

## Economics of AC-DC OPF Based Nodal Prices for Restructured Electric Power System

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**Abstract:** In the last few years, electricity markets have significantly restructured in both developed and developing countries. This restructuring have enabled a transition from a vertically integrated private or public monopoly market structure to one of competitive wholesale and retail mechanism. Under it, electricity nodal pricing has emerged as an efficient tool. In many developing countries, transmission congestions and investments problems in transmissions sector have further reduces the consumer benefits. Recent trend in these countries is to incorporate High Voltage Direct Current (HVDC) transmission in the AC transmission systems to gain its techno-economical advantages. This study aims at motivations and relevance of electricity nodal pricing, formulating AC-DC OPF based nodal pricing, simulating the methodology for IEEE 30-bus system and IEEE 118 bus test system and computing nodal prices for real system of India and evaluating the economic impact of HVDC transmission, generation addition and transmission loading on electricity nodal prices. Study finally concluded that proposed methodology are more suitable for developing countries to fulfill their objectives to develop wholesale electricity market.

**Key words:** Electricity market, HVDC link, AC-DC optimal power flow, nodal prices, economic, transmission loading

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### INTRODUCTION

In the last few years, electricity markets have significantly restructured throughout the world with a process of breaking up vertically integrated electricity utilities, introduction of regulations and commercial interfaces between the functions of generation, transmission, distribution of electrical energy. The aim of this restructuring is to promote competition and to make the electricity market more efficient. Recently, the electric power industry has entered in an increasingly competitive environment under, which it becomes more realistic to improve economics and reliability of power systems by enlisting market forces (Ray and Alvarado, 1988). In developing countries, the trend of electricity market is heading towards Transmission Open Access (TOA) whereby transmission providers will be required to offer the basic transmission service (operational and/or ancillary services) and pricing (Rivier and Perez-Ariaga, 1993). Electricity nodal pricing in this context is an effective scheme for providing techno economic benefits to the market participants. Nodal prices contain valuable information useful for Poolco operation and hence the scheme is to accurately determine them, continue to be an active area of research. In coming years, power consumption in developing countries have expected to

more than double compared to 35-40% increase in developed countries. Besides, many developing countries are facing the problems of transmission congestions, infrastructure investment especially in transmission and distribution segment due to inadequate investments incurred in the past. To reduce the gap between transmission capacity and power demand, trend is now to adopt HVDC transmission system in the existing AC networks to gain advantages of the investment. This trend has therefore needed to address in formulating nodal pricing scheme.

In developing country like in India, the Electricity Act (EA) 2003 has implemented to undertake comprehensive market reforms in the electricity sector. TOA and National Tariff Policy by Ministry of Power, Government of India (GoI) seeks to achieve the objectives to ensure optimal development of the transmission network: to promote efficient utilization of generation and transmission assets; to attract investments in the transmission sector and to provide adequate returns. The proposed pricing scheme can be more suitable for similar developing countries.

The electricity nodal pricing literature studies have reviewed as a background of the present work but with no intention to cover all the published work. Rivier and Perez-Ariaga (1993) developed spot price model by describing the meaning and numerical properties of the

generation and transmission components of spot price based on slack bus and system lambda. Finney *et al.* (1997) introduced an OPF based decomposition of spot prices to perform the operation of the Poolco model. Results have derived using the decomposition of the Lagrangian multipliers corresponding to power balance equations into components that represented the sum of generation, losses and system congestion. Schweppe *et al.* (1988) introduced the concept of spot price into power systems and provided the foundation and starting point for most successive research. Xie *et al.* (2000) presented an integrated spot pricing model by modifying existing Newton OPF by Interior Point algorithms. It included derivation of optimal nodal specific real-time prices for active and reactive powers and the method to decompose it into generation, loss and ancillary services such as spinning reserve, voltage control and security control. Ward *et al.* (2000) investigated pricing behavior at New Zealand spot market, called Price Inversion by DC power flow and a full AC power flow. It was shown dependent on the physical characteristics of the power system. Chen *et al.* (2001) provided a detailed description and components of nodal prices i.e., generation, transmission congestion, voltage limitations and other constraints. Meliopoulos *et al.* (2003) presented a model for efficient calculation of spot prices, animation and visualization of spot prices based on the quadratized power flow approach. Eugene Litvinov *et al.* (2004) discussed the pricing of marginal transmission network losses in the Locational Marginal Pricing (LMP) deployed in the ISO new England standard market design. Study achieved market clearing results by introducing loss distribution factors to balance explicitly the consumed losses in the lossless DC power system. The distributed market slack references have discussed. Wu *et al.* (2005) provided explicit formulas to calculate components of LMPs i.e., reference price, congestion price and loss price based on the single slack power flow formulation. Hugo *et al.* (2006) presented an approach for the allocation of transmission network costs by controlling the electricity nodal prices. It introduced generation and nodal injection penalties into the economic dispatch to create nodal price differences that recover the required transmission revenue from the resulting congestion rent. The prices reflect both the marginal costs and the capital costs of the network. Li *et al.* (2007) presented electricity price forecasting techniques, can be useful at different time horizons for electricity price forecasting in LMP spot markets. Fuzzy inference system, least-squares estimation, and the combination of both have proposed to improve the short-term forecasting performance. Li and Bo (2007)

provided an iterative DCOPF-based algorithm to calculate LMPs and to analyze the sensitivity of LMP with respect to the system load.

After this introduction, nodal prices have simulated for modified IEEE-30 Bus system and IEEE-118 Bus system computed prices for real system of India. The impacts of HVDC transmission, generation addition and transmission loading on electricity nodal prices have also evaluated.

**Need of modeling electricity nodal prices:** Restructuring within electricity markets worldwide have resulted in competition in generation, transmission and distribution segments. In addition, the reductions in regulation and government price setting in the market have led to wholesale electricity market prices became much more volatile. This resulted market participants facing increased risk in terms of volumes of electricity they can produce and sell and the prices they will receive for it. To facilitate market participants in terms of operations, risk management and investments, need is to model accurately nodal price behaviour. Aside from generators, investors and regulators, customers also require models of nodal prices in order to study market behaviour. Also, often forecast and models of nodal prices serve various applications in the operation of electricity markets. For example, in short-run generating companies have to make decisions regarding unit commitment. They will only want their generators is to be dispatched if it is profitable and as these decisions are often required hours or days in advance, so they require forecast of future nodal prices in order to determine profitability. In the medium term, scheduled maintenance of generating plants have to be decided based on nodal price forecast to manage offline period that will have the least impact on profitability. In the longer term, potential investors need forecasts of nodal prices in order to determine the potential profitability of their investment. Many industries use and pay for electricity as an important input in their operations, they also require forecasts of nodal prices to determine their own profitability. In many markets around the world, users are able to purchase contracts for electricity at a fixed price over a specified time. The valuation of such financial derivatives requires estimation of both the likely levels and volatility of nodal prices in order to determine what that fixed price should be, as well as fair price for the contract itself.

