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On Line Identification and Model Reference Adaptive Control of Dynamic Systems

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Abstract: This study describes on line, the identification process and the model reference adaptive control. The developed method uses, in a complementary manner, both parametric and no parametric identification techniques. Our algorithm permits the identification of discrete systems with an optimal search of their orders. The method performance has been tested on simulated systems of order 1, 2 and 3 with a pseudo-random binary sequence (PRBS) as an excitation input. After the identification, a model reference adaptive control is used. The performance of the developed adaptive control is tested by means of a simulated example.

Key words: Identification, discrete systems, Z transform, discrete fourier transform, model reference adaptive control, stability, positive real constraint, simple adaptive control

INTRODUCTION

Several techniques have been developed to predict the dynamic behavior of a system in search of the most representative model. Among existing methods, there are parametric and no parametric ones. In the amplitude and phase of transfer function where as in the temporal method (parametric one determines the coefficient of the transfer function). Our work is a combination of the two complementary methods. Indeed, ones the real and imaginary parts are found, they are used in the computation of the parameters of the transfer function. After the identification, an adaptive control technique is used to develop an adaptive model reference adaptive control.

The simple adaptive control approach to direct MRAC of multi-input multi-output plants was first proposed by Sobel *et al*. (1982). This approach uses a control structure which is a linear combination of feedforward of the model states and inputs and feedback of the error between plant and model outputs. This class of algorithms requires neither full state access nor satisfaction of the perfect model following conditions. Asymptotic stability is ensured provided that the plant is almost strictly positive real (ASPR). Barkana (1987) extended the original algorithm (which required the plant to satisfy the ASPR condition), to a class of plants which violates this condition. This approach involved designing a supplementary feedforward filter to be included in parallel with the original plant resulting in a new augmented plant which had to satisfy the same strictly positive real condition, unfortunately, the tracking error was not the true difference between the plant and the model outputs since it included the contribution of the supplementary feedforward filter. Thus, the approach was susceptible to a steady state error. Neat *et al*. (1992) suggested the incorporation of the feedforward filter of Barkana (1987) into the reference model's output as well as the plant's output in a manner so as to yield asymptotic tracking. Barkana (1991, 2005a) gives more studies about the ASPR condition and the convergence of the adaptive gains.

This study describes the on line identification and the adaptive control process for linear dynamic systems using a modified Z transform.

FAST FOURIER TRANSFORM (FFT)

In view of the importance of the DFT in various digital signal processing applications, such as linear filtering, correlation analysis and spectrum analysis (Doulon, 1984; Povy, 1975), its efficient computation is a topic that has received considerable attention by many mathematicians, engineers and applied scientists. From this point, Let X(k), represents the Fourier coefficients of x(n). Basically, the computational problem for the DFT is to compute the sequence {X(k)} of N complex-valued numbers given another sequence of data {x(n)} of length N, according to the formula

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$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn},\ 0 \le k \le N-1$$

$$W_N = e^{-j2\pi/N}$$

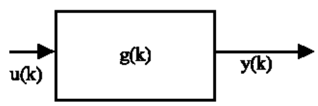
In general, the data sequence X(n) is also assumed to be complex valued. Similarly, the IDFT becomes

$$x(n) = \frac{1}{N}\sum_{n=0}^{N-1} X(k) W_N^{-nk},\ 0 \le n \le N-1$$

The DFT and IDFT involve basically the same type of computations.

DISCUTION OF THE IDENTIFICATION TECHNIQUE

Let us consider the following system which is assumed to be both discrete and linear:



Where:
u(k) : The input of system.
y(k) : The output.
g(k) : The response to a pulse signal.

The two signals u(k) and y(k) are related by the convolution product in the case of a linear time invariant system:

$$y(k) = u(k) * g(k) \tag{1}$$

One can estimate frequency-response models and visualize the responses on a Bode plot, which shows the amplitude change and the phase shift as a function of the sinusoid frequency.

For a discrete-time system sampled with a time interval T, the frequency-response model G(z) relates the Z-transforms of the input U(z) and output Y(z):

$$G(z) = y(z)/U(z) \tag{2}$$

The frequency-response command describes the steady-state response of a system to sinusoidal inputs (Ljung, 1987; Lucas and Beat, 1990). For a linear system, a sinusoidal input of a specific frequency results in an output that is also a sinusoid with the same frequency, but with a different amplitude and phase. The frequency response describes the amplitude change and phase shift as a command of frequency.

In other words, the frequency-response command, $G(e^{j\omega T})$, is the Laplace transform of the impulse response that is evaluated on the imaginary axis. The frequency-response command is the transfer command G(z) evaluated on the unit circle.

