

## Theoretical Determination of a Thermoelectric Module and Heat Sinks Sizing

A.A. Adeyanju and W. Compton  
Department of Mechanical and Manufacturing Engineering,  
University of West Indies, St. Augustine, Trinidad

**Abstract:** The study shows the theoretical determination of a thermoelectric module and heat sinks sizing and it was discovered that for a cooling time of 2 min and a beverage size of 474 mL (16 oz), 6 TEC modules and hence 12 heat sinks (one for each side of the TEC module) were needed. However, by increasing cooling time to 4 min and decreasing beverage size to 325 mL, 4 TEC modules and hence 8 heat sinks could be used.

**Key words:** Thermoelectric, module, heat sinks, sizing, beverage, Trinidad

### INTRODUCTION

Thermoelectric modules are solid state heat pumps that utilize the Peltier effect. During operation, DC current flows through the thermoelectric module causing heat to be transferred from one side of the thermoelectric device to other creating a cold and hot side. A single-stage thermoelectric module can achieve a temperature difference up to 70°C or can transfer heat at an utmost rate of 125 W under the extreme conditions. A thermoelectric cooling system incorporates a power source to provide a direct current through the electrical circuit. The basic arrangement of a thermoelectric module in a cooling system is shown in Fig. 1. Thermoelectric technology has existed for about 40 years and thermoelectric systems are

employed as cooling devices in many applications including military, aerospace, industrial and commercial. The main drawback of this technology, however is low Coefficient of Performance (COP), particularly in larger capacity applications. The interaction between thermal and electric phenomena (Seebeck effect (1821), Peltier effect (1834), Joule effect (1841) and Thomson effect (1857)) was known since the 19th century (Rowe and Bhandari, 1983). In 1885, the English Physicist J.W Rayleigh outlined the possibility of using thermoelectric devices as electricity generators but his development was totally stopped because of the low efficiency achieved. However, the major advance was made in the 1950's with the introduction of semiconductors as thermoelectric materials. It was observed that they had a high Seebeck

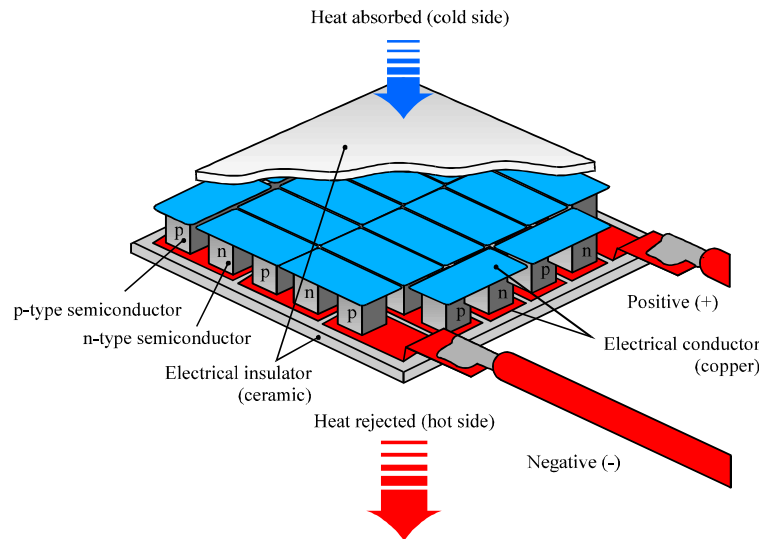


Fig. 1: Conventional arrangement for thermoelectric cooler

**Corresponding Author:** A.A. Adeyanju, Department of Mechanical and Manufacturing Engineering,  
University of West Indies, St. Augustine, Trinidad

coefficient, good electrical conductivity and low thermal conductivity. In those moments thermoelectric refrigeration began to look more promising and Peltier devices were developed for refrigeration applications mostly for the military field. Work on semiconductor thermocouples also led to the construction of thermoelectric generators with a high enough efficiency for special applications. There was little improvement in thermoelectric materials from the time of the introduction of semiconductor thermo-elements until the end of the 20th century. However, in recent years, several new ideas for the improvement of materials have been put forward and significant advances are being made (Goldsmid, 2009).