## **MATERIALS AND METHODS**

**Electricity nodal price formulation:** To induce efficient use of the transmission grid and generation resources by

providing correct economic signals, a nodal price theory for the restructured electricity markets is developed (Schweppe *et al.*, 1988). It is a method to determine market-clearing prices for several locations on the transmission grid (node). The price at each node reflects cost of the energy and the cost of delivering it. Nodal prices provided information about locational pricing, value of locating new generation, transmission upgradation etc. This enhances functions and efficiency of wholesale electricity market and improves the systems' ability to meet electricity demand.

**Problem formulation**

**AC system equations:** Let  $P = (p_1, \dots, p_n)$  and  $Q = (q_1, \dots, q_n)$  for  $n$  bus system, where  $p_i$  and  $q_i$  be active and reactive power demands of bus- $i$ , respectively. The variables in power system operation to be  $X = (x_1, \dots, x_m)$ , i.e., real and imaginary bus voltages. Then the operational problem of a power system for given load  $(P, Q)$  can be formulated as OPF problem;

$$\text{Minimize } f(X, P, Q) \text{ for } X \quad (1)$$

$$\text{Subject to } S(X, P, Q) = 0 \quad (2)$$

$$T(X, P, Q) \leq 0 \quad (3)$$

Where,  $S(X) = (s_1(X, P, Q), \dots, s_{n_1}(X, P, Q))^T$  and  $T(X) = (t_1(X, P, Q), \dots, t_{n_2}(X, P, Q))^T$  have  $n_1$  and  $n_2$  equations, respectively and are column vectors. Here,  $A^T$  represents the transpose of vector  $A$ .

$f(X, P, Q)$  is a scalar, short term operating cost, such as fuel cost. The generator cost function  $f_i(P_{Gi})$  in \$/MWh is considered to have cost characteristics represented by,

$$f = \sum_{i=1}^{NG} a_i P_{Gi}^2 + b_i P_{Gi} + c_i \quad (4)$$

Where,

- $P_{Gi}$  = The real power output
- $a_i, b_i, c_i$  = The cost coefficient of the  $i$ th generator
- NG = The generation buses

The constraints to be satisfied during optimization are,

- Vector of equality constraint such as power flow balance (i.e. Kirchoff's laws) has represented as:

$$S(X, P, Q) = 0 \text{ or}$$

$$P_G = P_D + P_{DC} + P_L \text{ and} \quad (5)$$

$$Q_G = Q_D + Q_{DC} + Q_L$$

Where:

- D = The demand
- G = The generation
- DC = DC terminal
- L = The transmission loss

The vector of inequality constraints includes all variables and function limits, such as upper and lower bounds of transmission lines, generation outputs, stability and security limits may be represented as:

$$T(X, P, Q) = 0 \quad (6)$$

- The maximum and minimum real and reactive power outputs of the generating sources have given by:

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad (i \in G_B) \text{ and} \quad (7)$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max} \quad (i \in G_B)$$

Where:

- $P_{Gi}^{\min}, P_{Gi}^{\max}$  = The minimum and maximum real power outputs of the generators
- $Q_{Gi}^{\min}, Q_{Gi}^{\max}$  = The minimum and maximum reactive power outputs

- Voltage limits (Min/Max) signals the system bus voltages to remain within a narrow range. It is denoted by the following constraints:

$$|V_i^{\min}| \leq |V_i| \leq |V_i^{\max}| \quad (f = 1, \dots, N_B) \quad (8)$$

Where:

- $N_B$  = Number of buses

- Power flow limits is the transmission line's thermal or stability limits capable of transmitting maximum power (MVA) flow through the lines and it is expressed by the following constraints:

$$P_f^{\min} \leq P_f \leq P_f^{\max} \quad (f = 1, \dots, N_{oele}) \quad (9)$$

Where:

- $N_B$  = No. of transmission lines connected to grid

Then, operating conditions of a combined AC-DC electric power system may described by the vector:

$$X = [\delta, V, x_c, x_d]^t \quad (10)$$

Where:

- $\delta$  and  $V$  = The vectors of the phases and magnitude of the phasor bus voltages
- $x_c$  = The vector of control variables
- $x_d$  = The vector of dc variables

**DC system equations:** The following relationship is for the dc variables. Using the per unit system (Lu *et al.*, 1988), the average value of the dc voltage of a converter connected to bus (i) is:

$$V_{di} = a_i V_i \cos \alpha_i - r_{ci} I_{di} \quad (11)$$

Where:

$\alpha_i$  = The gating delay angle for rectifier operation or the extinction advance angle for inverter operation

$r_{ci}$  = The commutation resistance

$a_i$  = The converter transformer tap setting

By assuming a lossless converter, the equation of the dc voltage written as:

$$V_{di} = a_i V_i \cos \varphi_i \quad (12)$$

where,  $\varphi_i = \delta_i - \xi_i$  and  $\varphi$  is the angle by which the fundamental line current lags the line-to-neutral source voltage.

The real power flowing in or out of the dc network at terminal (i) may be expressed as:

$$P_{di} = V_{i1} I_{d1} \cos \varphi_i \text{ or } P_{di} = V_{di} I_{di} \quad (13)$$

The reactive power flow into the dc terminal is:

$$Q_{di} = V_{i1} I_{d1} \sin \varphi_i \text{ or } Q_{di} = V_{di} I_{di} \sin \varphi_i \quad (14)$$

The Eq. 13, 14 can substituted into the Eq. 5 to form part of the equality constraints. Based on these relationships, the operating condition of the DC system can describe by the vector:

$$X_d = [V_{d1}, I_{d1}, a, \cos \alpha, \varphi]^T \quad (15)$$

The DC currents and voltages have related by the DC network equations. In AC case, a reference bus is specified for each separate dc system; usually the bus of the voltage controlling DC terminal operating under constant voltage (or constant angle) control is chosen as the reference bus for that DC network equation.

Here, Eq. 1 and 3 are an OPF problem for the demand (P, Q). There are many approaches that can be used to get an optimal solution such as linear programming, Newton method, quadratic programming, nonlinear programming, interior point method, artificial intelligence (i.e., a rtificial neural network, fuzzy logic, genetic algorithm, evolutionary programming,

ant colony and particle swarm optimization etc.) methods (Chen *et al.*, 2001; Pandya and Joshi, 2008).