Passing from the Z transform to the discrete Fourier transform we get:

$$G(e^{j\Omega}) = \frac{DFT(y(k))}{DEF(u(k))} \tag{3}$$

Where, DFT stands for the discrete Fourier transform and $\Omega = \omega T$. On the order hand knowing that any system can be written as:

$$G(z) = \frac{b_0 + b_1 z^1 + b_2 z^2 + \ldots + b_n z^n}{z^T (a_0 + a_1 z^1 + \ldots + a_n z^n)}$$

With:
T : The system's delay.
α_0 : Generally taken to be equal to 1.

Moreover G (z) is complex and can be written as:

$$Re + j\,Im = \frac{b_0 + b_1 z^1 + b_2 z^2 + \ldots + b_n z^n}{z^T (a_0 + a_1 z^1 + \ldots + a_n z^n)} \tag{4}$$

We have

$$z = e^{j\Omega} = \cos(\Omega) + j\sin(\Omega)$$

Substituting into Eq. (4), one gets:

$$Re + j\,Im = \frac{b_0 + b_1(\cos\Omega + j\sin\Omega) + \ldots + b_n(\cos n\Omega + j\sin n\Omega)}{(\cos T\Omega + j\sin T\Omega) + a_1(\cos(1+T)\Omega + j\sin(1+T)\Omega) + \ldots}$$

$$+ \ldots + a_n(\cos(n+T)\Omega + j\sin(n+T)\Omega)$$

That means

$$\begin{aligned} & Re[\cos T\Omega + a_1\cos(1+T)\Omega + \ldots + \\ & \quad + a_n\cos(n+T)\Omega] \\ & - Im[\sin T\Omega + a_1\sin(1+T)\Omega + \ldots + \\ & \quad + a_n\sin(n+T)\Omega] \\ & = b_0 + b_1\cos\Omega + \ldots + b_n\cos n\Omega \end{aligned} \tag{5a}$$

$$\begin{aligned}&\mathrm{Re}[\sin T\Omega + a_1 \sin(1+T)\Omega + \dots + \\ &\qquad + a_n \sin(n+T)\Omega] \\ &- \mathrm{Im}[\cos T\Omega + a_1 \cos(1+T)\Omega + \dots + \\ &\qquad + a_n \cos(n+T)\Omega] \\ &= b_1 \sin\Omega + \dots + b_n \sin\Omega\end{aligned} \tag{5b}$$

$$\begin{aligned}&\mathrm{Re}\cos T\Omega - \mathrm{Im}\sin T\Omega = \\ &-a_1[\mathrm{Re}\cos(1+T)\Omega - \mathrm{Im}\sin(1+T)\Omega] + \dots + \\ &\qquad + a_n[\mathrm{Re}\cos(n+T)\Omega - \mathrm{Im}\sin(n+T)\Omega] \\ &= b_0 + b_1\cos\Omega + \dots + b_n \cos n\Omega\end{aligned} \tag{6a}$$

$$\begin{aligned}&\mathrm{Re}\sin T\Omega - \mathrm{Im}\cos T\Omega = \\ &-a_1[\mathrm{Re}\sin(1+T)\Omega - \mathrm{Im}\cos(1+T)\Omega] + \dots + \\ &\qquad a_n[\mathrm{Re}\sin(n+T)\Omega - \mathrm{Im}\cos(n+T)\Omega] \\ &= 0 + b_1\sin\Omega + \dots + b_n \sin n\Omega\end{aligned} \tag{6b}$$

To simplify let us take :

X_j = $-\mathrm{Re}\cos(j+T)\Omega + \mathrm{Im}\sin(j+T)\Omega$

Y_j = $\mathrm{Cos}(j\Omega)$

Z_i = $\mathrm{Re}_i \cos T\Omega_i - \mathrm{Im}_i \sin T\Omega_i$

Where:

i : The number of samples needed to characterise the system.

j : A counter ranging from 1 to the order of the system.

