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Design of Discrete Controller via a Novel Model Order Reduction Technique

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Abstract: Modeling physical systems usually results in system of higher order whose order is >2. Design of controllers for the working system becomes tedious when the system order is high. It is often desirable to approximate these models by reduced order models. In this study, a computationally simple approach is proposed for order reduction of linear time invariant discrete systems. The controller is designed for the reduced order model and is connected in cascade with the original system to obtain the desired specifications. The proposed method assures the stability of the system under the reduced order model case. The validity of the proposed method is illustrated by solving few numerical examples and the results are compared with the existing techniques.

Key words: Differentiation technique, order reduction, routh-hurwitz stability, integral square error, PID controller, error index (J)

INTRODUCTION

Order reduction is a common theme within the simulation, control and optimization of complex physical processes. The mathematical models used in these computations often result in large scale systems. For example, such systems arise due to accuracy requirements on the spatial discretization of fluids or structures, in the context of lumped-circuit approximations of distributed electronic circuit elements, such as the interconnect or package of VLSI chips, or in the simulations of Micro Electromechanical Systems (MEMS), which have both electrical and mechanical components. Order reduction is often crucial to accelerate the simulation of such large scale systems.

In the advent of new and more complex technology, engineers often encounter large-scale systems, which are numerically demanding, structurally spacious and are not very practical. These large scale systems in turn create a demand for smaller, less spacious and computationally faster systems. To achieve this goal, engineers rely on model reduction technique. The problem must be approached from a realistic point of view in order to preserve the characteristic of the original system and reserve case for troubleshooting and maintenance. A large number of publications on model order reduction have been published. Chen (1974), Genesio and Milanses (1976), Elrazaz and Sinha (1981), Pal and Prasad (1990) and many others have studied extensively different methods of model reduction.

A popular approach, known as Pade approximation method for deriving reduced order models has been based on matching the time moments of original and reduced order systems represented in Shamash (1973), Baskar (1975) and Bultheel and Barel (1986). This technique has a number of useful properties, such as, computational simplicity, fitting of the initial time moments and the steady state values of the output of original and reduced order systems being the same for input of the form $\sum \alpha_i t^i$. This simple technique usually gives good results and is not computationally demanding. A well known drawback of this method, however, is that an unstable reduced order might arise from a stable model. To remedy this situation, several variants of the method have been proposed. One such technique suggested by Shoji *et al.* (1985) using a least-squares time moment fit to obtain a reduced transfer function denominator and then obtain the numerator by exact time matching. A suggestion to make the Shoji *et al.* (1985) method less sensitive to the pole distribution of the original system, was proposed by Lucas and Beat (1990), in which the linear shift point was about a general point 'a', where $a \approx (1-\alpha)$ and $-\alpha$ is the real part of the smallest magnitude pole.

A method proposed by Selvaganesan (2007) uses the generalized routh table and factor division method in combination to obtain the reduced order model. The steady state accuracy is achieved in this method multiplying the gain correction factor along with the reduced order model. The method of model order

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reduction by least squares moment matching was generalized in Lucas and Munro (1991) by including the Markov parameters in the process to cope with a wider class of transfer functions. Further, Parmer *et al.* (2008) suggested the concept of order reduction by least-square moment matching and generalized least square methods has been extended about a general point 'a' in order to have better approximations of higher order linear, time-invariant dynamic systems.

In this study, the differentiation method is used to obtain the constant term of the reduced order denominator polynomial along with the model reduction method. The characteristics of reduced order model are similar to the original higher order system when using the proposed model reduction method. The controller is designed based on the reduced order model and the same is applied to the original system. The stability of the reduced order is assured in this method.