**Electricity nodal price:** The real and reactive power prices at bus i is the Lagrangian multiplier value of the equality and in-equality constraints. These values have calculated by solving the first order condition of the Lagrangian, partial derivatives of the Lagrangian with respect to every variable concerned. Therefore the Lagrangian function (or system cost) of equation defined as:

$$\begin{aligned} L = & \sum_{i=1}^{NG} a_i P_G^2 + b_i P_G + c_i + \sum_{i \in LB} \lambda_{pi} (P_{Di} - P_{Gi} + P_{DCi} + P_{Li}) \\ & + \sum_{i \in LB} \lambda_{qi} (Q_{Di} - Q_{Gi} + Q_{DCi} + Q_{Li}) + \sum_{i \in LB} \lambda_{ppli} (P_{Gi}^{min} - P_{Gi}) \\ & + \sum_{i \in GB} \rho_{pu} (P_{Gi} - P_{Gi}^{max}) + \sum_{i \in GB} \rho_{qu} (Q_{Gi}^{min} - Q_{Gi}) \\ & + \sum_{i \in GB} \rho_{ui} (Q_{Gi} - Q_{Gi}^{max}) + \sum_{i=1}^{NB} \rho_{Vli} (|V_i^{min}| - V_i) \\ & + \sum_{i=1}^{NB} \rho_{Vui} (|V_i| - |V_i^{max}|) + \sum_{i=1}^{NB} \rho_{\theta li} (\theta_i^{min} - \theta_i) \\ & + \sum_{i=1}^{NB} \rho_{\theta ui} (\theta_i - \theta_i^{max}) + \sum_{i=1}^{Noele} \rho_{Pi} (P_{fi}^{min} - P_{fi}) \\ & + \sum_{i=1}^{Noele} \rho_{Qui} (P_{fi} - P_{fi}^{max}) \end{aligned} \quad (16)$$

Where:

l and u = The lower and upper limits

$\lambda(\lambda_1, \dots, \lambda_n)$  = The vector of Lagrange multipliers concerning the equality constraints

$\rho(\rho_1, \dots, \rho_n)$  = The Lagrange multipliers concerning to the inequality constraints

Then, at an optimal solution (X,  $\lambda$ ,  $\rho$ ) and for a set of given (P, Q), the nodal price of real and reactive power for each bus is expressed below for  $i = 1, \dots, n$ .

$$\pi_{p,i} = \frac{\partial L(X, \lambda, \rho, P, Q)}{\partial p_i} = \frac{\partial f}{\partial p_i} + \lambda \frac{\partial S}{\partial p_i} + \rho \frac{\partial T}{\partial p_i} \quad (17)$$

$$\pi_{q,i} = \frac{\partial L(X, \lambda, \rho, P, Q)}{\partial q_i} = \frac{\partial f}{\partial q_i} + \lambda \frac{\partial S}{\partial q_i} + \rho \frac{\partial T}{\partial q_i} \quad (18)$$

Here,  $\pi_{p,i}$  and  $\pi_{q,i}$  are active and reactive nodal prices at bus I, respectively. The difference i.e.,  $\pi_{p,i} - \pi_{p,j}$  and  $\pi_{q,i} - \pi_{q,j}$  shows active and reactive power transmission charges from bus-j to bus-i. Equation 17 can be view as the system marginal cost created by an increment of real power load at bus i. The above formulations have

simulated in MATLAB programming using the fmincon function available in the optimization toolbox based on interior point algorithms. An advantage of it is that the constraints can be directly evaluated as functions of the state variables, which can be separate modules reducing programming complexity.

**RESULTS AND DISCUSSION**

**Modified IEEE 30 bus system:** The given scheme is simulated for modified IEEE 30-bus system. The system consists of 6 generators and 42 transmission lines (Fig. 1). A HVDC link has connected between bus 1 and bus 30. The ratings of the converter at these buses are 1.0 Per Unit (PU). The upper and lower bounds (real power) for generators  $G_{1}$ ,  $G_{2}$ ,  $G_{13}$ ,  $G_{22}$ ,  $G_{23}$  and  $G_{27}$  and their fuel cost functions expressed as  $(f_i = a_i P_{Gi}^2 + b_i P_{Gi} + c_i)$  in (\$/MWh) and real and reactive power demand is shown in Table 1. The upper and lower bounds (reactive power) for all generators are in the range of  $-0.5 \leq Q_{Gi} \leq 0.5$ . The voltage values for all buses have bounded between 0.95 and 1.05. All of the values have indicated by PU. The transmission lines capacity (in MVA) has given in Table 2.

For this system there are  $2 \times 24$  equalities constraints of S corresponding to their respective real and reactive power balances of the buses without a generator and 72 inequalities constraints of T corresponding of 30 pairs of voltage,  $2 \times 6$  pairs of generation output, one pair of line flow upper and lower bounds, respectively.

The nodal prices of real power with and without HVDC links have simulated and compared shown in the Table 3. The result indicates that with the incorporation of HVDC link in existing AC transmission system, electricity

nodal prices at several buses are improved. This study further extended to simulate nodal price behaviour under transmission line loading conditions as shown in Fig. 2. The nodal prices obtained are lower for design capacity compared to 75 and 65% transmission loading capacity.

**Modified IEEE-118 bus system:** The given scheme has simulated on a large-scale power system, i.e., IEEE 118-bus system (You *et al.*, 2004). The system consists of 54 generators and 186 branches (Fig. 3). A HVDC link has connected between bus 12 and bus 44. The ratings of the converter at these buses are 1.0 Per Unit (PU). The voltage values for all buses have bounded between 0.95 and 1.05. All of the values have indicated by PU.

The nodal prices of real power with and without HVDC links have simulated and compared shown in the Table 4. The result indicates that with the incorporation of HVDC link in existing AC transmission system, electricity nodal prices at several buses have reduced.

**Indian electricity market and real transmission network**

**Indian electricity market:** India's electricity sector has grown to 143,061 MW as on 31st March, 2008 with a compound annual growth rate of 8-9%. This sector is been characterized by shortage of supply vis-à-vis demand. In order to improve its performances, GoI initiated electricity sector restructuring in 1991. The EA 2003 is been brought to facilitate private investments and to help cash strapped State Electricity Boards to meet electricity demand. On the electricity transmission front, the Indian grid is divided into five sub grids namely, Northern, Western, Southern, Eastern and North-Eastern called regional grids. Each one has number of constituent sub grids formed by state and private utility networks. All

