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Flow of Blood in a Porous Medium and its Effect on Heat Transfer Rate

[1]Bimal Kumar Mishra, [2]Priyabrata Pradhan and [3]T.C. Panda
[1]Department of Applied Mathematics, Birla Institute of Technology, Mesra, 835215 Ranchi, India
[2]Department of Mathematics, Manitara Science College, Ganjam, 761118 Orissa, India
[3]Department of Mathematics, Orissa Engineering College, 752050 Bhubaneswar, India

Abstract: Large number of applications is associated with heat transfer in muscle and skin tissues and thermal therapy applications. Blood flow plays an important role in determining the effectiveness of thermal therapy. In general, the effect of blood flow on temperature distribution in tissue is considered as cooling due to both thermally significant large vessels and smaller microvasculature during thermal treatments. Transportation of particles (tissue cells) and interconnected voids that contain either arterial or venous blood through porous media has significant applications of biomedical systems such as biological tissues which include flow, heat and mass transfer through porous media. By applying the porous medium model to describe the heat transfer and effect of shear stress in living tissue, an analytical solution is obtained. It is observed that as the porous parameter is increased, the wall shear stress and heat transfer rate also increases. It has been also investigated that as the heat transfer rate is increased, the wall shear stress also increases.

Key words: Heat Transfer, porous medium, blood flow, wall shear stress, Darcy model, India

INTRODUCTION

A porous medium basically consists of a bed of many relatively closely packed particles or some other form of solid matrix which remains at rest and through which a fluid flows. If the fluid fills all the gaps between the particles, the porous medium is said to be saturated with the fluid that is, with saturated porous medium it is not possible to add more fluid to the porous medium without changing the conditions at which the fluid exists. Porous media can be characterized by their specific surface (s) and porosity (ε), respectively defined as:

$$S = \frac{\text{Total interface area}}{\text{Total volume}}$$

$$\varepsilon = \frac{\text{Void volume}}{\text{Total volume}}$$

Some porous media such as biological tissues, are deformable under mechanical loads. Biological tissues are heterogeneous, in which the porosity is equal to the void fraction of local interstitial fluid. There are three compartment in biological tissues; blood and lymph vessels, cells and interstitium. The interstitial space can be further divided into the extra cellular matrix and the interstitial fluid. The extra cellular region (cells and interstitium) can be considered a porous medium, with pores saturated with interstitial fluid. Tissues can be treated as a porous medium as it is composed of dispersed cells separated by connective voids which allow for flow of nutrients, minerals, etc. to reach all cells within the tissue (Fig. 1). Different examples of pore structure in tissues is depicted in Fig. 2. It is well known from flow studies in porous media, where the pore network shows strong similarities with the capillary network that spatial correlation plays an important role for the macroscopic flow behavior. Therefore, in analogy with flow in porous media, the blood flow in the thin vessels is described by macroscopic equations, i.e. by Darcy's law and mass

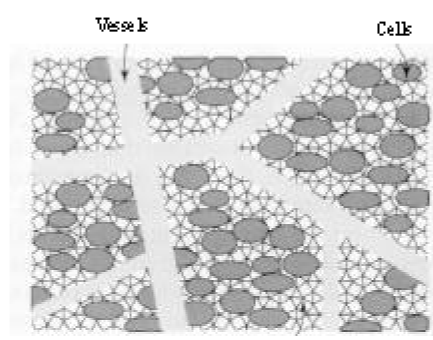

Fig. 1: A schematic of biological tissues interstitial space (Truskey *et al*., 2004)

Corresponding Author: Bimal Kumar Mishra, Department of Applied Mathematics, Birla Institute of Technology, Mesra, 835215 Ranchi, India

