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Effect of the Collisional Frequency Variations on the Transmitted Power in Argon Plasma

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Abstract: The investigation of the variations of the electromagnetic power transmitted in a WR 340 waveguide partially filled with argon plasma over a range of frequencies is carried out. Unlike previous approaches where the permittivity of the plasma is either considered constant, or evaluated at a fixed frequency or where the collision frequency is completely neglected, our analysis takes into account the variations of this parameter in determining the power absorbed by the plasma. Results show that at frequencies close to the plasma frequency the amount of power delivered to the plasma is minimal due to maximal attenuation.

Key words: Microwave power, collisional plasma, complex permittivity, wave complex constant, skin depth, rectangular waveguide

INTRODUCTION

Material processing is rapidly growing because of its numerous applications and the advantage of using microwave energy (Oktay and Akman, 2003). Besides their main usage as a guiding wave apparatus, waveguides are widely used in electronic structures. When filled or partially filled with an ionized gas, they can be used for electromagnetic material characterization to determine the complex permittivity of nobles gases (Motta *et al.*, 2001; Jose *et al.*, 2003; Zulkifly *et al.*, 2001; Santra and Limaye, 2005) and produce and sustain a plasma (MIP: Microwave-Induced Plasma) (Karyn *et al.*, 2001). These structures can be used in a number of applications: deposition, cutting, welding and cleaning (waste treatment) (Mostofa *et al.*, 2005). Since all the power transmitted by a source is guided within the waveguide, the losses are small and the efficiency becomes important. The structure investigated in this study was the subject of many publications (Oktay and Akman, 2003; Motta *et al.*, 2001; Jose *et al.*, 2003; Zulkifly *et al.*, 2001; Santra and Limaye, 2005; Karyn *et al.*, 2001; Mostofa *et al.*, 2005; Phadungsak *et al.*, 2002 a, b; Giovanni, 2005).

In this research the power transmitted at frequencies close to the plasma frequency and collision rate is investigated through the determination of the wave complex constant. The dispersion equation is then used to determine the range of frequencies enabling the propagation of only the fundamental mode TE_{10}. The power delivered to the collisional plasma is then determined for different collision frequencies.

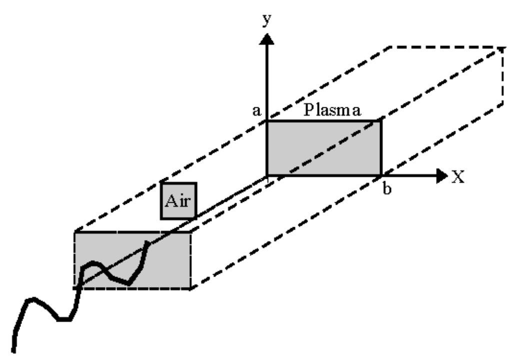

Fig. 1: Rectangular waveguide partially filled with argon plasma

PROBLEM DESCRIPTION

A microwave is transmitted into a WR 340 waveguide (Fig. 1) partially filled with an argon plasma. The power delivered by the source travels in the part filled with air with no attenuation and the incident wave is damped in the collisional plasma. The electromagnetic wave ($\vec{E}, \vec{B}$) propagates in mode (m, n) depending on the frequency of the the medium permittivity according to the wave dispersion equation (Perez *et al.*, 1997).

$$K^2 = \frac{\omega^2}{c^2}\varepsilon_{rc} - \left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{a}\right)^2 \qquad (1)$$

where • = 2• f is the wave angular frequency, c the speed of light, •. the complex permittivity of the medium, (m, n) the wave mode and K the wave constant.

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In non-magnetic mediums and using the usual $e^{-i\bullet t}$ time dependence in harmonic propagation, the Maxwell's Equations can be reduced to:

$$\mathrm{div}\vec{E} = 0 \qquad (2)$$

$$\mathrm{div}\vec{B} = 0 \qquad (3)$$

$$\overrightarrow{\mathrm{Rot}}\,\vec{E} = i\omega\vec{B} \qquad (4)$$

$$\overrightarrow{\mathrm{Rot}}\,\vec{B} = -i\mu_0\varepsilon_0\omega\left(\varepsilon_r + i\frac{\sigma}{\omega\varepsilon_0}\right)\vec{E} \qquad (5)$$

Where μ_0 and $\bullet_0$ are, respectively the magnetic permeability and the dielectric constant of vacuum and $\bullet_r$ and • are the relative permittivity and the conductivity of the given medium.

Wave propagation in the volume of the waveguide filled with air: In the volume filled with air, • = 0 and $\bullet_r$ is considered close to unity. The wave equation for $\vec{E}$ is then:

$$\Delta\vec{E} + \frac{\omega^2}{c^2}\varepsilon_r\vec{E} = 0 \qquad (6)$$

where

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

is the speed of light in vacuum.
The wave constant K is such that:

$$K^2 = \frac{\varepsilon_r}{c^2}\left(\omega^2 - \omega_{cmn}^2\right) \qquad (7)$$

Where

$$\omega_{cmn} = \frac{c}{\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{a}\right)^2}$$

is the cut off angular frequency of the waveguide. Equation (7) shows that, when $\bullet_r$ is real positive (which is the case for all materials except plasmas), the wave constant is then: real positive if $\bullet > \bullet_{cmn}$ or pure imaginary K = i |K| if $\bullet > \bullet_{cmn}$. For the first case: $\vec{E} = \vec{E}_0(x,y)e^{i(Kz-\omega t)}$ and the wave travels in the z direction without damping. In the second case: $\vec{E} = \vec{E}_0(x,y)e^{-|K|z}e^{-i\omega t}$. The wave does not propagate in the waveguide (evanescent wave).