STATEMENT OF PROBLEM

Consider an nth order linear time invariant dynamic system described in transfer matrix,

$$G(z)=\frac{N(z)}{D(z)}=\frac{a_0+a_1z+a_2z^2+........+a_{n-1}z^{n-1}}{b_0+b_1z+b_2z^2+.........+b_{n-1}z^{n-1}+b_nz^n} \quad (1)$$

$$=\frac{\sum_{k=0}^{n-1}a_kz^k}{\sum_{k=0}^{n}b_kz^k}$$

Where,

$$\alpha_k=\left[a_{ij}^k\right]_{pxq} \quad i=1,2,\cdots.p$$

$$j = 1, 2, \ldots.q$$

$$k = 0, 1, 2, \ldots n-1$$

$$\text{and } b_k = \alpha_{0,k} \ (k = 0, 1, 2, \ldots.. n-1)$$

The corresponding r^{th} ($r<n$) order reduced order is of the form

$$G_r(z)=\frac{N_k(z)}{D_k(z)}=\frac{d_0+d_1z+d_2z^2+........+d_{r-1}z^{r-1}}{e_0+e_1z+e_2z^2+.........+e_{r-1}z^{r-1}+e_rz^r} \quad (2)$$

$$=\frac{\sum_{k=0}^{r-1}d_kz^k}{\sum_{k=0}^{r}e_kz^k}$$

where,

$$d_k=\left[a_{ij}^k\right]_{pxq} \quad i=1,2,\cdots.p$$

$$j = 1, 2, \ldots.q$$

$$k = 0, 1, 2, \ldots r-1$$

$$\text{and } e_k \ (k = 0, 1, 2, \ldots.. r-1, r)$$

In this study, the reduced order model from Eq. (2) is obtained from the Eq. (1) and the reduced model retains the important characteristics of the original system and approximates its response as closely as possible for the same type of inputs.

PROPOSED METHOD OF MODEL REDUCTION

This method consists of 4 steps.

Step 1: Convert the discrete transfer function from z-domain in to s-domain by using linear transformation.

Step 2: Determination of the reduced order denominator constant term:

The reduced order denominator polynomial constant term e_0 is obtained by using the differentiation method. In this method the reduced order denominator polynomial constant term is obtained form the Eq. (3).

The general representation of k^{th} order denominator is,

$$D_k(z)=\sum_{i=1}^{k+1}b_{i-1}\frac{{}^{n-i+1}C_{n-k}}{{}^{n}C_{n-k}}z^{i-1} \quad (3)$$

Where, k = 1, 2, 3,….n-1

For k = 1,

$$D_1(z)=b_0+\frac{{}^{n-1}C_{n-1}}{{}^{n}C_{n-1}}b_1z$$

and for k = 2,

$$D_2(z)=b_0+\frac{{}^{n-1}C_{n-2}}{{}^{n}C_{n-2}}b_1z+\frac{{}^{n-2}C_{n-2}}{{}^{n}C_{n-2}}b_2z^2,$$

respectively.

The constant term b_0 value in Eq. (3) is assigned to e_0 and is used in the step 2 to obtain the reduced order model.

Step 3: Determination of reduced order transfer function by using the constant term e_0.

The nth order original system given in Eq. (1) in s-domain is equated to the rth order reduced model represented by the Eq. (2) in s-domain.

$$\frac{a_0 + a_1 s + a_2 s^2 + ... + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + ... + b_{n-1} s^{n-1} + b_n s^n} = \frac{d_0 + d_1 s + d_2 s^2 + ... + d_{r-1} s^{r-1}}{e_0 + e_1 s + e_2 s^2 + ... + e_{r-1} s^{r-1} + e_r s^r} \quad (4)$$

On cross multiplying and rearranging the Eq. (4)

$$a_0 e_0 + (a_0 e_1 + a_1 e_0)s + (a_0 e_2 + a_1 e_1 + a_2 e_0)s^2 + ...$$
$$+ a_{n-1} e_k s^{n-1+k} = b_0 d_0 + (b_0 d_1 + b_1 d_0)s + ... + b_n d_{k-1} s^{n-1+k} \quad (5)$$