Now-a-days, in the civil market, thermoelectric refrigeration has a place in medical applications and scientific mechanisms and devices where accurate temperature control is needed. Nevertheless, there are other applications with great potential in which companies are starting to show interest e.g., dehumidifiers (Vian *et al.*, 2002), domestic and automobile air conditioning systems, portable iceboxes, domestic refrigerators, devices to transport perishable products and computer processor coolers etc.

For these applications, thermoelectric refrigeration competes with conventional refrigeration systems like vapor compression refrigeration. For a typical conventional refrigeration system, a temperature difference between the ambient and the cabinet of about 25-30 K at  $T_h = 300$  K is usually required to achieve satisfactory cooling performance. This indicates that the maximum COP of a thermoelectric refrigerator comprised of a commercially available module is around 0.9-1.2.

However, the practical COP of a thermoelectric refrigerator is much lower than this because the temperature difference between the hot and cold side of the thermoelectric module is larger than the temperature difference between the ambient and the cabinet. In other words, the hot side temperature is higher than the ambient and the cold side temperature is lower than the cabinet temperature. For a practical thermoelectric cooling system, the hot side heat exchanger rejects the heat produced on the hot side of the thermoelectric module to the ambient and so reduces the hot side temperature. The cold side heat exchanger removes the heat from the cold region to the cold side of thermoelectric module and so increases the temperature of the cold side. Because the thermoelectric module is very high heat intensity equipment, the high efficiency thermoelectric heat exchangers is necessary.

In principle, heat exchangers in thermoelectric cooling systems should be designed to minimize their thermal resistance under restrictions such as the size of the

system and heat transfer method and system design method because as the thermal resistance of the heat exchangers increases, the efficiency of thermoelectric cooling systems decreases. Typical heat exchanger designs include natural convection and forced convection heat exchangers for heat rejection to air and forced convection heat exchangers for heat rejection to water flow. Of these common types of heat exchanger, the liquid cooled system is the most efficient. The typical heat exchanger thermal resistance for a 45 mm<sup>2</sup> thermoelectric module is:

**Natural convection:** About 0.853-13.075 m<sup>2</sup> KkW<sup>-1</sup> depending on the fin density and the ratio of the heat exchanger base plate area to the thermoelectric module area. Higher ratios of the heat exchanger base plate area to the thermoelectric module area result in a lower thermal resistance.

**Forced air convection:** About 0.531-5.759 m<sup>2</sup> KkW<sup>-1</sup> depending on the air flow rate. Larger air flow rates result in a lower thermal resistance.

**Water-cooled exchanger:** About 0.348-0.737 m<sup>2</sup> KkW<sup>-1</sup> depending on the water flow rate. Larger water flow rates result in a lower thermal resistance. A ducted, forced-air, convection system has a higher performance than an un-ducted system.

Thermoelectric data shows the typical allowances of temperature difference between the hot side and ambient with respect to the heat exchange mode. That is:

- Natural convection 20-40°C
- Forced air convection: 10-15°C
- Liquid exchangers: 2-5°C above liquid temperature

Since the heat flux densities on the cold side of the system are considerably lower than those on the hot side, an allowance of about 50% on the afore-mentioned hot side data can be used. Various types of heat pipe may be used to conduct heat from the small ceramic area to the convection surface which is an alternate to the metal heat spreader plate. Use of a heat pipe will not be of benefit for natural convection because the dominant thermal resistance in this case is the convection resistance (Webb, 1998).

Water-cooled forced convection heat exchangers have excellent performance. The main drawback of a water-cooled heat exchanger is that it needs a convenient source of cooling water. Without a source of cooling water, a forced convection water heat exchanger would require a pump and radiator and associated fittings and

tubing. The added resistance of the radiator would increase the overall resistance. Air-cooled systems are therefore often more desirable. Many heat exchange systems based on the afore-mentioned forced air convection exchangers and the use of heat pipes have been reported.

Sofrata (1996) reported that using a double fan in an appropriate position could significantly increase the efficiency of the forced air exchanger compared to using the single fan in a refrigerator.