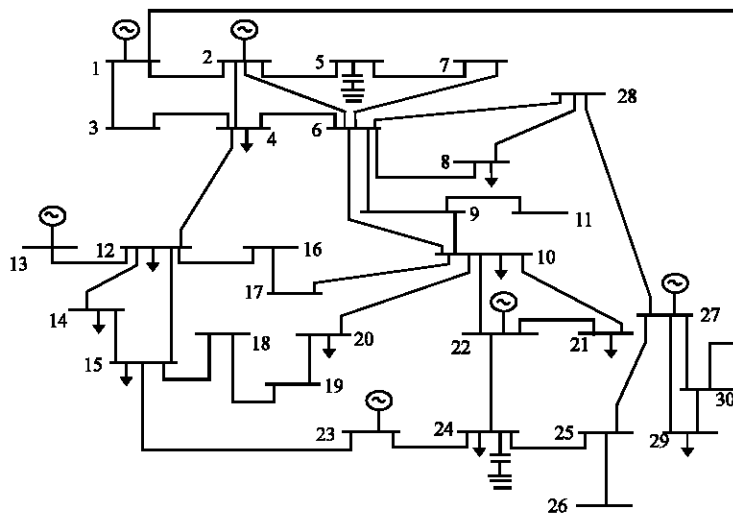


Fig. 1: Modified IEEE 30-bus test system

Table 1: Generator: real power, fuel cost and demand

Bus no.	Lower limit (real power)	Upper limit (real power)	Demand ( $p_i + jq_i$ )	Generation cost		
				$a_i$	$b_i$	$c_i$
1	1.0	1.5	0.0+j0.0	0.14	20.4	5.0
2	0.0	0.5	-0.217+j0.13	0.20	19.3	5.0
3	-	-	0.024+j0.012	-	-	-
4	-	-	0.076+j0.016	-	-	-
5	-	-	0.942+j0.019	-	-	-
6	-	-	0.0+j0.0	-	-	-
7	-	-	0.228+j0.109	-	-	-
8	-	-	0.30-j0.30	-	-	-
9	-	-	0.0+j0.0	-	-	-
10	-	-	0.058+j0.0	-	-	-
11	-	-	0.0-j0.177	-	-	-
12	-	-	0.112+j0.0	-	-	-
13	0.5	1.0	0.00-j0.155	0.14	20.4	5.0
14	-	-	0.062+j0.016	-	-	-
14	-	-	0.082+j0.025	-	-	-
16	-	-	0.035+j0.018	-	-	-
17	-	-	0.090+j0.058	-	-	-
18	-	-	0.032-j0.009	-	-	-
19	-	-	0.095+j0.034	-	-	-
20	-	-	0.022+j0.007	-	-	-
21	-	-	0.175+j0.112	-	-	-
22	0.5	1.5	-0.00+j0.00	0.20	19.3	5.0
23	0.1	0.5	0.032+j0.016	0.14	20.4	5.0
24	-	-	0.087+j0.067	-	-	-
25	-	-	0.0+j0.0	-	-	-
26	-	-	0.035+j0.023	-	-	-
27	0.2	0.6	0.0-j0.10	0.20	19.3	5.0
28	-	-	0.0+j0.0	-	-	-
29	-	-	0.024+j0.009	-	-	-
30	-	-	0.106+j0.0	-	-	-

Table 2: Transmission capacity for IEEE 30-bus system

Line no.	From bus to bus	Line capacity (MVA)	Line no.	From bus to bus	Line capacity (MVA)
1	1-2	130	22	15-18	16
2	1-3	130	23	18-19	16
3	2-4	65	24	19-20	32
4	3-4	130	25	10-20	32
5	2-5	130	26	10-17	32
6	2-6	65	27	10-21	32
7	4-6	90	28	10-22	32
8	5-7	70	29	21-22	32
9	6-7	130	30	15-23	16
10	6-8	32	31	22-24	16
11	6-9	65	32	23-24	16
12	6-10	32	33	24-25	16
13	9-11	65	34	25-26	16
14	9-10	65	35	25-27	16
15	4-12	65	36	28-27	65
16	12-13	65	37	27-29	16
17	12-14	32	38	27-30	16
18	12-15	32	39	29-30	16
19	12-16	32	40	8-28	32
20	14-15	16	41	6-28	32
21	16-17	16	42	1-30	130

these sub grids and networks have connected to form a 400 kV national grid. The constituent systems have their own generation in addition to generation by central government undertakings in different parts of the country and feeds power in the grid at different locations. The central and state transmission utility is responsible for the

national and regional transmission system development and is also providing open access on its inter and intra-state transmission system. The Power Grid Corporation of India Limited is set up for establishment and operation of regional and national power grids to ease transfer of power within and across the regions with reliability, security and economy, on sound commercial principles. Power Trading Corporation is helping market participants to find counterparts. To increase efficiency and competition in the sector, Central Electricity Regulatory Commission (CERC) allowed open access to market participants, National Grid formation and development of inter-regional electricity transmission linkages and implemented availability based tariff for real time balancing market. To promote power trading in a free power market, CERC recently setup Indian Energy Exchange (IEX). It has developed as market based institution for providing price discovery and price risk management to the electricity generators, distribution licensees, electricity traders, consumers and other stakeholders. At present, IEX offers day-ahead contracts whose time line is set in accordance with the operations of Regional Load Dispatch Centers (RLDCs). IEX coordinates with the National Load Dispatch Centers/RLDCs and State LDCs for scheduling of traded contracts.

Table 3: IEEE 30-bus test system: results for bus voltage and nodal price

Bus no.	Bus voltage (PU)		Nodal price (\$/MWh)		Bus no.	Bus voltage (PU)		Nodal price (\$/MWh)	
	Without HV/DC	With HV/DC	Without HV/DC	With HV/DC		Without HV/DC	With HV/DC	Without HV/DC	With HV/DC
1	1.05	1.02	20.8	15.7	16	0.98	1.02	21.4	15.7
2	1.05	1.02	21.0	15.6	17	0.98	1.02	21.8	16.0
3	1.05	1.02	21.1	15.6	18	0.97	1.01	21.6	16.1
4	1.05	1.03	21.2	15.6	19	0.97	1.01	21.8	16.2
5	1.05	1.03	22.6	15.3	20	0.97	1.01	21.8	16.2
6	1.05	1.02	21.5	15.7	21	0.98	1.02	22.7	16.2
7	1.04	1.02	22.1	15.7	22	0.98	1.02	19.5	16.2
8	1.05	1.02	21.6	15.8	23	0.99	1.02	20.2	16.0
9	1.03	1.00	21.7	15.9	24	0.99	1.02	19.6	16.1
10	1.03	1.02	21.8	16.0	25	1.02	1.03	15.5	16.0
11	1.05	0.95	21.7	15.9	26	1.02	1.02	15.2	15.9
12	1.04	1.04	20.9	15.2	27	1.05	1.05	15.1	15.9
13	1.05	1.05	15.1	15.2	28	1.02	1.02	21.5	15.8
14	1.03	1.03	21.1	15.5	29	1.03	1.04	15.5	15.8
15	1.02	1.02	21.0	15.7	30	1.03	1.04	15.7	15.5