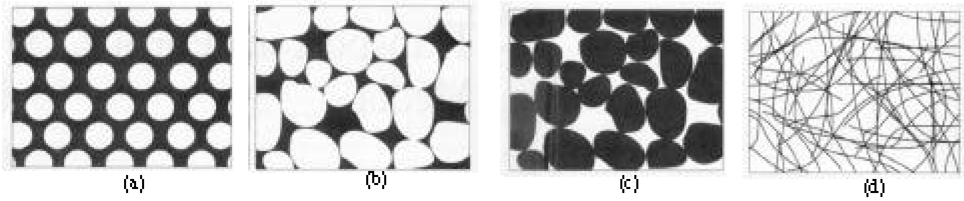

Fig. 2: Examples of pore structure (a) a regular array of cylindrical pores, b) foam structure of pores (c) a granular structure of pores (d) a fiber matrix (Truskey *et al.*, 2004)

conservation (Truskey *et al*., 2004). The real application of the porous media models and Bio-heat transfer in human tissues is relatively recent. Bio-heat is usually referred to heat transfer in the human body. The development of transport models in porous media have a bearing in the progress of several applications such as transport of macromolecules in aortic media, blood flow through contracting muscles, interstitial fluid flow in axisymmetric soft connective tissue, heat transfer in muscle and skin tissues, thermal therapy applications and others. Biological tissues contain dispersed cells separated by voids. Blood enter these tissues through vessels referred to as arteries and perfuse to the tissue cells via blood capillaries as shown in Fig. 1.

Returned blood from the capillaries is accumulated in veins where the blood is pumped back to the heart. Energy transport in tissues is due to thermal conduction, blood perfusion and heat generation (e.g. metabolic heat generation).

MATERIALS AND METHODS

Darcy flow model: The interaction between solid and liquid phases in porous media was first quantified by Darcy (1856). When the fluid flows through a porous media, the solid particles exert a force on the fluid equal and opposite to the drag force on the solid particles. This force must be balanced by the pressure gradient in the flow i.e., for flow through a control volume for any chosen direction: Difference between rate fluid momentum leaves and the rate fluid momentum enters control volume = Net viscous force on surface of control volume + Net pressure force on control volume-Drag force on particles in control volume + Net buoyancy force.

In the Darcy model of flow through a porous media, it is assumed that the flow velocities are low and that momentum changes and viscous forces into the fluid are consequently negligible compared to the drag force on the particles. In such flows, the drag force on a body is proportional to the velocity over the body and to the viscosity of the fluid.

$$u = -\frac{KG}{\eta}$$

where:

u = Velocity of blood
G = Pressure gradient
η = Viscosity of blood
K = Porous parameter

This derivation reveals that Darcy's law neglects the friction within the fluid and exchange of momentum between the fluid and solid phases. Therefore, Darcy's law has been widely used in the analysis of interstitial fluid flow. An important domain that deals with the application of the Darcy model to flow through tissues is the blood flow in tumors (abnormal mass of tissue that results from excessive cell division that is uncontrolled and progressive). It was found that models for convective transport through porous media are widely applicable in the simulation of blood flow of tumors and muscles and in modeling blood flow when fatty plaques of cholesterol and artery clogging clots are formed in the lumen (Khaled and Vafai, 2003).

Typical applications of Bio-heat transfer include human thermoregulation, cryopreservation of living cells and thermal burn injury. Thermal therapy utilizes the cytotoxic effect of high temperature to destroy pathological tissues. Therefore, it is important to understand the transient temperature behavior in heated tissues during the heating process. The real application of the porous media models and Bio-heat transfer in human tissues is relatively recent. Xuan and Roetzel (1997, 1998) used the transport through porous media concepts to model the tissue blood system composed mainly of solid particles (tissue cells) and interconnected voids that contain either arterial or venous blood.

The energy transport in a biological system is usually expressed by the Bio-heat equation. The Bio-heat equation developed by Pennes (1948) is one of the earliest models for energy transport in tissues. Pennes assumed that the arterial blood temperature TB is uniform throughout the tissue (Fig. 1), while he considered the vein temperature to be equal to the tissue temperature which is denoted by T at the same point. The equation that Pennes utilized is summarized as follows in its simplest form:

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + C_{pb} W_b (T_b - T) + q_m$$

where:

- x = Space coordinate
- ρ = Tissue density
- C_p = Tissue specific heat
- C_{pb} = Blood specific heat
- W_b = Blood volumetric perfusion rate
- k = Tissue thermal conductivity
- q_m = Heat generation within the tissue
- T_b = Arterial blood temperature
- T = Tissue temperature, respectively

Our body is made up of cells and organs that are very sensitive to changes in temperature. While the exterior surface of our skin can adapt and tolerate rather large changes in skin temperature, our internal organs cannot. That is why, our body has a very sophisticated heat regulating system that allows our internal body temperature to stay at a constant 98.6°F. Once our internal temperature begins to deviate from 98.6°F, the body reacts to counter the heat gain or loss.