Substituting in Eq. (6a) we get:

$$-a_1X_1^i \dots - a_nX_n^i + b_0 - b_1Y_1^i \dots - b_nY_n^i = Z_i \tag{7}$$

Thus one gets the following matrix from :

$$\begin{bmatrix} Z_1 \\ . \\ . \\ . \\ . \\ . \\ Z_n \end{bmatrix} = \begin{bmatrix} -X_1^1 & \dots & -X_n^1 & 1 & -Y_1^1 & \dots & -Y_n^1 \\ X_1^2 & \dots & -X_n^2 & 1 & -Y_1^2 & & -Y_n^2 \\ . & \dots & & & & & . \\ . & \dots & & & & & . \\ . & \dots & & & & & . \\ . & \dots & \dots & \dots & \dots & \dots & . \\ -X_1^N & & -X_n^N & 1 & -Y_1^N & & -Y_n^N \end{bmatrix} \begin{bmatrix} a_1 \\ . \\ a_n \\ b_0 \\ b_1 \\ . \\ b_n \end{bmatrix} \tag{8}$$

If the delay T is known then the system (8) is written as:

$$B = A \times \theta \tag{9}$$

When θ is an unknown vector, *A* is rectangular matrix and B is a vector of measurement.

The optimal solution of Eq. (9.a) is given by :

$$\theta = (A^{Tr}A)^{-1}A^{Tr}B \tag{10}$$

Which is the method of least squares applied to our system, where, Tr : Is the transpose of the matrix.

OPTIMAL SEARCH OF ORDER

The search of system's order (Lucas and Beat, 1990) requires the use of very complex methods. In our case we choose the following approach: from a linear system, part of data. We compute the parameters of transfer function for each measured interval with an order varying from 1 to 5 and then we validate the model in another interval. For each order, the variance of all the parameters is computed. The optimal order is then obtained from the minimal variance.

DISCUTION OF THE ADAPTIVE CONTROL TECHNIQUE

The linear time invariant model reference adaptive control is considered for the plant

$$\begin{aligned}\dot{x}_p(t) &= A_p x_p(t) + B_p(t)u_p(t) \\ y_p(t) &= C_p x_p(t)\end{aligned} \tag{11}$$

where, $x_p(t)$ is the $(n \times 1)$ state vector, $u_p(t)$ is the $(m \times 1)$ control vector, $y_p(t)$ is the $(q \times 1)$ plant output vector and A_p and B_p are matrices with appropriate dimensions. The range of the plant parameters is assumed to be known and bounded with

$$\underline{a}_{ij} \le a_p(i,j) \le \bar{a}_{ij},\ i,\ j = 1,\dots,n \tag{12}$$

$$\underline{b}_{ij} \le b_p(i,j) \le \bar{b}_{ij},\ i,\ j = 1,\dots,n \tag{13}$$

The objective is to find, without explicit knowledge of A_p, B_p, the control $u_p(t)$ such that the plant output vector $y_p(t)$ follows the reference model

$$\begin{aligned}\dot{x}_m(t) &= A_m x_m(t) + B_m(t)u_m \\ y_m(t) &= C_m x_m(t)\end{aligned} \tag{14}$$

The output y_m is the desired response to the set point command u_m. The model incorporates the desired behavior of the plant, but its choice is not restricted. In particular, the order of the plant may be much larger than the order of the reference model.

Then the adaptive control law basen the command generator tracker (CGT) approach is given as (Kaufman *et al.*, 1998):

$$u_p(t) = K_e(t).e_y(t) + K_x(t).x_m(t) + K_u(t).u_m(t) \quad (15)$$

where,

$e_y(t) = y_m(t) - y_p(t)$ and $K_e(t)$, $K_x(t)$ and $K_u(t)$

are adaptive gains and concatenated into the matrix K(t) as:

$$K(t) = \left[K_e(t) \;\; K_x(t) \;\; K_u(t)\right] \quad (16)$$

Defining the vector r(t) as:

$$r(t) = \begin{bmatrix} y_m(t) - y_p(t) \\ x_m(t) \\ u_{m(t)} \end{bmatrix} \quad (17)$$

The control $u_p(t)$ is written in a compact form as:

$$u_p(t) = K(t).r(t) \quad (18)$$

where,

$$K(t) = K_p(t) + K_i(t) \quad (19)$$

$$K_p(t) = \left[y_m(t) - y_p(t)\right].r^T(t)T_p,\; T_p \geq 0 \quad (20)$$

$$K_i(t) = \left[y_m(t) - y_p(t)\right].r^T(t)T_i,\quad T_i \succ 0 \quad (21)$$

The sufficiency conditions for asymptotic tracking are:

- There exists a solution to the CGT problem .
- The plant is almost strictly positive real (ASPR); that is there exists a positive definite constant gain matrix, not needed for implementation, such that the closed loop transfer function

$$G(s) = \left[I + G_p(s)\tilde{K}_e\right]^{-1} G_p(s) \quad (22)$$

is strictly positive real (SPR), (Povy, 1975).