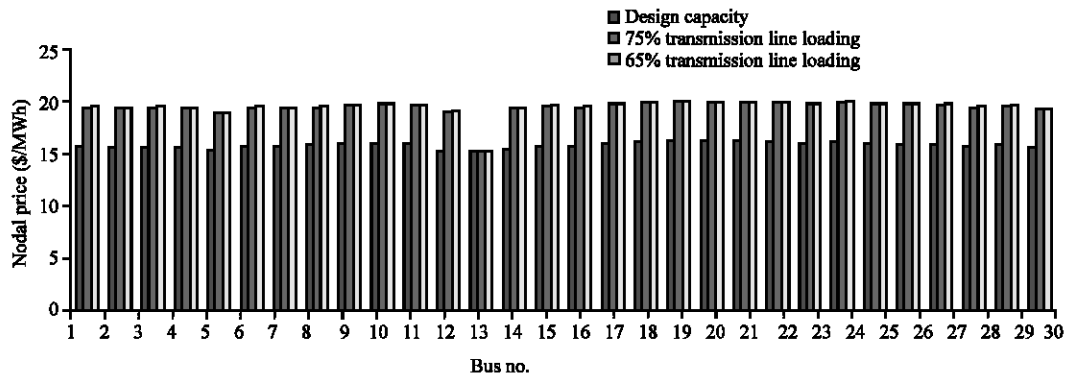


Fig. 2: IEEE 30-bus system: nodal price behaviour in transmission line loading condition

**Real transmission network of India:** Maharashtra State Electricity Board is the largest installed capacity of 15,580 MW in India. In 2005, it was unbundled into Generation, Transmission and Distribution Company. Maharashtra State Electricity Transmission Company Limited (MSETCL) infrastructure consists of ±500 kV HVDC, 400 kV, 220 kV, 132 kV, 110 kV, 486 EHV sub-stations and 35626 circuit km lines. Due to inadequate investment, transmission sector is also feeling the strain. The current transformation capacity is of 22,168 MVA, which have proposed to enhance to 68,182 MVA to bring down line loading with an investment of over Rs. 200 billion. It has also proposed to offer most of the new evacuation lines and sub-stations to private sectors on tariff-based bidding.

Present study considered a real network of 400 kV MSETCL shown in Fig. 4. It consists of 19 intra-state buses (i.e., Bus No. 1-19) and 8 inter-state buses. Additional power to fulfill real power demands has imported from inter-state generators namely BHILY, KHANDWA, SDSRV, BOISR, BDRVT, TARAPUR and SATPR. The generator installed capacity, upper and lower

bounds for generators, fuel cost function for intra and inter-state generators expressed as ( $f_i = a_i P_{Gi}^2 + b_i P_{Gi} + c_i$ ) in (\$/MWh) and real power demand is shown in Table 5. The voltages at all buses have bounded between 0.96 and 1.04 PU. The operating data for HVDC link has shown in Table 6. The transmission line loading capacity expressed in MVA has shown in Table 7. CHDPUR selected as a reference bus.

This system have  $2 \times 16$  equalities constraints of S of their respective real and reactive power balances of the buses without a generator and 48 inequalities constraints of T of 27 pairs of voltage,  $2 \times 11$  pairs of generation output and one pair of line flow upper and lower bounds.

**System simulation without HVDC link (case-1):** Electricity nodal prices obtained at various buses are high for 400 kV MSETCL system without considering HVDC link as shown in Table 8. It is due to huge active power deficit at KOYNA-4, CHDPUR and KORDY, congestions in the transmission lines namely, CHDPUR-KORDY, BHSWL2-ARGBD4 and CHDPUR-PARLY2, rising power demand and costly power imported to fulfill present demand. The state of Maharashtra has been facing severe

Table 4: IEEE 118-bus test system: simulated results for bus voltage and nodal price

Bus no.	Bus voltage (PU)		Nodal price (\$/MWh)		Bus no.	Bus voltage (PU)		Nodal price (\$/MWh)	
	Without HV/DC	With HV/DC	Without HV/DC	With HV/DC		Without HV/DC	With HV/DC	Without HV/DC	With HV/DC
1	0.96	0.95	136	101	60	0.98	0.96	96	91
2	0.98	0.97	98	98	61	0.95	0.97	94	92
3	0.95	0.96	115	100	62	0.95	0.97	95	88
4	0.97	0.98	101	100	63	0.95	0.96	101	95
5	0.97	0.99	109	100	64	0.97	0.99	114	96
6	0.96	0.99	96	98	65	0.97	0.99	72	66
7	0.98	0.99	100	97	66	1.05	1.01	73	73
8	1.03	1.01	62	54	67	1.01	1.00	82	80
9	1.01	1.03	74	54	68	1.02	1.00	69	67
10	0.98	1.01	53	53	69	1.04	1.02	67	68
11	0.97	0.98	93	97	70	1.05	1.04	62	67
12	0.97	0.99	99	95	71	1.05	1.01	55	59
13	1.05	0.98	101	93	72	1.05	1.01	65	60
14	1.03	1.00	114	90	73	1.05	1.02	70	59
15	1.02	1.04	65	74	74	1.03	1.01	64	68
16	1.03	1.00	90	88	75	1.03	1.01	77	69
17	1.01	1.05	117	70	76	1.04	1.02	72	69
18	1.02	1.05	83	71	77	1.04	1.02	62	68
19	0.98	1.05	92	73	78	1.02	1.01	83	69
20	1.05	1.04	84	72	79	1.05	1.01	78	69
21	1.04	1.03	69	71	80	1.04	1.02	77	68
22	1.05	1.04	68	69	81	1.02	1.01	79	67
23	0.95	1.05	76	66	82	1.03	1.03	68	66
24	1.03	1.05	70	64	83	1.01	1.03	76	66
25	1.01	1.05	68	65	84	1.02	1.04	86	65
26	0.95	1.05	75	65	85	1.01	1.05	71	64
27	1.01	1.05	87	68	86	0.99	1.04	63	62
28	1.05	1.04	67	69	87	1.05	1.05	60	60
29	0.97	1.05	87	69	88	1.01	1.04	66	64
30	0.95	1.05	82	65	89	0.98	1.05	69	64
31	1.03	1.01	87	69	90	1.05	1.04	62	64
32	1.05	1.02	88	68	91	1.05	1.02	66	61
33	1.03	1.03	91	73	92	1.04	1.01	70	64
34	1.05	1.01	91	69	93	1.03	1.00	63	65
35	1.05	1.05	92	69	94	1.03	1.01	69	66
36	1.05	1.02	91	69	95	0.98	1.01	67	67
37	1.05	1.02	88	69	96	0.97	1.02	66	67
38	1.04	1.05	89	67	97	1.04	1.01	59	68
39	1.05	1.04	68	70	98	1.05	1.02	77	67
40	1.03	1.02	63	70	99	1.05	1.01	99	64
41	1.02	1.01	96	71	100	1.04	1.02	88	64
42	1.03	1.02	98	72	101	1.03	1.00	68	64
43	1.02	1.00	82	72	102	1.04	1.00	66	64
44	1.01	1.01	78	76	103	1.05	1.01	69	62
45	1.01	1.01	82	76	104	1.05	1.01	60	62
46	0.98	1.02	74	74	105	1.05	1.05	62	62
47	1.05	1.03	77	75	106	1.02	1.04	60	63
48	1.04	1.02	79	76	107	1.02	1.05	59	61
49	1.05	1.03	79	76	108	1.05	1.01	60	61
50	1.05	1.01	72	78	109	1.05	1.01	60	61
51	1.02	0.99	83	82	110	1.05	1.03	64	60
52	1.03	0.98	82	83	111	1.05	1.01	54	59
53	0.97	0.98	88	85	112	1.05	1.02	72	59
54	0.98	0.99	89	85	113	1.05	1.03	79	69
55	0.98	1.01	86	85	114	1.04	1.04	68	68
56	0.98	0.99	87	85	115	1.04	0.99	67	68
57	0.97	0.99	83	83	116	1.03	1.00	63	67
58	0.96	0.98	84	84	117	1.03	0.97	98	97
59	0.95	0.98	90	91	118	1.05	1.02	66	69