By equating the coefficients of the same power of 's' on both sides in the Eq. (5), the following relations are obtained:

$$a_0 e_0 = b_0 d_0$$
$$a_0 e_1 + a_1 e_0 = b_0 d_1 + b_1 d_0$$

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$$a_0 e_{r-1} + a_1 e_{r-2} + a_2 e_{r-3} + ... = b_0 d_{r-1} + b_1 d_{r-2} + b_2 d_{r-3} + ...$$
$$a_0 e_r + a_1 e_{r-1} + a_2 e_{r-2} + ... = b_1 d_{r-1} + b_2 d_{r-2} + b_3 d_{r-3} + ...$$

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$$a_{n-1} e_r = b_n d_{r-1}$$

By solving above equations, the unknown parameters of reduced order transfer function in Eq. (2) are calculated with the e_0 value obtained in step 1.

Step 4: The reduced order transfer function in s-domain is converted in to z-domain by substituting s = z-1.

The stability of an original and reduced order models are check by using Routh-Hurwitz stability criterion method.

PID CONTROLLER DESIGN

To meet the designer's specifications, the PID controller is designed. PID controller consists of 3 types of control, Proportional, Integral and Derivative control.

The transfer function of the PID controller is defined as follows:

$$G_c(z) = K_P + K_I T\left(\frac{z}{z-1}\right) + K_D\left(\frac{z-1}{Tz}\right) \quad (6)$$

where:

K_P = The proportional gain

K_I = The integral gain

K_D = The derivative gain

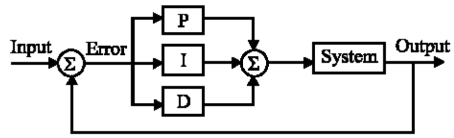

Fig. 1: PID controller in closed loop

First, let's take a look at the effect of a PID controller on the closed-loop system using the schematic above. To begin, the variable e is the tracking error or the difference between the desired reference value and the actual output. The controller takes this error signal and computes both its derivative and its integral. The signal, which is sent to the actuator is now equal to the proportional gain (K_P) times the magnitude of the error plus the integral gain (K_I) times the integral of the error plus the derivative gain (K_D) times the derivative of the error (Fig. 1).

ALGORITHM FOR THE DESIGN OF PID CONTROLLER

Step 1: Read the open loop transfer function of the given higher order system.

Step 2: Form the closed loop transfer function.

Step 3: Obtain the step response of closed loop system.

Step 4: Check the response for the required specifications.

Step 5: If the specifications are not met, get a reduced order model (by using the proposed method reduction) and design a controller for the reduced order model.

Step 6: Obtain the initial values of the parameter KP, K_I and K_D by pole-zero cancellation method.

Step 7: Cascade the controller with reduced order model and get the closed loop response with the initial values of the controller parameters.

Step 8: Find the optimum values for the controller parameters, which satisfy the required specifications.

Step 9: By applying the optimum values, cascade this controller with the original system.

Step 10: Obtain the closed loop step response of the system with the controller.

Step 11: If the specification is met, exit; else tune the parameters of the controller till it meets the required specifications.

ILLUSTRATIVE EXAMPLE

Consider the transfer function of the plant from Mukherjee and Mishra (1988) as:

$$G(z) = \frac{0.1625z^7 + 0.125z^6 - 0.0025z^5 + 0.00525z^4 - 0.02263z^3 - 0.00088z^2 + 0.003z - 0.000413}{z^8 - 0.6307z^7 - 0.4185z^6 + 0.078z^5 - 0.057z^4 + 0.1935z^3 + 0.09825z^2 - 0.0165z + 0.00225}$$

Step 1: By applying the linear transformation (z = s + 1) suggested by Shamash (1974) in the equation, we get the transfer function in s-domain as:

$$G_r(s) = \frac{0.1625s^7 + 1.263s^6 + 4.16s^5 + 7.555s^4 + 8.161s^3 + 5.225s^2 + 1.829s + 0.2693}{s^8 + 7.369s^7 + 23.17s^6 + 40.32s^5 + 41.98s^4 + 26.3s^3 + 9.595s^2 + 1.997s + 0.2493}$$

Step 2: Determination of the reduced order denominator constant term:

By using the proposed method, the reduced order denominator constant term e_0 is obtained as 0.00225 and is used in step 3.