The discretization of the dynamic system is given by the following equation which is called generalized method:

$$S = \frac{Z-1}{\alpha TZ + (1-\alpha)T} \quad (23)$$

where $0 \leq \alpha \leq 1$ and T is the sampling interval.

This method can generalize many others methods. Among them we cite:

- The backward transformation (Doulon, 1984; Ljung, 1987): obtained by $\alpha = 0$ and replacing in Eq. (23) by the following expression:

$$S = \frac{Z-1}{T}$$

- The bilinear transformation (Povy, 1975): obtained by $\alpha = 1/2$ and given by the following expression:

$$S = \frac{2(Z-1)}{T(Z+1)}$$

- Te forward transformation (Doulon, 1984), obtained by $\alpha = 1$

$$S = \frac{Z-1}{TZ}$$

The generalized method is an approximate discrete transfer function H(S). This method presented by this note gives the equivalent transfer function simply by submitting the Eq. (23) for S into the know H(S), the result is the desired H(Z).

The required calculus by this method can't be solved by the hand. Nest, we present a computer algorithm for accomplishing the transformation when the transfer function is written as the ratio of two polynomials.

SIMULATION RESULTS

The simulated systems are, respectively of order 1, 2 and 3. The excitation is a pseudo random binary sequence (PRBS) of length 127 and amplitude [-1, 1]. In order to test the method much further, we have injected a Gaussian noise of a standard deviation σ_b equals to 0, 1 and 2.

Example 1: Consider the first order system given by the transfer function

Table 1a: Real and estimated values of the denominator

	n	α_0	α_1
Real system	1	1	-2
Model with $\sigma_b = 0$	1	1	-2.00
Model with $\sigma_b = 1$	1	1	-1.7560
Model with $\sigma_b = 2$	1	1	-1.2568

Table 1b: Real and estimated values of the nominator

	n	b_0	b_1
Real system	1	12	-4
Model with $\sigma_b = 0$	1	12.00	-4.00
Model with $\sigma_b = 1$	1	-10.8007	-3.3975
Model with $\sigma_b = 2$	1	8.3076	-2.2332

Table 2a: Real and estimated values of the denominator

	n	α_0	α_1	α_2
Real system	2	1	-3.4686	2.8571
Model with $\sigma_b = 0$	2	1	-3.4286	2.8571
Model with $\sigma_b = 1$	2	1	-3.3841	2.8067
Model with $\sigma_b = 2$	2	1	-3.0262	2.4279

Table 2b: Real and estimated values of the nominator

	n	b_0	b_1	b_2
Real system	2	68.5714	-40.000	5.7143
Model with $\sigma_b = 0$	2	68.5714	-40.000	5.7143
Model with $\sigma_b = 1$	2	67.3610	-39.130	5.8163
Model with $\sigma_b = 2$	2	53.9363	-34.736	5.9760

$$G(z) = \frac{-6+2z}{-0.5+z} = \frac{12-4z}{1-2z} = \frac{b_0+b_1z}{1+a_1z}$$

Which give the following Table 1 of results, where *n* and σ_b are, respectively the order and the standard deviation.

Example 2: Consider the second order simulated system given by the transfer function:

$$G(z) = \frac{24-14z+2z^2}{0.35-1.2z+z^2} = \frac{b_0+b_1z+b_2z^2}{1+a_1z+a_2z^2}$$

Which give the following Table 2 of results, where n and σ_b are, respectively the order and the standard deviation.

Example 3: Consider the third order simulated system given by the transfer function Table 3:

$$G(z) = \frac{-120+94z-21z^2+2z^3}{-0.21+1.07z-1.8z^2+z^3} = \frac{b_0+b_1z+b_2z^2+b_3z^3}{1+a_1z+a_2z^2+a_3z^3}$$

Example 4: In this study, we try to plot the response of the system and the model of example 3. Figure 1 shows the time response of both the system and the model with $\sigma_b = 2$. Figure 2 and 3 show the frequency response, amplitude and phase. We can see the identification is absolutely well performed.

