

## Calculations of Short-Length Permanent Magnets

Emad Said Addasi

Department of Electrical and Electronics Engineering Technology,  
Yanbu Industrial College, P.O. Box 30436, Yanbu El-Senaiya, Saudi Arabia

**Abstract:** The current study focuses on the permanent magnet calculations, particularly the cylindrical short-length (small height) permanent magnet. The used model of this permanent magnet takes into account the leakage flux. A procedure for permanent magnet calculations has been developed. Simple equations have been carried out in this study for permanent magnet calculations. These equations show that for short-length permanent magnet, unlike long cylindrical permanent magnet, the value of permeance of the useful (required) magnetic flux path and the permeance of the leakage flux path do not depend on the length of the permanent magnet, but they depend only on the magnet diameter. And for permanent magnet design these equations will be useful.

**Key words:** Short-length magnet, leakage flux, dimensions, cylindrical form, hysteresis loop

### INTRODUCTION

Permanent magnet is a piece of ferromagnetic material with high coercive magnetomotive force, which has been magnetized and is used as a source of permanent magnetic field. Permanent magnets are used in wide aspects of industrial applications. They are used in electrical machines (Osin and Shakaryan, 1990), in some measuring devices, transducers, speakers and many other applications. For evaluation of permanent magnet parameters it is useful to use demagnetization curve (the curve in the second quadrant of the hysteresis loop) as shown in Fig. 1. Permanent magnets have several important parameters such as maximum magnetic energy ( $w_{max}$ ), saturation magnetic field density ( $B_s$ ), residual magnetic field density ( $B_r$ ), coercive magnetomotive force ( $H_c$ ) and other parameters. The maximum specific magnetic energy ( $w_{max}$ ) yields when the permanent magnet gives maximum energy per unit volume as shown in Fig. 2 (Osin and Shakaryan, 1990).

Usually for permanent magnets used in industry  $B_r = (0.5-1.4) \text{ T}$ ,  $H_c = (40-800) \text{ kA m}^{-1}$ ,  $w_{max} = (10-135) \text{ kJ m}^{-3}$  (Osin and Shakaryan, 1990). Other important parameters of the permanent magnet are the form and dimensions. One of the common used forms of permanent magnets is the cylindrical form. Figure 3 illustrates cylindrical permanent magnets, which are used in electrical machines. Permanent magnets are used in many other applications (Lukaniszyn *et al.*, 2003; Mendrela *et al.*, 2003; Lukaniszyn *et al.*, 2002; Mendrela, 2000; Wrobel and Mellor, 2003). Studies of permanent magnets may be found in many studies Addasi (2008), Hawkins (1997),

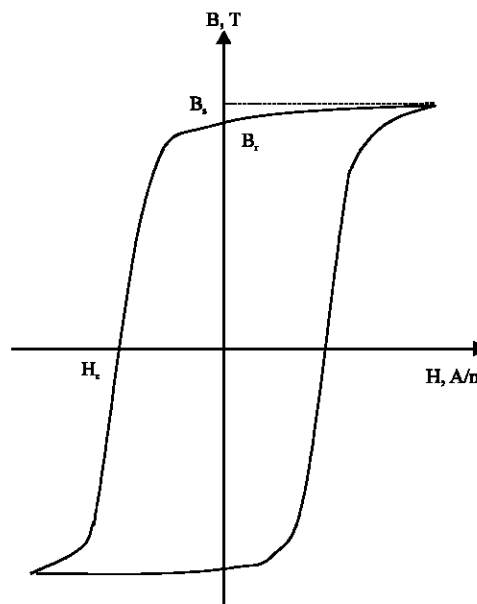


Fig. 1: The hysteresis loop of a permanent magnet

Andrew *et al.* (2003), Jagiela and Wrobed (2005), Bruskin *et al.* (1990) and Sochnyov (1970). In the study, Addasi (2008) provided derivation and calculations for determination the ratio of the length to the diameter of the permanent magnet, at which the ratio of the leakage flux to the desired flux through the magnet body is minimized. But this study considered only the cylindrical permanent magnet with height much larger than the diameter. On the other hand, many applications use short-length permanent magnets. Therefore, the results in