Step 3: Equate the transfer function $G_r(s)$ with the general second order transfer function.

$$\frac{0.1625s^7 + 1.263s^6 + 4.16s^5 + 7.555s^4 + 8.161s^3 + 5.225s^2 + 1.829s + 0.2693}{s^8 + 7.369s^7 + 23.17s^6 + 40.32s^5 + 41.98s^4 + 26.3s^3 + 9.595s^2 + 1.997s + 0.2493}$$

$$= \frac{d_1 s + d_0}{e_2 s^2 + e_1 s + e_0}$$

Cross multiplying and rearranging above arrangement for the same powers of 's', the set of equations are obtained as:

$$0.1625e_2 = d_1 \tag{7}$$

$$0.1625e_1 + 1.263e_2 = d_0 + 7.369d_1 \tag{8}$$

$$1.263e_0 + 4.16e_1 + 7.555e_2 = 23.17d_0 + 40.32d_1 \tag{9}$$

$$4.16e_0 + 7.555e_1 + 8.161e_2 = 40.32d_0 + 41.98d_1 \tag{10}$$

Table 1: Error index comparison

Method of order reduction	Error index (J)
Routh-hurwitz method	0.0104
Selvaganesan (2007)	0.3397
Proposed method	0.0011

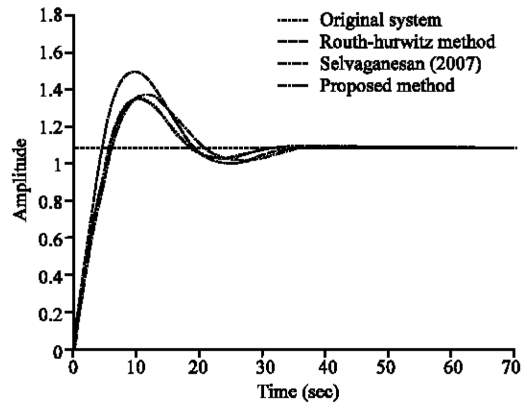

Fig. 2: Comparison of unit step responses

$$7.555e_0 + 8.161e_1 + 5.225e_2 = 41.98d_0 + 26.3d_1 \tag{11}$$

$$8.161e_0 + 5.225e_1 + 1.829e_2 = 26.3d_0 + 9.595d_1 \tag{12}$$

$$5.225e_0 + 1.829e_1 + 0.2693e_2 = 9.595d_0 + 1.997d_1 \tag{13}$$

$$1.829e_0 + 0.2693e_1 = 1.997d_0 + 0.2493d_1 \tag{14}$$

$$0.2693e_0 = 0.2493d_0 \tag{15}$$

By solving these equations with the value e_0 = 0.00225, the values of d_0, d_1, e_2 and e_1 are calculated as:

$$d_0 = 0.002431$$
$$d_1 = 0.004756$$
$$e_1 = 0.007209 \text{ and}$$
$$e_2 = 0.029270$$

$$G_r(s) = \frac{0.004756s + 0.002431}{0.02927s^2 + 0.007209s + 0.00225} \tag{16}$$

The transfer obtained in s-domain is compared with the routh-hurwitz method and the combinational method of routh-hurwitz and factor division method proposed by the Selvaganesan (2007). It is clear that accuracy of the proposed method along with the original system is comparatively more with these methods and is shown in Fig. 2 and an error index values calculated for these methods in z-domain is listed in Table 1.

