_{mn} \rangle \end{matrix} \quad k = 1,2,...,K \tag{6}$$

Step 2: Estimate the degree of non-membership of the criteria $C_j \in C$.

Similarly, assume that the decision maker $P_k(k = 1, 2, ..., K)$ construct an IFS $W^k_j = \{\langle A_j, \rho^k_j, \tau^k_j \rangle\}$ where, ρ^k_j and τ^k_j are the degree of membership and the degree of non-membership of the criteria $C_j \in C$ and $0 \le \rho^k_j, \le 1$, $0 \le \tau^k_j \le 1$, and $0 \le \rho^k_j + \tau^k_j \le 1$. So, in fact the weights lies in the closed interval $[\omega^{kl}_j, \omega^{ku}_j] = [\rho^k_j, \rho^k_j + \eta^k_j]$ where, $\omega^{kl}_j = \rho^k_j$ and $\omega^{ku}_j = \rho^k_j + \eta^k_j = 1 - \tau^k_j$. Obviously, $0 \le \omega^{kl} \le \omega^{ku} \le 1$ for each criteria $C_j \in C$. Then, weight vector of all attributes can be concisely expressed in the following format:

$$\omega^k = ([\omega^{kl}_1, \omega^{ku}_1], [\omega^{kl}_2, \omega^{ku}_2], ..., [\omega^{kl}_n, \omega^{ku}_n]), \quad k = 1,2,...,K \tag{7}$$

Step 3: Estimate the degree of non-membership of the decision maker, P_k.

In a similar fashion, assume that α_k and β_k be the degree of importance (or membership) and the degree of non-importance (or non-membership) for the decision maker $P_k \in P(k = 1, 2,..., K)$, respectively where, $0 \le \alpha_k \le 1$, $0 \le \beta_k \le 1$ and $0 \le \alpha_k + \beta_k \le 1$. So, in fact the weight of P_k lies in the closed interval $[\omega^l_k, \omega^u_k] = [\alpha_k, \alpha_k + \gamma_k]$ where, $w^l_k = \alpha_k$ and $w^u_k = \alpha_k + \gamma_k = 1-\beta_k$. Obviously, $0 \le \omega^l_k \le \omega^u_k \le 1$ for each $P_k \in P$. Then, a weight vector of all decision makers can be concisely expressed in the following format:

$$W = ([\omega^l_1, \omega^u_1], [\omega^l_2, \omega^{ku}_2], ..., [\omega^l_K, \omega^u_K]) \tag{8}$$

Step 4: Find the optimal solution $\omega^k = (\omega^k_1, \omega^k_2, ..., \omega^k_m)^T$.

Find the total the degree of membership μ^{kl}_{ij} and non membership μ^{ku}_{ij} of the criteria $C_j \in C$.

Solving the linear programming:

$$\max\left\{ z = \frac{\sum_{j=1}^{n}\sum_{i=1}^{m}(\mu^u_{ij} - \mu^l_{ij})\omega_i}{n} \right\} \tag{9}$$
$$\begin{cases} \omega^l_i \le \omega_i \le \omega^u_i \quad (i = 1,2,...,m), \\ \sum_{i=1}^{m} \omega_i = 1. \end{cases}$$

Step 5: Find the relative closeness coefficient of alternative $A_i \in A$.

$$z^{kl}_j = \sum_{i=1}^{m} \mu^{kl}_{ij} \omega^k_i = \sum_{i=1}^{m} \mu_{ij} \omega^k_i \tag{10}$$

and

$$z^{ku}_j = \sum_{i=1}^{m} \mu^{ku}_{ij} \omega^k_i = 1 - \sum_{i=1}^{m} v_{ij} \omega^k_i \tag{11}$$

For each j = 1, 2,...,n.

Step 6: Find the relative closeness coefficient interval of alternative $A_i \in A$.

$$\xi_i = [\xi^l_i, \xi^u_i] = \sum_{k=1}^{K}([w^l_k, w^u_k][z^{kl}_i, z^{ku}_i]) \\ \left[\sum_{k=1}^{K} w^l_k z^{kl}_i, \sum_{k=1}^{K} w^u_k z^{ku}_i\right], \\ i = 1,2,...,m \tag{12}$$

where, $w_k = [w^i_k, w^u_k]$ (k = 1, 2,..., k) is the relative weight of the decision maker $P_k \in P$.