A long chimney for a natural-convection heat exchanger may also improve the performance of the refrigerator without the need to use fans that of course, require the electrical power input. A novel, air-cooled thermosyphon reboiler-condenser system has been reported (Webb, 1998) and has been used as a heat exchanger of a thermoelectric refrigerator (Gilley and Webb, 1999).

This system is capable of providing very low heat sink resistance values with air cooling and a thermal resistance as low as  $0.0194\text{-}0.0505\text{ KW}^{-1}$  was obtained for cooling a  $45\text{ mm}^2$  module. The system promises significantly higher COP for thermoelectric coolers than is possible using existing heat exchange technology.

Riffat *et al.* (2001) have reported a thermoelectric refrigeration system which employs a Phase Change Material (PCM) as a cold side heat exchanger for cooling storage and improvement of the COP. The refrigeration system was first fabricated and tested using a conventional heat sink system (bonded fin heat sink system) at the cold heat sink.

In order to improve the performance and storage capability, the system was reconstructed and tested using an encapsulated Phase Change Material (PCM) as a cold sink.

Both configurations used heat pipe embedded fins as the heat sink on the hot side. Results of tests on the latter system showed an increased performance. This was because the PCM had a large storage capacity allowing most of the cooling energy to be absorbed by the PCM and therefore the cold side temperature fell more slowly than when the PCM was not used. During the phase change process, the temperature of the refrigeration system was almost constant until the phase change process was complete.

This helped to keep the temperature difference across the thermoelectric module to a minimum, thus improving its performance. In general, thermoelectric modules are very high heat intensity equipment which need high efficiency heat exchangers to lower the hot side temperature and increase the cold side temperature in order to improve the COP.

Use of a greater number of modules would also improve the COP of the system. Use of more modules would reduce the heat load on each module and so lower the heat flux densities of both the hot and cold side of each module.

## MATERIALS AND METHODS

### Thermoelectric modules parameters selection:

- Desired temperature of cold compartment/beverage =  $6^\circ\text{C}/279.15\text{ K}$
- Ambient temperature ( $T_{\text{amb}}$ ) =  $31^\circ\text{C}/304.15\text{ K}$
- Required time for desired cooling from  $T_{\text{amb}}$  to  $4^\circ\text{C}$  =  $120\text{ sec}$
- Volume of bevarage to be cooled =  $16\text{ oz} = 474\text{ cc}$
- Mass of bevarage to be cooled =  $474 \times 0.001\text{ kg} = 0.5\text{ kg}$
- NB: Properties of water is used in place of bevarage for calculation

Using Newton's law of cooling:

$$Q = \dot{m} C_p \Delta T$$

$$C_{p(\text{water})} = 4186\text{ J/kg.K}$$

$$Q_{\text{beverage}} = \frac{0.5}{120} \times 4186 \times (304.15 - 279.15) = 436.04\text{ W}$$

Where:

$Q_{\text{beverage}}$  = Heat load supplied by bevarage

Accounting for heat transfer through panels of cold compartment:

- Top and bottom panel dimensions =  $0.32 \times 0.155\text{ m}$
- Vertical side panel dimension =  $0.44 \times 0.155\text{ m}$
- Front and back panel dimensions =  $0.32 \times 0.44\text{ m}$

Thermal conductivity (K) of materials:

$$K_{\text{Al},6061} = 167\text{ W/m.K}$$

$$K_{\text{Styrofoam}} = 0.033\text{ W/m.K}$$

To find hot-side temperature of TEC modules,  $T_{\text{hot}}$ : keeping heat sink at  $15^\circ\text{C}$  above ambient temperature:

$$T_{\text{hot}} = T_{\text{amb}} + 15^\circ\text{C} = 31 + 15 = 46^\circ\text{C}$$

TEC module parameters and performance graphs

Type	Couples	I <sub>max</sub> (A)	V <sub>max</sub> (V)	Q <sub>cmax</sub> (w) ΔT = 0	ΔT <sub>max</sub> (°C) Q <sub>c</sub> =0	R (Ω)
CP1-12726	127	26	15.4	243.5	68	0.45±0.05