Table 3a: Real and estimated values of the denominator

	n	α_0	α_1	α_2	α_3
Real system	3	1	-5.0952	8.5714	-4.7619
Model with $\sigma_b = 0$	3	1	-5.0952	8.5714	-4.7619
Model with $\sigma_b = 1$	3	1	-5.0599	8.5165	-4.7294
Model with $\sigma_b = 2$	3	1	-5.0546	8.4869	-4.7140

Table 3b: Real and estimated values of the nominator

	n	b_0	b_1	b_2	b_3
Real system	3	571.4280	-447.6189	114.2866	-9.5238
Model with $\sigma_b = 0$	3	571.4280	-447.6189	114.2866	-9.5238
Model with $\sigma_b = 1$	3	567.4156	-444.1917	113.1358	-9.1779
Model with $\sigma_b = 2$	3	566.1417	-443.4795	114.2427	-10.7569

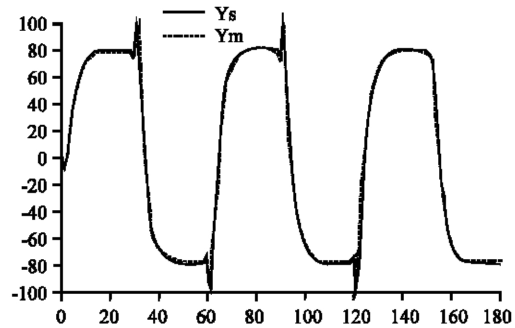


Fig. 1: Response of the system and model

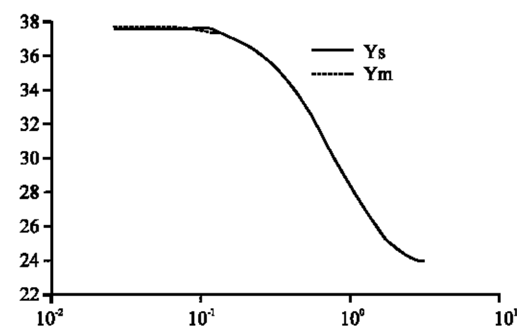


Fig. 2: Amplitude of the system and model

Example 5: In this example we try the show the performance of the developed model reference adaptive algorithm. The dynamic system is given by:

$$y = \frac{1}{s^2+s+1}u_p$$

Which has been identified in the example two in the discrete case.

The reference model de is given by:

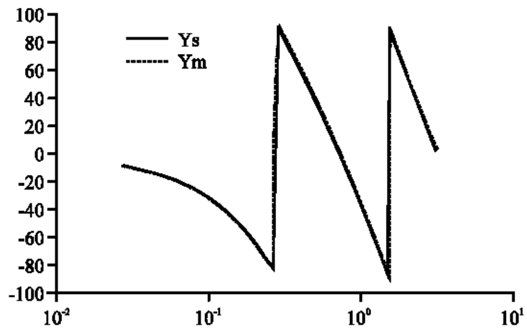


Fig. 3: Phase of the system and model

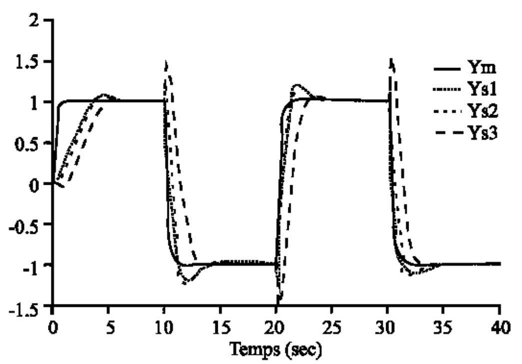


Fig. 4: Outputs of the system and the model for $\alpha = \{0, 1, 5\}$

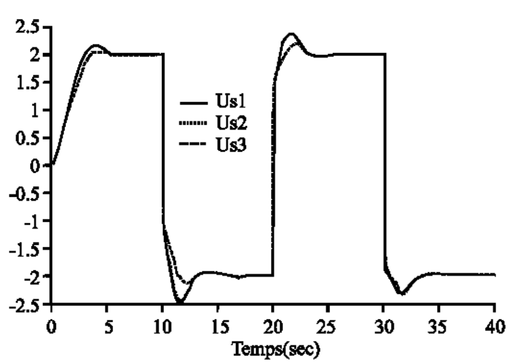


Fig. 5: Input to the system model for $\alpha = \{0, 1, 5\}$

$$y_m = \frac{3}{s+3} u_m$$

The model input is a square wave of amplitude $u_m = \pm 1$ and a period of 20 sec.

For different values of α we have the following responses.

Figure 4 shows the response of the system and the model. We see that the error goes to zero after a few seconds. The input signal is depicted in the Fig. 5, which is smoothing and has a limited value.

CONCLUSION

The simulation performed on systems of order 1, 2 and 3 shows that the identification has been realized in a satisfactory manner regarding to the negligible temporal and frequentiel errors. The developed algorithm permits the identification of discrete systems of an arbitrary order with an optimal search of this order. The efficiency of the method has been tested on noise systems of order 1, 2 and 3. The performance of this method is much better if the noise variance is low.

After the process of identification, a model reference adaptive control is developed. The method's performance has been tested on simulated systems of a second order.

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