Step 7: Estimate the pairwise comparison of alternative $A_i \in A$.

$$p(X_s \succ X_t) = p(\xi_s \ge \xi_t) \\ = \max\left\{1 - \max\left\{\frac{\xi^u_t - \xi^l_s}{L(\xi_s) + L(\xi_t)}, 0\right\}, 0\right\} \tag{13}$$

Where:

$$\xi_s = [\xi^l_s, \xi^u_s], \ \xi_t = [\xi^l_t, \xi^u_t] \text{ and } L(\xi_t) = \xi^u_t - \xi^l_t$$

Thus, the pairwise comparison can be obtained in matrix as follows:

$$P = (p_{st})_{m\times m} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_3 \end{array}\begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{pmatrix} \end{array} \quad (14)$$

where, $P_{sl} = p(X_s \geq X_t)$ for alternatives A_s and A_t in A.

Step 8: Find an optimal degree of membership for alternative $A_i \in A (i = 1, 2, ..., m)$.

$$A_i = \frac{(\sum_{s=1}^{m} p_{is})^{1/m}}{\sum_{s}^{m}\sum_{t}^{m} p_{st}} \quad (15)$$

Step 9: Ranking the alternative $A_i \in A (i = 1, 2, ..., m)$. Obviously, $A_i \in [0, 1]$ for alternative $A_i \in A (i = 1, 2, ..., m)$ is generated according to the decreasing order of A_i.

RESULTS AND DISCUSSION

The example from Li *et al.* (2008) is used to illustrate the proposed method. Assume that there be a group consisting of three experts (or decision makers) $P_k (k = 1, 2, 3)$, who are invited to assess three command and control systems $A_i = (i = 1, 2, 3)$ on three readiness indexes such as information accuracy C_1, system availability C_2 as well as picture completeness C_3.

Denote $P = \{P_1, P_2, P_3\}$, $X = \{X_1, X_2, X_3\}$ and $A = \{A_1, A_2, A_3\}$ as expert opinions, alternatives and criteria. The decision to find the best alternative can be made through the following steps.

Step 1: Develop a hierarchical structure.

$$(\langle \mu_{ij}^1, \nu_{ij}^1 \rangle)_{3\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle 0.75, 0.10\rangle & \langle 0.81, 0.15\rangle & \langle 0.42, 0.48\rangle \\ \langle 0.62, 0.25\rangle & \langle 0.67, 0.21\rangle & \langle 0.76, 0.07\rangle \\ \langle 0.79, 0.21\rangle & \langle 0.45, 0.49\rangle & \langle 0.63, 0.31\rangle \end{pmatrix} \end{array}$$

$$(\langle \mu_{ij}^2, \nu_{ij}^2 \rangle)_{3\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle 0.71, 0.15\rangle & \langle 0.82, 0.11\rangle & \langle 0.31, 0.48\rangle \\ \langle 0.58, 0.35\rangle & \langle 0.58, 0.30\rangle & \langle 0.81, 0.15\rangle \\ \langle 0.84, 0.05\rangle & \langle 0.61, 0.30\rangle & \langle 0.65, 0.20\rangle \end{pmatrix} \end{array}$$

and

$$(\langle \mu_{ij}^3, \nu_{ij}^3 \rangle)_{3\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle 0.85, 0.10\rangle & \langle 0.75, 0.10\rangle & \langle 0.48, 0.32\rangle \\ \langle 0.75, 0.05\rangle & \langle 0.70, 0.15\rangle & \langle 0.65, 0.15\rangle \\ \langle 0.60, 0.30\rangle & \langle 0.56, 0.20\rangle & \langle 0.70, 0.16\rangle \end{pmatrix} \end{array}$$

respectively.

After adjusting the value of the intuitionistic index the matrix will be as follows:

$$(\langle \mu_{ij}^{1l}, \mu_{ij}^{1u} \rangle)_{3\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle 0.75, 0.90\rangle & \langle 0.81, 0.85\rangle & \langle 0.42, 0.52\rangle \\ \langle 0.62, 0.75\rangle & \langle 0.67, 0.79\rangle & \langle 0.76, 0.93\rangle \\ \langle 0.79, 0.79\rangle & \langle 0.45, 0.51\rangle & \langle 0.63, 0.69\rangle \end{pmatrix} \end{array}$$