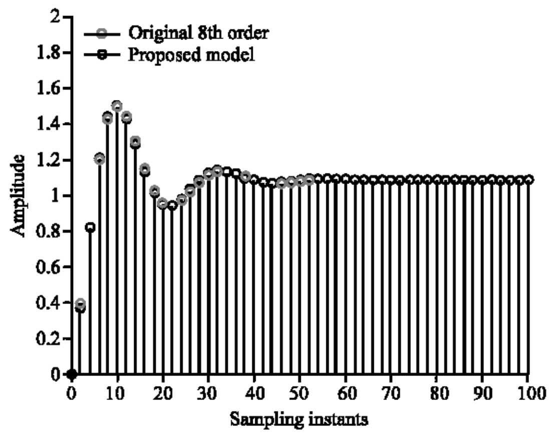

Fig. 3: Comparison of unit step responses

Furthermore, the stability of the reduced order is assured in s-domain. The reduced order $G_r(s)$ obtained is forwarded in to the next step to obtain the reduced order transfer function in z-domain.

Step 4: By applying the inverse transformation (s = z - 1) in the Eq. (16), the reduced order discrete transfer function is obtained as:

$$G_r(z) = \frac{0.004756z - 0.002325}{0.02927z^2 - 0.05133z + 0.02431} \quad (17)$$

The step response of the original and reduced order discrete time transfer functions is shown in Fig. 3.

By using the proposed method, the reduced order model most closely follows the original system response in transient and steady state conditions. The roots of the reduced order transfer function are 0.4889, 0.8768 + 0.2484i and 0.8768-0.2484i and it indicates that the reduced order system is stable and the roots are with in the unit circle.

PID CONTROLLER DESIGN

By using the proposed method, the reduced second order model is obtained and is given by,

$$G_r(z) = \frac{0.004756z - 0.002325}{0.02927z^2 - 0.05133z + 0.02431} \quad (18)$$

By using the Pole-zero cancellation technique the initial values of K_P, K_I, K_D are obtained from the reduced second order model as:

$$K_P = 0.00271,\ K_I = 0.00225,\ K_D = 0.02431$$

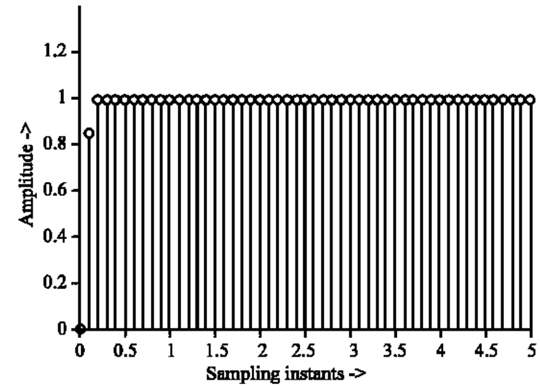

Fig. 4: Step response of reduced model with PID controller

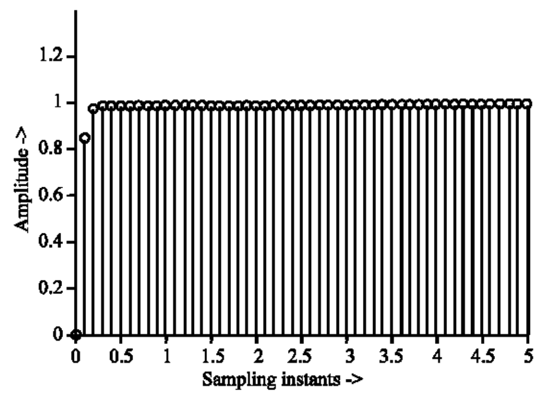

Fig. 5: Step response of original system with PID controller

The tuned values are,

$$K_P = 0.91,\ K_I = 0.0000096568,\ K_D = 0.431$$

The unit step response of original system with PID controller and reduced system with PID controller are shown in Fig. 4 and 5.

Thus, the design of PID controller using the proposed method is helps to obtain the designer's specifications in transient and as well as in steady state moments for the given original system.

CONCLUSION

The proposed model reduction method uses the differentiation technique in its procedure to derive stable reduced order models for linear time invariant dynamic systems. The algorithm has also been extended to the design of compensators and sub-optimal controllers for continuous and discrete systems. The algorithm is simple, rugged and computer oriented. The matching of step response is assured reasonably well in this method. The algorithm preserves more stability and avoids any error in between the initial or final values of the responses of original and reduced order models.

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