Effect of heat stress: Heat stress happens as a result of an increase in internal body temperature. As the internal temperature goes up, our body responds by increasing the circulation of blood flow to the surface of the skin. This blood flow increase is preceded by an increase in heart rate and an increase in the size of blood vessels. All of this combined, puts a strain on the heart and circulatory system.

When more blood is pumped close to the skin for cooling, less blood goes to the brain. Bending, squatting or standing up suddenly can result in dizziness or a momentary blackout, which could cause secondary injuries or accidents at a job site. If the temperature of the air and surrounding objects in the research area rises above body temperatures, then conduction, convection and radiation cause the body to gain heat instead of losing it.

The evaporation of sweat becomes the body's most important and sometimes only cooling method. But sweating can also make things worse by causing the body to lose fluids and minerals. Most people will lose about a quart of sweat an hour while working in extreme heat. This puts even more strain on the circulatory system since, it actually lowers the amount of blood in your body. And just because you're sweating, you may not be getting rid of heat, since sweat must evaporate from your skin to cool your body. Normally, the faster the air moves over the body, the more sweat evaporates. But if the air is too full of water vapor (humidity) to absorb anymore, you can work in front of a fan and still not lose heat. As a result of your natural cooling defense systems being ineffective, your internal temperature continues to rise to a dangerous level resulting in heat stroke (lack of adequate blood supply to the brain), permanent damage to the central nervous system or death.

When Surface blood vessels that enlarge to cool the blood, collapse form loss of body fluids and minerals then heat exhaustion happens in the body. Heat transfer in human tissues involves complicated processes such as heat conduction in tissues, heat transfer due to perfusion of the arterial venous blood through the pores of the tissue (blood convection), metabolic heat generation and external interactions such as electromagnetic radiation emitted from cellphones. Heat transfer plays an important role in living systems as it affects the temperature and its spatial distribution in tissues.

The primary role of temperature is the regulation of a plethora of rate processes that govern all aspects of the life process. These thermally driven rate processes define the differences between sickness and health injury and successful therapy, comfort and pain and accurate and limited physiological diagnosis. In recent years, the flow of fluids through porous media has become an important topic. The study of flow of an electrically conducting fluid has many applications in engineering problems such as Magneto Hydrodynamics (MHD), plasma studies and the boundary layer control in the field of aerodynamics (Kim, 2000). In the past few years, several simple flow problems associated with classical hydrodynamics have received new attention within the more general context of Magneto Hydrodynamics (MHD).

The study of the motion of Newtonian fluids in the presence of a magnetic field has applications in many areas including the handling of biological fluids, plasma and blood (Makinde 2001, 2003; Ramachandra Rao and Deshikachar, 1986). Raptis *et al.* (1982) have analyzed hydromagnetic free convection flow through a porous medium between two parallel plates. Aldoss *et al.* (1995) have studied mixed convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. We propose a mathematical model of blood flow in porous medium and try to find an analytical solution for the effect of porous parameter on shear stress and heat transfer rate.

RESULTS AND DISCUSSION

Mathematical formulation: We assume the flow of blood in the porous medium to be Newtonian and its viscosity and density to be constant. The flow of blood in an artery