$$(\langle \mu_{ij}^{2l}, \mu_{ij}^{2u} \rangle)_{3\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle 0.71, 0.85\rangle & \langle 0.82, 0.89\rangle & \langle 0.31, 0.52\rangle \\ \langle 0.58, 0.65\rangle & \langle 0.58, 0.70\rangle & \langle 0.81, 0.85\rangle \\ \langle 0.84, 0.95\rangle & \langle 0.61, 0.70\rangle & \langle 0.65, 0.80\rangle \end{pmatrix} \end{array}$$

$$(\langle \mu_{ij}^{3l}, \mu_{ij}^{3u} \rangle)_{3\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} \langle 0.85, 0.90\rangle & \langle 0.75, 0.90\rangle & \langle 0.48, 0.68\rangle \\ \langle 0.75, 0.95\rangle & \langle 0.70, 0.85\rangle & \langle 0.65, 0.85\rangle \\ \langle 0.60, 0.70\rangle & \langle 0.56, 0.80\rangle & \langle 0.70, 0.84\rangle \end{pmatrix} \end{array}$$

In a similar fashion, the degree ρ_j^k of membership and the degrees τ_j^k of non-membership for the three criteria given by each expert $P_k (k = 1, 2, 3)$ can be obtained and expressed in the matrix format as follows:

$$(\langle \rho_j^1, \tau_j^1 \rangle)_{1\times 3} = \begin{array}{ccc} C_1 & C_2 & C_3 \\ (\langle 0.35, 0.25\rangle, & \langle 0.25, 0.40\rangle, & \langle 0.30, 0.55\rangle), \end{array}$$

$$(\langle \rho_j^2, \tau_j^2 \rangle)_{1\times 3} = \begin{array}{ccc} C_1 & C_2 & C_3 \\ (\langle 0.25, 0.25\rangle, & \langle 0.30, 0.65\rangle, & \langle 0.35, 0.40\rangle) \end{array}$$

and

$$(\langle \rho_j^3, \tau_j^3 \rangle)_{1\times 3} = \begin{array}{ccc} C_1 & C_2 & C_3 \\ (\langle 0.31, 0.45\rangle, & \langle 0.22, 0.50\rangle, & \langle 0.28, 0.59\rangle) \end{array}$$

respectively.

Similarly, the degree α_k of membership and the degree β_k of non-membership for the three experts $P_k (k = 1, 2, 3)$ can be obtained and expressed in the matrix format as:

$$(\langle \alpha_k, \beta_k \rangle)_{1\times 3} = \begin{array}{ccc} P_1 & P_2 & P_3 \\ (\langle 0.25, 0.25\rangle, & \langle 0.35, 0.40\rangle, & \langle 0.30, 0.65\rangle) \end{array}$$

Step 2: Estimate the degree of non-membership of the criteria $C_j \in C$.

The weight vector of criteria given by the expert P_1, P_2, P_3 may be written in the interval format as follows:

$$(\langle \omega_j^{1l}, \omega_j^{1u} \rangle)_{1\times 3} = \begin{array}{ccc} C_1 & C_2 & C_3 \\ (\langle 0.35, 0.75\rangle, & \langle 0.25, 0.60\rangle, & \langle 0.30, 0.45\rangle) \end{array}$$

$$(\langle \omega_j^{2l}, \omega_j^{2u} \rangle)_{1\times 3} = \begin{matrix} C_1 & C_2 & C_3 \\ (\langle 0.25, 0.75 \rangle, & \langle 0.30, 0.35 \rangle, & \langle 0.35, 0.60 \rangle) \end{matrix}$$

$$(\langle \omega_j^{3l}, \omega_j^{3u} \rangle)_{1\times 3} = \begin{matrix} C_1 & C_2 & C_3 \\ (\langle 0.31, 0.55 \rangle, & \langle 0.22, 0.50 \rangle, & \langle 0.28, 0.41 \rangle) \end{matrix}$$

Step 3: Estimate the degree of non-membership of the decision maker, P_k.

Similarly, the weight vector of the experts P_1, P_2 and P_3 may be written in the interval format as follows:

$$(\langle \omega_k^{l}, \omega_k^{u} \rangle)_{1\times 3} = \begin{matrix} P_1 & P_2 & P_3 \\ (\langle 0.25, 0.75 \rangle, & \langle 0.35, 0.60 \rangle, & \langle 0.30, 0.35 \rangle) \end{matrix}$$

Step 4: Find the optimal solution of each decision maker $\omega^k = (\omega^k_1, \omega^k_2, ..., \omega^k_m)^T$.