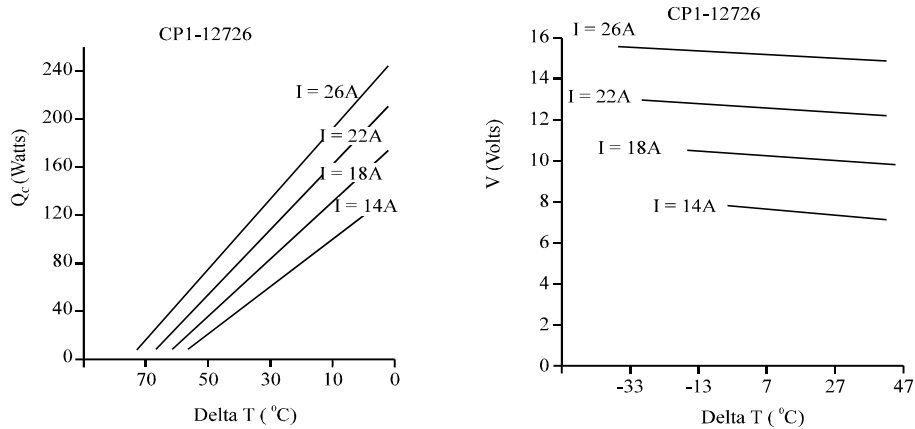


Fig. 2: TEC module performance graphs (obtained from TEC module spec. sheet)

Temperature difference ΔT across TEC can now be calculated as follows:

$$\Delta T = T_{hot} - T_{cold} = 46 - 6 = 40^{\circ}\text{C}$$

40°C will be used for design calculations parameters to be used for TEC module selection:

$$Q = 75 \text{ W, } \Delta T = 40^{\circ}\text{C}$$

From TEC module performance (Fig. 2):

- At Q = 75 Watts and delta T = 40°C
- Voltage = 12V and current, A = 21 A
- Heat produced internally by each TEC module = VI = 12×21 = 252 Watts

Total heat that must be dissipated by hot side heat sink = Heat produced internally by TEC module, VI + cold side cooling load, Q<sub>tec</sub> = 252 + 75 = 327 W. Thermal resistance of heat sink to be used:

$$R_{hs} = \frac{T_{hot} - T_{amb.}}{(VI) + Q_{tec}}$$

$$R_{hs} = \frac{46 - 31}{327} = 0.05 \text{ K/W}$$

A heat sink of rating 0.05 K/W or less must be used with each TEC module.

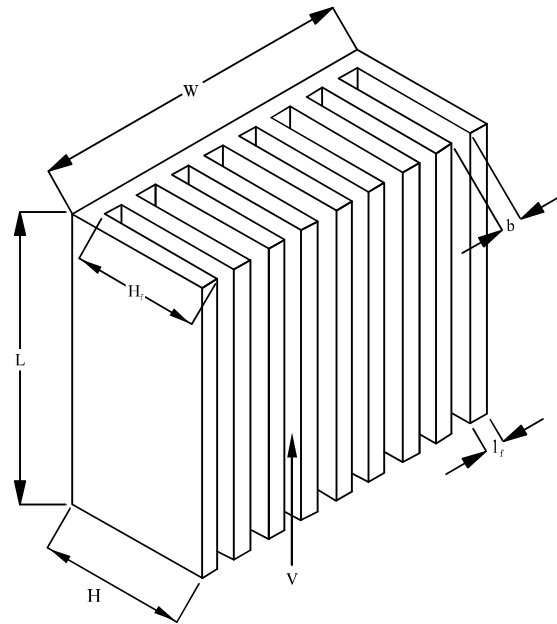


Fig. 3: Heat sink design parameters

**Heat sink sizing:** Heat sink design parameters (Fig. 3) are discussed as:

- Heat sink height, H = 0.085 m
- Fin width, L = 0.135 m
- Fin thickness, t<sub>f</sub> = 2×10<sup>-3</sup>

- Fin length,  $H_f = 0.08$  m
- Heat sink width,  $W = 0.12$  m
- Number of fins,  $N_{fin} = 25$

Volumetric flow rate of heat sink fan:

$$\dot{v} = 300 \text{ cfm} = 8.50 \text{ m}^3/\text{min} = 0.142 \text{ m}^3/\text{sec}$$