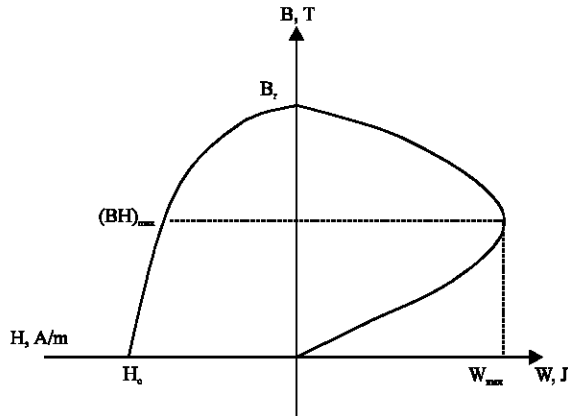


Fig. 2: Demagnetization and maximum energy curves of a permanent magnet

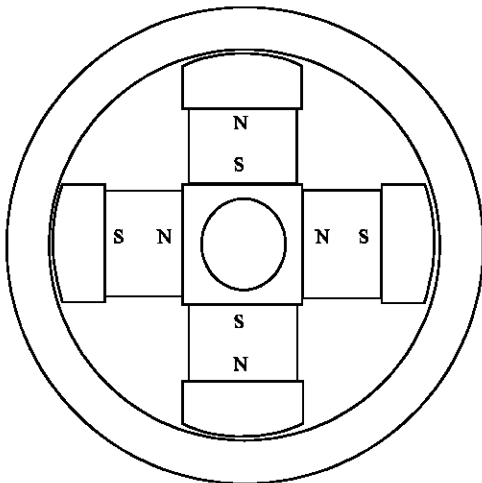


Fig. 3: The cylindrical permanent magnets used in electrical machines

Addasi (2008) are not acceptable for these cases. Therefore, short-length permanent magnets require additional study.

In the current study, the main aim is to develop simple procedure and calculations, which may be useful for design the cylindrical short-length permanent magnets.

### SHORT-LENGTH PERMANENT MAGNET

For short-length permanent magnets, as shown in Fig. 4,  $b/r < 1$ . For the assumed model of the permanent magnet the pole ends may be considered as 2 surfaces with equal magnetic potentials. The desired magnetic flux ( $\Phi_d$ ) passes through these surfaces as shown in Fig. 4. The magnetic flux through the side surface of the permanent magnet may be determined relative to the

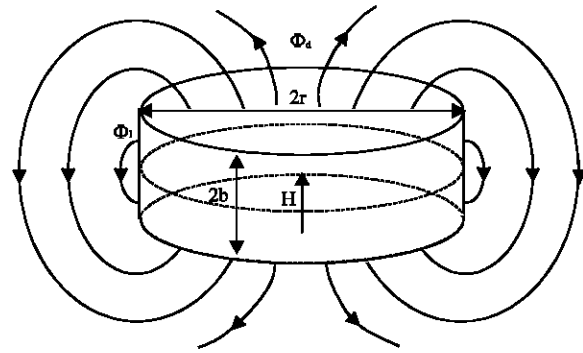


Fig. 4: A short-length cylindrical permanent magnet

desired magnetic flux. The total magnetic permeance of this permanent magnet may be calculated from the magnetic permeance of the leakage flux ( $\Phi_l$ ) from the side surface of the permanent magnet ( $\rho_{lf}$ ) and magnetic permeance of the desired magnetic flux ( $\Phi_d$ ) and the permanent magnet body permeance ( $\rho_b$ ). Using the analogy between electrical and magnetic fields (Bruskin *et al.*, 1990; Stephen, 2005) the magnetic permeance of the leakage flux may be calculated:

$$\rho_{lf} = \frac{\pi \cdot \mu_0}{2} \cdot 2 \cdot \pi \cdot r = \pi^2 \cdot \mu_0 \cdot r \quad (1)$$

The magnetic reluctance of the permanent magnet body may be found:

$$R_b = \frac{b}{\mu \cdot \pi \cdot r^2} \quad (2)$$

where:

$\mu$  = The magnetic permeance of the permanent magnet material at the point of maximum energy  $(BH)_{max}$

And the magnetic reluctance from the magnet faces:

$$R_f = \frac{1}{8 \cdot \mu_0 \cdot r} \quad (3)$$

And the magnetic reluctance of the permanent magnet, for the desired magnetic field, may be determined:

$$R_m = R_b + R_f = \frac{b}{\mu \cdot \pi \cdot r^2} + \frac{1}{8 \cdot \mu_0 \cdot r} = \frac{8 \cdot \mu_0 \cdot b \cdot \mu \cdot \pi \cdot r}{8 \cdot \mu_0 \cdot \mu \cdot \pi \cdot r^2} \quad (4)$$

Thus, the magnetic permeance:

$$\rho_m = \frac{1}{R_m} = \frac{8 \cdot \mu_0 \cdot \mu \cdot \pi \cdot r^2}{8 \cdot \mu_0 \cdot b + \mu \cdot \pi \cdot r} \quad (5)$$

Therefore, the total magnetic permeance of the studied magnetic circuit of this short-length permanent magnet may be determined:

$$\rho = \rho_{lf} + \rho_m = \pi^2 \cdot \mu_o \cdot r + \frac{8 \cdot \mu_o \cdot \mu \cdot \pi \cdot r^2}{8 \cdot \mu_o \cdot b + \mu \cdot \pi \cdot r} \quad (6)$$

And finally, the last Eq. 7 gives:

$$\rho = \mu \cdot r \left[ \pi^2 + \frac{8}{\left( \frac{8 \cdot b \cdot \mu_o}{r \cdot \mu} + 1 \right)} \right] \quad (7)$$

But for short-length permanent magnets  $b/r < 1$  and in real permanent magnets  $\mu/\mu_o \approx 25 \ll 1$  (Osin and Shakaryan, 1990; Sochnyov, 1970) thus,

$$\rho \approx \pi^2 \cdot \mu_o \cdot r + 8 \cdot \mu_o \cdot r = \mu_o \cdot r \cdot (\pi^2 + 8) \quad (8)$$

Therefore, the magnetic permeance of the permanent magnet body and the magnetic pole faces found by Eq. 5 may be simplified as following (assuming that  $b/r < 1$  and  $\mu_o/\mu \ll 1$ ):

$$\rho_m = \frac{8 \cdot \mu_o \cdot \mu \cdot \pi \cdot r^2}{\pi \cdot \mu \cdot r \left( \frac{8 \cdot \mu_o \cdot b}{\pi \cdot \rho \cdot r} \right)} \approx 8 \cdot \mu_o \cdot r \quad (9)$$

The ratio of the magnetic permeance through side surface to the magnetic permeance through magnet pole faces using the last approximated Eq. 10 will be:

$$\frac{\rho_{lf}}{\rho_m} = \frac{\pi^2 \cdot \mu_o \cdot r}{8 \cdot \mu_o \cdot r} = 1.234 \quad (10)$$

From the last Eq. 10, it is clear that for short-length permanent magnet with  $b/r < 1$  the ratio of the magnetic permeance through side surface to the magnetic permeance through magnet pole faces ( $\rho_{lf}/\rho_m$ ) does not depend on the permanent magnet dimensions.

### CONCLUSION

A cylindrical short-length permanent magnet has been studied. The permanent magnet has been modeled by an electric equivalent circuit. The used model of this permanent magnet takes into account the leakage flux. A procedure for permanent magnet calculations has been developed. Simple equations have been carried out in

this study for permanent magnet calculations. These equations show that for short-length permanent magnet, unlike for long permanent magnet, the value of permeance through the useful (desired) magnetic flux path and through the leakage magnetic flux path do not depend on the length of the permanent magnet. But they depend only on the magnet radius. And for permanent magnet design these equations will be useful.

The ratio of the magnetic permeance through side surface to the magnetic permeance through magnet pole faces does not depend on the permanent magnet dimensions for short-length permanent magnets ( $b/r < 1$ ).

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