power shortages at peak hours. The state utility is making all efforts to fulfill demand by power purchases and load shedding. To make wholesale electricity market more competitive, need is to

pour large value of active power in the state. Similarly new investment in transmission segment in above transmission lines are needed to remove congestions.



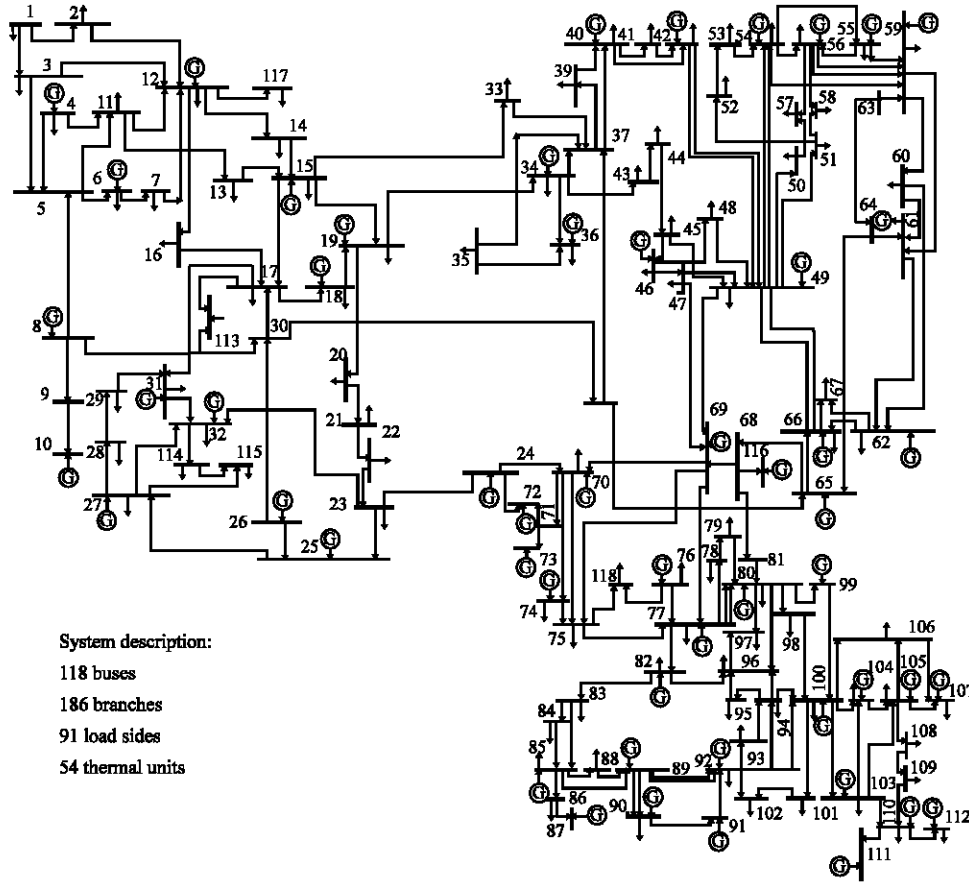


Fig. 3: IEEE-118 bus test system

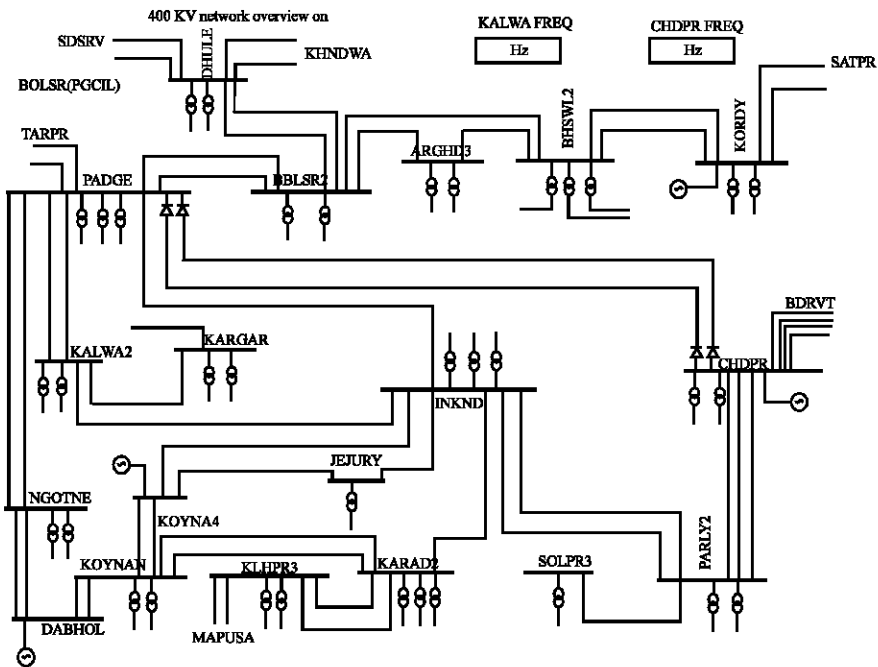


Fig. 4: 400 kV MSETCL, India

Table 5: Generation capacity and costs characteristics and real peak demand

Bus/generator	Gen. cap. (PU)	P <sub>G</sub> (PU)		Generation cost (\$/Mwh)			Real load (PU)
		Max	Min	a <sub>i</sub>	b <sub>i</sub>	c <sub>i</sub>	
<b>Intra-state buses/generators</b>							
CHDPUR	2.3	1.72	0.2	0.20	20.4	10.2	0.3
KORDY	1.06	0.54	0.2	0.20	22.4	10.2	0.3
BHSQL2	-	-	-	-	-	-	0.2
ARGBD4	-	-	-	-	-	-	0.5
BBLSR2	-	-	-	-	-	-	0.5
DHULE	-	-	-	-	-	-	0.2
KALWA	-	-	-	-	-	-	0.3
KARGAR	-	-	-	-	-	-	0.3
LONKAND	-	-	-	-	-	-	0.6
NGOTNE	-	-	-	-	-	-	0.3
DABHOL	1.50	1.44	0.2	1.02	71.4	10.2	0.0
KOYNA-N	-	-	-	-	-	-	0.4
KOYNA-4	1.50	0.19	0.1	0.20	20.4	10.2	0.0
KLHPR3	-	-	-	-	-	-	0.4
JEJURY	-	-	-	-	-	-	0.3
KARAD2	-	-	-	-	-	-	0.5
SOLPR3	-	-	-	-	-	-	0.3
PARLY2	-	-	-	-	-	-	0.05
PADGE	-	-	-	-	-	-	0.4
<b>Inter-state buses/generators</b>							
BHILY	-	0.6	0.1	0.20	36.9	10.2	-
KHANDWA	-	0.7	0.2	1.02	36.9	10.2	-
SDSRV	-	0.5	0.1	1.02	77.6	10.2	-
BOISR	-	0.2	0.1	1.02	55.7	10.2	-
BDRVT	-	1.7	0.2	0.20	22.5	10.2	-
TARAPUR	-	0.4	0.1	1.02	58.6	10.2	-
SATPR	-	0.2	0.1	1.02	55.7	10.2	-

Table 6: Data: ±500 kV CHDPUR-PADGE HVDC link

Particulars	Data	Particulars	Data
<b>Power flow rating</b>	1500 MW	<b>Thyristor valves</b>	
		Max. voltage	7 kV
		Rated current	1700 A
<b>Converter X'mer</b>		<b>Resistance</b>	
Voltage of each pole	500 kV	(1-Pole)	7.5Ω
Rated power of unit	298.6 MVA	(2-Pole)	7.5Ω
		Metallic Return	15Ω
<b>HVDC Line</b>		<b>Operation</b>	12.5-15°
Length of line	753 Km	CHDPUR-	
Number of poles	2	Converter/Rectifier	17-22°
Nominal DC voltage	±500 kV	PADGE-Inverter	

**System simulation with HVDC link (case-2):** The electricity demand in the Maharashtra state is concentrated in the Western region and the major generation in the Eastern region. The state's share is of 1,700 MW from central generating station is received at CHDPUR. The AC transmission network comprising of three 400 kV circuits between CHDPUR and Mumbai can safely transmit around 1200 MW of power without any contingency outage. Therefore, it was necessary to provide additional transmission capacity of around 1,500 MW. Expansion of 400 kV lines was not feasible due to sever constraints of right of way and cost considerations.

The option of ±500 kV HVDC bipole has found most viable. The electricity nodal prices have simulated with HVDC link shown in Table 9. The obtained nodal prices