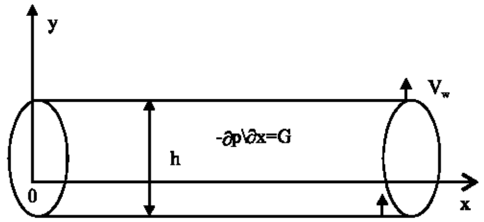

Fig. 3: Geometry of the problem

with saturated porous medium is depicted in Fig. 3. We further assume the pressure gradient in x-direction to be constant. The equation of governing the motion is given as Spurk (1997):

$$\upsilon\frac{d^2u}{dy^2} - V_w\frac{du}{dy} = -\frac{G}{\rho} \quad (1)$$

where,

$$u = 0;\ y = 0;\ u = 0;\ y = h \quad (2)$$

Method of solution: In order to solve the Eq. 1, we consider the homogeneous and particular solution. Solution of the homogenous part of Eq. 1 is given as:

$$\upsilon\frac{d^2u}{dy^2} - V_w\frac{du}{dy} = 0$$

Let,

$$u(y) = Ce^{\lambda y}$$

We obtain the characteristic polynomial, where λ and C are constant $\upsilon\lambda_2 - V_w\lambda = 0$, with the roots, $\lambda_1 = 0$, $\lambda_2 = V_w/\upsilon$. So,

$$u_h = C_1 + C_2 e^{\frac{V_w y}{\upsilon}} \quad (3)$$

Particular solution of given equation is given as: $u_p = C_3y + C_4$ is inserted in the given equation and the constants are found as:

$$C_3 = \frac{G}{\rho V_w} \text{and} C_4 = 0 \quad (4)$$

where C_1, C_2, C_3 and C_4 are constants. So general solution of Eq. 1 is given by:

$$u(y) = u_p + u_h = C_1 + C_2 e^{\frac{V_w y}{\upsilon}} + \frac{G}{\rho V_w} y$$

Now using the boundary Eq. 2, we get

$$-C_1 = C_2 e^{V_w y} = \frac{Gh}{\rho V_w}\left(\frac{1}{1 - e^{\frac{Vwh}{\upsilon}}}\right)$$

So, the velocity of blood is given by:

$$u(y) = \frac{Gh}{\rho V_w}\left(\frac{y}{h} - \frac{1 - e^{\frac{V_w y}{\upsilon}}}{1 - e^{\frac{V_w h}{\upsilon}}}\right) \quad (5)$$

Now applying the lim $V_w \to 0$, the expression of velocity is thus given by:

$$u(y) = \frac{Gh^2}{2\eta}\left(\frac{y}{h} - \frac{y^2}{h^2}\right) \quad (6)$$

The momentum balance equation for the flow through porous medium is:

$$u = -\frac{KG}{\eta} \quad (7)$$

For a Newtonian fluid, the shear stress is expressed as:

$$\tau = -\eta e \quad (8)$$

From Eq. 7 and 8, we get:

$$\tau = \frac{KG}{u}\frac{du}{dy}$$

Now, putting the value of u and du/dy from Eq. 6, we get:

$$\tau = \frac{KG}{y}\left(1 - \frac{y}{h} - \frac{2y^2}{h^2}\right) \quad (9)$$

So the expression of shear stress for the artery is given as:

$$\tau = \frac{KG}{y}\int\left(1 - \frac{y}{2r} - \frac{2y^2}{4r^2}\right)dr$$

$$\tau = \frac{KG}{y}\left(y - \frac{y}{2}\ln r + \frac{y^2}{2r}\right) \quad (10)$$

The rate of heat transfer across the artery's wall is given as:

$$Nu = -\frac{\partial\theta}{\partial y} \quad (11)$$

where:

$$\theta = \frac{T - T_0}{T_w - T_0}, Gr = \frac{g\beta(T - T_0)\alpha^2}{\upsilon u}, Da = \frac{K}{\alpha^2} \quad (12)$$

From Eq. 6, 11 and 12, we get the expression for the heat transfer rate as:

$$Nu = \frac{2g\beta K(T - T_0)\rho}{GrDaGhy^2} \quad (13)$$

From Eq. 9 and 13, we find the relation between heat transfer rate and shear stress given as:

$$Nu = \frac{2g\beta K(T - T_0)\tau\rho}{GrDaG^2hy}\left(1 + \frac{y}{h} + \frac{2y^2}{h^2}\right) \quad (14)$$

In order to get a physiological insight into the effect of porous parameter on the wall shear stress and heat transfer rate the following values are taken:

Gr (Grashoff number)	=	1
ρ (Density of blood)	=	1.056 g cm^{-3}
η (Viscosity of blood)	=	0.04 dyne cm^{-2}
h (Diamter of artery)	=	3 cm

Porous media are usually characterized on the macroscopic level by the introduction of macro parameters like the porosity and the permeability. So we aim to find the relation between microscopic pore structures and the macroscopic pore parameters (porosity, viscosity and shear stress). This study investigates the effect of porous parameter on shear stress and heat transfer rate of the blood flow in an artery filled with porous medium. From the study, we observe that as we increase porous parameter the wall shear stress and heat transfer rate also increases (Fig. 4a, b). Also as we increase the wall shear stress heat transfer rate also increases (Fig. 4c).

Nomenclature:

u	=	Velocity of blood
G	=	Pressure gradient
V_W	=	Normal velocity component at the wall of artery
h	=	Diameter of the artery
r	=	Radius of the artery
Nu	=	Heat transfer rate
T	=	Fluid temperature
g	=	Gravitational force
T0	=	Fluid temperature at the inner wall
TW	=	Fluid temperature at the upper wall
Gr	=	Grashoff number
Da	=	Darcy number
e	=	(= -du dy^{-1}) Strain rate
υ	=	Kinematics viscosity of blood
η	=	Viscosity of blood
ρ	=	Density of blood
τ	=	Shear stress

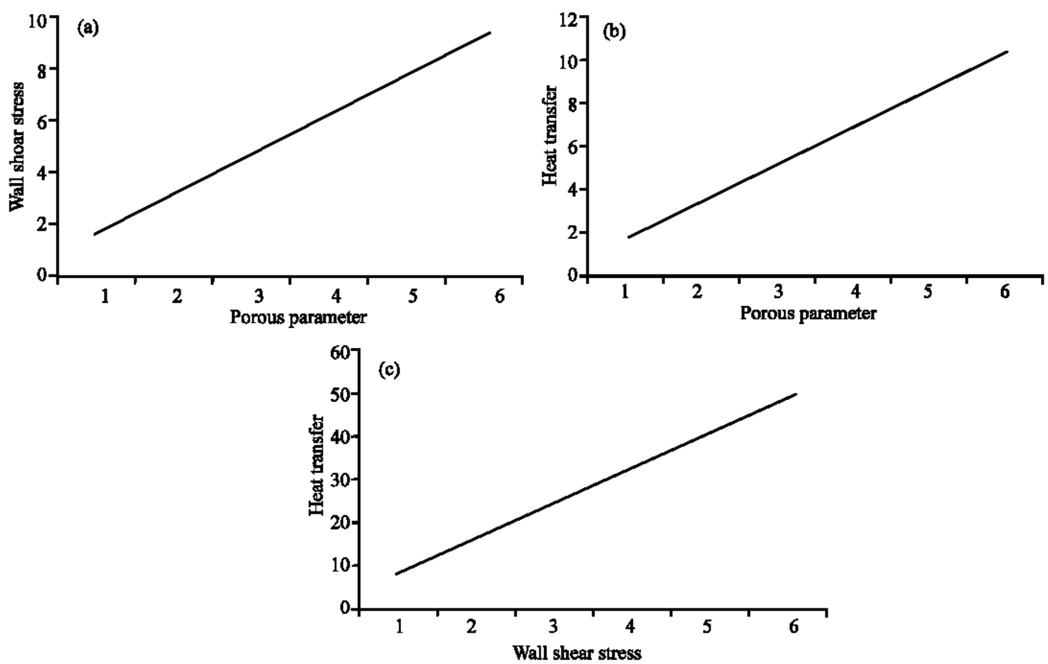

Fig. 4: Variation of porous parameters, a) wall shear stress against porous parameters, b) heat transfer against porous parameters, c) heat transfer rate against wall shear stress

β = Coefficient of volume expansion due to temperature

θ = Non dimensional temperature

CONCLUSION

In this study, the results can be used in wide range of application such as thermal simulations within the brain, hyperthemic sessions, heat transfer in muscle and skin tissues and thermal therapy applications.

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