Air velocity between fins,

$$V = \frac{\dot{v}}{N_{fin} \cdot b \cdot H_f}$$

and  $b \cdot H_f =$  cross sectional area perpendicular to air flow:

$$\therefore V = \frac{0.142}{25 \times 2.917 \times 10^{-3} \times 0.080} = 24.48 \text{ m/sec}$$

Spacing between fins:

$$b = \frac{W - N_{fin} t_f}{N_{fin} - 1} = \frac{0.12 - (25 \times 0.002)}{25 - 1} = 2.917 \times 10^{-3} \text{ m}$$

Properties of air at 1 atm and 28°C:

$$\mu = 184.6 \times 10^{-7} \text{ N s/m}^2 \quad C_p = 1.007 \text{ KJ/kg.K}$$

$$K = 26.3 \times 10^{-6} \text{ KW/m.K} \quad \rho = 1.1614 \text{ kg/m}^3$$

$$Pr = \frac{C_p \mu}{K} = \frac{1.007 \times 184.6 \times 10^{-7}}{26.3 \times 10^{-6}} = 0.707$$

The composite model for forced convection for the plate fin heat sink is shown by the equation:

$$Nu_i = \left[ \frac{1}{\left( \frac{R_e^* \cdot Pr}{2} \right)^3} + \frac{1}{\left( 0.664 \sqrt{R_e^*} \cdot Pr^{0.33} \sqrt{1 + \frac{3.65}{\sqrt{R_e^*}}} \right)^3} \right]^{-1/3}$$

Where  $Nu_i =$  Ideal Nusselt's number (Teertstra *et al.*, 2001). Teertstra *et al.* (2001) modelled the heat sink as  $(N_{fin}-1)$  parallel plate 2-dimensional channels where channel width (space between fins,  $b$ ) is used as the characteristic length. The Reynold's number,  $R_e$  is thus defined by:

$$R_e = \frac{\rho \cdot V \cdot b}{\mu}$$

The channel's width, length and Reynold's number are combined to obtain the channel Reynold's number,  $R_e^*$ , analogous o the channel or Elenbaas Reyleigh number in natural convection:

$$R_e^* = R_e \cdot \frac{b}{L}$$

$$\therefore R_e^* = \frac{1.1614 \times 24.34 \times (2.9 \times 10^{-3})^2}{184.6 \times 10^{-7} \times 0.135} = 96.51$$

and

$$Nu_i = \left[ \frac{1}{\left( \frac{96.51 \times 0.707}{2} \right)^3} + \frac{1}{\left( 0.664 \sqrt{96.51} \times 0.707^{0.33} \times \sqrt{1 + \frac{3.65}{\sqrt{96.51}}} \right)^3} \right]^{-1/3} = 6.67$$

Average heat transfer co-efficient:

$$h = Nu_i \cdot \frac{K_{air}}{b} = 6.67 \times 26.3 \times 10^{-3} / 0.0029 = 60.11 \text{ W/m} \cdot \text{K}$$

Fin efficiency (Incropera *et al.*, 2007):

$$\eta_f = \frac{\tanh(m \cdot H_f)}{m \cdot H_f}$$

Where:

$$m = \sqrt{\frac{hP}{k_{fin} A_c}}$$

$P =$  Fin parameter  $= 2t_f + 2L$

$A_c =$  Fin c/s area  $= t_f L$

$$P = (2 \times 0.002) + (2 \times 0.135) = 0.274 \text{ m}$$

$$A_c = 0.002 \times 0.135 = 270 \times 10^{-6} \text{ m}^2$$

$$m = \sqrt{\frac{60.11 \times 0.274}{167 \times 270 \times 10^{-6}}} = 19.11$$

Hence,

$$\eta_f = \frac{\tanh(19.11 \times 0.08)}{19.11 \times 0.08} = 0.60$$

Heat sink thermal resistance (Fig. 4),  $R_{hs}$  is then given by (Incropera *et al.*, 2007):