are lower as compared to previous case. Moreover, congestions in the above mentioned transmission lines reduced. Bus voltages are improved and it is possible to transmit sufficient and cheaper power available at BDRVT to fulfill the demand. This link is vital in peak load condition to maintain the prices within limit.

**System simulation with HVDC link and generation addition (case-3):** The nodal prices can reduce with addition of hydro generation KOYNA-4 at its full extent. The prices obtained are the lowest and uniform at several buses shown in Table 10. At peak load, hydro generation plays a key role to reduce the prices and avoids costly inter-state power purchases. This move can promotes competition in the wholesale electricity market from strategically located lower-cost units and demand response can benefit to MSETCL, as the transmission grid can utilize more efficiently. Figure 5 shows improved bus voltages at several buses with the addition of HVDC link and Hydro Generation at peak load.

**System simulation with HVDC link and transmission line loading (case-4):** Figure 6 shows the comparison of nodal price behaviour in transmission line loading environment. Higher prices obtained at several buses at average transmission line loading. MSETCL at present

Table 7: Transmission line loading (MVA) of 400 kV MSETCL system

Line no.	Name of Xn. Line	Design MVA	ATL	Line no.	Name of Xn. Line	Design MVA	ATL
1	CHDPUR-KORDY	500	403	26	LONKAND- KOYNA4	500	352
2	KORDY-BHILY	1000	636	27, 28	KOYNA4- KOYNA-N	1000	900
3	KORDY-SATPR	500	436	29, 30	NGOTNE-DABHOL	1000	900
4,5	KORDY-BHLSWL2	1200	1022	31, 32	KOYNA-N- DABHOL	1000	900
6	BHLSWL2- ARGBD3	500	352	33, 34	KOYNA-N- KARAD2	1000	900
7	BHLSWL2- BBLSR2	500	416	35, 36	KLHPR3-MAPUSA	1000	900
8	ARGBD3- BBLSR2	500	142	37, 38	KLHPR3-KARAD2	1000	900
9,10	BBLSR2-DHULE	1000	522	39	KOYNA4- JEJURY	500	450
11,12	DHULE- KHNDWA	2000	1040	40	LONKAND- JEJURY	500	450
13,14	DHULE- SDSRV	1000	226	41	LONKAND-KARAD2	500	310
15,16	BBLSR2-PADGE	1000	542	42	KARAD2-SOLPR3	500	340
17	PADGE- BOISR	1000	900	43, 44	LONKAND-PARLY2	1400	1060
18	PADGE-LONKAND	500	274	45	SOLPR3-PARLY2	1000	544
19	PADGE- KARGAR	500	198	46, 47, 48	PARLY2-CHDPUR	2100	1716
20, 21	PADGE- KALWA	1000	573	49, 50, 51, 52	BDRVT- CHDPUR	4000	2188
22,23	PADGE- NGOTNE	1000	900	53, 54	PADGE-TARAPUR	1000	450
24	KALWA-KARGAR	500	177	55, 56	CHDPUR-PADGE	3000	2800
25	KALWA-LONKAND	500	420				

\*ATL: Average Transmission Line Loading (MVA); (www.mahatransco.com)

Table 8: Electricity nodal price (\$/MWh) and bus voltages without HVDC link at peak load condition

Bus no.	Bus name	P <sub>G</sub> (PU)	Nodal price	Bus no.	Bus name	P <sub>G</sub> (PU)	Nodal price
1	CHDPUR	1.44	24	10	LONKND	-	168
2	KORDY	0.54	46	11	NGOTNE	-	105
3	BHLSWL2	-	90	12	DABHOL	1.44	60
4	ARGBD4	-	104	13	KONA-N	-	288
5	BBLSR2	-	118	14	KONA-4	0.19	279
6	DHULE	-	116	15	KLHPR3	-	309
7	PADGE	-	140	16	JEJURY	-	223
8	KALWA	-	142	17	KARAD2	-	309
9	KARGAR	-	142	18	SOLPR3	-	526
-	-	-	-	19	PARLY2	-	20

Inter state Power Purchase-P<sub>G</sub> (PU): BHILY: 0.6; KHNDWA: 0.89; SDSRV: 0.3; BOISR: 0.12; BDRVT: 0.1; TARAPR: 0.49; SATPR: 0.09