For expert P_1.

The optimal solution for expert P_1 is $\omega^1 = (\omega^1_1, \omega^1_2, \omega^1_3)^T$ - $(0.35, 0.35, 0.30)^T$.

For expert P_2.

Thus, the optimal solution for expert P_2 is $\omega^2 = (\omega^2_1, \omega^2_2, \omega^2_3)^T = (0.35, 0.30, 0.35)^T$.

For expert P_3.

Thus, the optimal solution for expert P_3 is $\omega^3 = (\omega^3_1, \omega^3_2, \omega^3_3)^T = (0.31, 0.41, 0.28)^T$.

Step 5: Find the relative closeness coefficient of alternative $A_i \in A$

$$[z_1^{1l}, z_1^{1u}] = [0.7165, 0.8145],$$
$$[z_2^{1l}, z_2^{1u}] = [0.6530, 0.7270],$$
$$[z_3^{1l}, z_3^{1u}] = [0.6020, 0.7040]$$
$$[z_1^{2l}, z_1^{2u}] = [0.7165, 0.8250],$$
$$[z_2^{2l}, z_2^{2u}] = [0.6745, 0.7665],$$
$$[z_3^{2l}, z_3^{2u}] = [0.4705, 0.7170]$$
$$[z_1^{3l}, z_1^{3u}] = [0.7390, 0.8645],$$
$$[z_2^{3l}, z_2^{3u}] = [0.6763, 0.8515],$$
$$[z_3^{3l}, z_3^{3u}] = [0.6023, 0.7413]$$

Step 6: Find the relative closeness coefficient interval of alternative $A_i \in A$.

$$\xi_1 = \sum_{k=1}^{3}([w_k^l, w_k^u][c_1^{kl}, c_1^{ku}]) = [0.6448, 1.3974]$$

$$\xi_2 = \sum_{k=1}^{3}([w_k^l, w_k^u][c_2^{kl}, c_2^{ku}]) = [0.6022, 1.3021]$$

$$\xi_3 = \sum_{k=1}^{3}([w_k^l, w_k^u][c_3^{kl}, c_3^{ku}]) = [0.5375, 1.2188]$$

Step 7: Estimate the likelihoods of pairwise comparison of alternative $A_i \in A$.

$$p(\xi_1 \geq \xi_1) = 0.5,$$
$$p(\xi_1 \geq \xi_2) = 0.5475,$$
$$p(\xi_1 \geq \xi_3) = 0.5997$$
$$p(\xi_2 \geq \xi_1) = 0.4525,$$
$$p(\xi_2 \geq \xi_2) = 0.5,$$
$$p(\xi_2 \geq \xi_3) = 0.5536$$
$$p(\xi_3 \geq \xi_1) = 0.4003,$$
$$p(\xi_3 \geq \xi_2) = 0.4464,$$
$$p(\xi_3 \geq \xi_3) = 0.5$$

Thus, the matrix format as follows:

$$P = (p_{st})_{3\times 3} = \begin{matrix} & X_1 & X_2 & X_3 \\ X_1 & 0.5 & 0.5475 & 0.5997 \\ X_2 & 0.4525 & 0.5 & 0.5536 \\ X_3 & 0.4003 & 0.4464 & 0.5 \end{matrix}$$

Step 8: Find an optimal degree of membership for alternative $A_i \in A (i = 1, 2, ..., m)$

$$A_1 = 0.3663, A_2 = 0.3347, A_3 = 0.3000$$

Step 9: Ranking the alternative $A_i \in A(i = 1, 2, ..., m)$. Then, the best alternative is A_1 and the ranking order of the three alternatives is given by $A_1 \succ A_2 \succ A_3$.

CONCLUSION

AHP is inherently a subjective process, which involves uncertainties in the evaluation and affects the process in decision making. Meanwhile, the notion IFS can handle vagueness type of uncertainties. This study has proposed a new decision making method in MAGDM by considering the uniqueness of pairwise comparison in AHP and the complementary of memberships in IFS. The concept of IFS in AHP has been introduced through pairwise comparisons. A ranking order has been obtained via IF-AHP evaluation process by using positive and negative components of IFS in the concept of AHP. The numerical evaluation validated the decision process using IF-AHP method. The application of IF-AHP certainly can help a decision maker to make more realistic and informed decisions based on available information.

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