$$R_{hs} = \frac{1}{\eta_o h A_t}$$

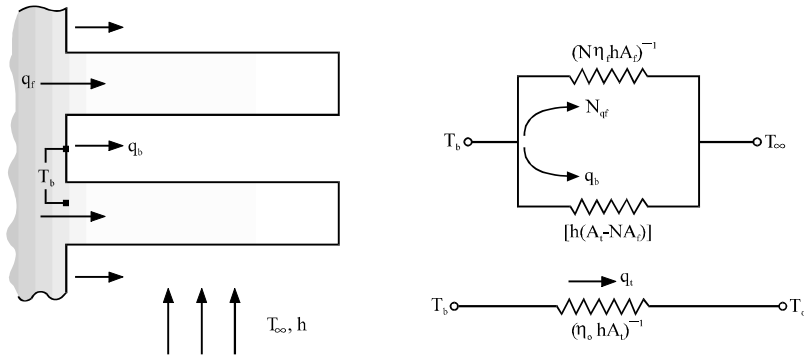


Fig. 4: Thermal resistance across heat sink fins

Where  $\eta_o$  is overall surface efficiency which characterizes an array of fins and the base to which they are attached (Incropera *et al.*, 2007):

$$\eta_o = 1 - \frac{NA_f}{A_t}(1 - \eta_f)$$

Where:

$A_f$  = Fin surface area =  $2 H_f L$

$A_t$  = Area associated with both the fins and exposed surface of the base (prime area)

$$A_t = NA_f + A_b$$

Where:

$A_b$  = Prime surface/base area of spaces between fins

$$A_b = (N_{fin} - 1) \cdot b \cdot L = (25 - 1) \times 0.0029 \times 0.135 = 0.009 \text{ m}^2$$

$$A_f = 2 \times 0.08 \times 0.135 = 0.022 \text{ m}^2$$

$$A_t = (25 \times 0.022) + 0.009 = 0.559$$

$$\therefore \eta_o = 1 - \frac{25 \times 0.022}{0.559}(1 - 0.60) = 0.606$$

Hence;

$$R_{hs} = \frac{1}{\eta_o h A_t} = \frac{1}{0.606 \times 60.11 \times 0.559} = 0.049 \text{ K/W}$$

$$0.049 \text{ K/W} \gg 0.05 \text{ K/W}$$

(maximum allowable heat sink resistance). Therefore, heat sink design meets requirements.

## RESULTS AND DISCUSSION

The theoretical analysis showed a supply voltage of 7V and current of 14 A obtained for the TEC modules from

the TEC module performance curves (Fig. 2). Which gives a wattage of  $(7 \times 14) = 98\text{W}$ . However, to gain this power rating of 98W, 11.69 volts had to be supplied which caused a corresponding current of 8.4 A in the TEC module. Design calculations stipulated a volume flow rate at the beginning of each heat sink of 150 cubic feet per minute (cfm). This design requirement was met. For the left side warm compartment, the flow rate at the beginning of the first heat sink was 530 and 160 cfm at the beginning of the second heat sink. For the right side warm compartment, the flow rate at the beginning of the first heat sink was 880 and 450 cfm at the beginning of the second heat sink. The difference in the flow rates on the left and right side can be explained by the fact that the left side had a longer duct with an extra  $45^\circ$  angle than the right side. Therefore, pressure loss along the duct that supplies the left side would have been greater than that along the duct supplying the right side.

The dimensions and different airflows for heat sink thermal resistance was obtained from the mathematical analysis.

The use of excel was instrumental in speeding up this process and the above calculations only represent the final result with the desirable values. Variables that were found to decrease the heat sink's thermal resistance are as follows: increasing air flow-rate across heat sinks, increasing fin length ( $H_f$ ), increasing fin width ( $L$ ), increasing number of fins ( $n_f$ ) while keeping heat sink Width ( $W$ ) the same, hence decreasing fin thickness ( $t_f$ ) also; increasing fin thickness ( $t_f$ ) while keeping heat sink width ( $W$ ) the same, hence decreasing space between fins ( $b$ ) also.

## CONCLUSION

The theoretical determination of TEC module and heat sinks shows that for a cooling time of 2 min and a beverage size of 474 cc (16 oz) mL, 6 TEC modules and

hence 12 heat sinks (one for each side of the TEC module) were needed. However, by increasing cooling time to 4 min and decreasing beverage size to 325 mL, 4 TEC modules and hence, 8 heat sinks could be used.

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