Table 9: Electricity nodal price (\$/MWh) with HVDC link for peak load

Bus no.	Bus name	P <sub>G</sub> (PU)	Nodal price	Bus no.	Bus name	P <sub>G</sub> (PU)	Nodal price
1	CHDPUR	1.72	23.55	10	LONKND	-	58.99
2	KORDY	0.54	30.18	11	NGOTNE	-	64.24
3	BHLSWL2	-	43.67	12	DABHOL	1.10	70.19
4	ARGBD4	-	48.01	13	KONA-N	-	73.62
5	BBLSR2	-	52.13	14	KONA-4	0.19	72.70
6	DHULE	-	50.99	15	KLHPR3	-	82.54
7	PADGE	-	58.99	16	JEJURY	-	66.07
8	KALWA	-	58.99	17	KARAD2	-	82.30
9	KARGAR	-	58.99	18	SOLPR3	-	137.86
-	-	-	-	19	PARLY2	-	11.89

Inter state power import/purchase; P<sub>G</sub> (PU): BHILY: 0.4; KHNDWA: 0.89; SDSRV: 0.27; BOISR: 0.12; BDRVT: 0.84; TARAPR: 0.29; SATPR: 0.01

Table 10: Electricity nodal price (\$/MWh) with HVDC link and generation addition

Bus no.	Bus name	P <sub>G</sub> (PU)	Nodal price	Bus no.	Bus name	P <sub>G</sub> (PU)	Nodal price
1	CHDPUR	2.00	23.78	10	LONKND	-	25.92
2	KORDY	0.64	24.30	11	NGOTNE	-	26.00
3	BHLSWL2	-	25.67	12	DABHOL	0.1	25.83
4	ARGBD4	-	26.12	13	KONA-N	-	25.77
5	BBLSR2	-	26.14	14	KONA-4	1.5	25.71
6	DHULE	-	26.19	15	KLHPR3	-	25.93
7	PADGE	-	25.98	16	JEJURY	-	25.94
8	KALWA	-	26.02	17	KARAD2	-	25.89
9	KARGAR	-	26.05	18	SOLPR3	-	25.62
-	-	-	-	19	PARLY2	-	25.02

Inter state power purchase; P<sub>G</sub> (PU): BHILY: 0.05; KHNDWA: 0.10; SDSRV: 0.01; BOISR: 0.01; BDRVT: 1.7; TRAPR: 0.05; SATPR: 0.01

reviewing the investments in transmission segment to reduced transmission loading. The present transmission infrastructure is inadequate to encourage competition in

this segment and to create wholesale electricity market. Huge investments have needed to reduce price volatility and to encourage competition.

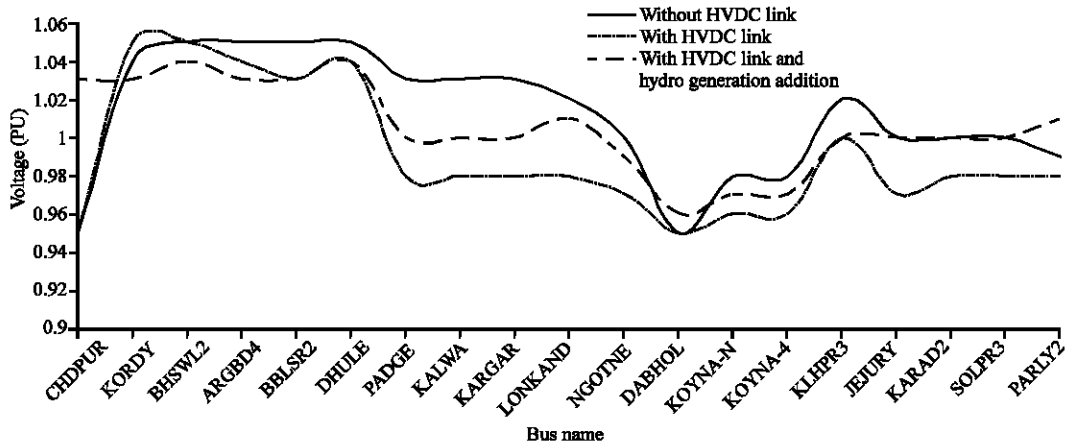


Fig. 5: Impact of addition of HVDC link and hydro generation on bus voltage behaviour

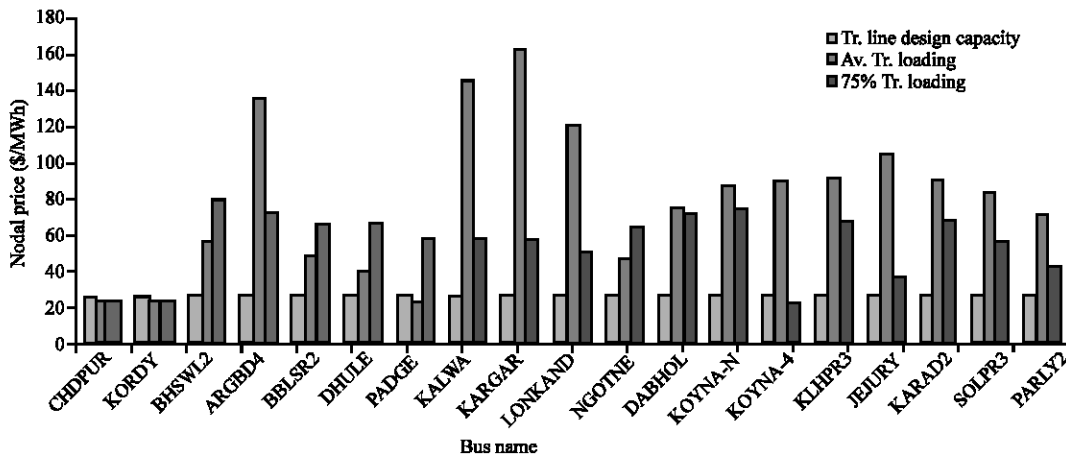


Fig. 6: Nodal price behaviour in transmission loading environment

**CONCLUSION**

In several developing countries, due to electricity transmission congestions and lack of investments further reduces the consumer benefits. To overcome these barriers and to meet consumer benefits, countries now adopting HVDC transmission in the existing AC transmission systems to gain techno-economical advantages of it. In addition, a common element of restructuring in transmission has opened for use by all eligible market participants under open access regime. In open access, economic analysis of electricity nodal prices can play a vital role to motivate investors and the utility to develop wholesale power market. This study presented basics of optimal nodal pricing based on AC-DC based OPF simulation. The suggested methodology is more suitable for developing countries due to its simplicity. It suggested importance of nodal pricing information to

system operator, regulatory commissions and utilities about transmission investments, power quality and nodal prices for development of wholesale electricity markets.

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