Wave propagation in the plasma: In this medium $\bullet_{re}$ and • are complex quantities given, respectively by:

$$\varepsilon_{rc} = \varepsilon_r + i\varepsilon''_r \qquad (8)$$

And:

$$\sigma = \sigma' + i.\sigma'' \qquad (9)$$

The expressions for • and $\bullet_{re}$ are deduced using the cold plasma, one fluid Drude Model (Perez *et al.*, 1997) for a collisional plasma with a collision frequency and a plasma frequency $\bullet_p$ such as:

$$\sigma = \sigma_0\left(\frac{\nu^2}{\omega^2+\nu^2} + i\frac{\omega\nu}{\omega^2+\nu^2}\right) \qquad (10)$$

$$\varepsilon_{rc} = 1 - \frac{\omega_p^2}{\omega^2+\nu^2} + i\frac{\nu}{\omega}\frac{\omega_p^2}{\omega^2+\nu^2} \qquad (11)$$

Where $\bullet_0$ is the DC conductivity of the plasma.

Typical density n and collision frequency • parameters used for argon plasma, as reported in (Mostofa *et al.*, 2005) fall in the ranges of: $3\times10^{16}<n<2.3\times10^{17}\,m^{-3}$ and $6.23\times10^{9}<\bullet<14.4\times10^{11}$ rd $s^{\bullet1}$. In our analysis we selected n = 7×10^{16} m^{-3} and • = 12×10^{9} rd $s^{\bullet1}$ which are consistent with the published results. The calculated plasma frequency $\bullet_p$ is thus equal to 15×10^{9} rd $s^{\bullet1}$.

The plot of the complex permittivity (Fig. 2) shows that for angular frequencies less than 9×10^{9}(rd $s^{\bullet1}$), the real permittivity becomes negative and the material acquires a high imaginary part. In the plasma, the wave constant is complex such that:

$$K = K' + i.K'' \qquad (12)$$

where K' is the phase constant and K'' is the attenuation constant. The two parameters are computed as:

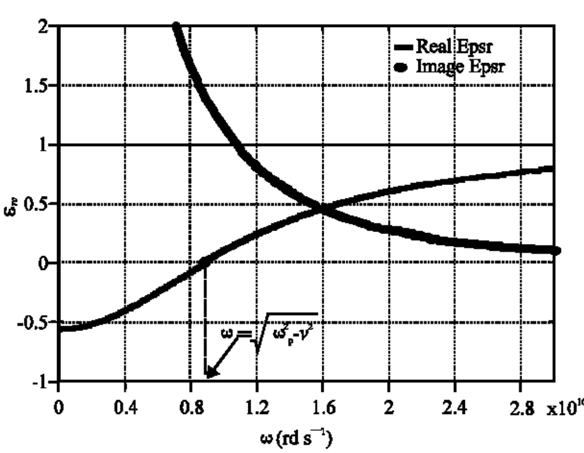

Fig. 2: Permittivity versus angular frequency

$$K'' = \frac{1}{\sqrt{2}}\left(\sqrt{X^2+Y^2} - X\right)^{1/2}$$
$$K' = \frac{Y}{2K''} \text{ with } K'' \neq 0 \qquad (13)$$

where:

$$X = \frac{\omega^2}{c^2}\varepsilon_r' - \left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{a}\right)^2$$
$$Y = \frac{\omega^2}{c^2}\varepsilon_r'' \qquad (14)$$

The case K" = 0 for which ε_r" = 0, describes a collisionless plasma having a real permittivity:

$$\varepsilon_{rc} = \varepsilon_r' = 1 - \frac{\omega_p^2}{\omega^2} \qquad (15)$$

For $\omega > \omega_p$, ε_{rc} is positive and the wave propagates without damping. This result explains the transparence of metals to ultra-violet and X-rays. It shows also the possibility to communicate via satellites, through the ionosphere, by using frequencies of few MHz since the ionosphere plasma frequency is of the order of 10^6 rd/s (Perez *et al*., 1997).

If $\omega < \omega_p$, then $\varepsilon_r' < 0$ (left handed material) and the wave does not propagate in the plasma. It is totally reflected.

For the general $K' \neq 0$ case and $K'' \neq 0$ the TE_{10} wave travels within the plasma in an attenuated form and the electric field is then given by:

$$\vec{E} = E\sin\left(\frac{\pi}{b}x\right)e^{-K''z}e^{-i\left(K'z-\omega t\right)}\vec{j} \qquad (16)$$

E is the magnitude of the incident wave which depends on the input power transmitted to the plasma. The wave is damped by a factor

$$\delta = \frac{1}{K''}$$

which describes the skin depth in the plasma. Figure 3 shows that the attenuation constant approaches zero as the frequency is increased. The wave will propagate in the plasma without damping with a large wave number.

NUMERICAL METHOD ANALYSIS

Applying the weak Galerkin formulation to Eq. 6, then evaluating the resulting integrals by parts over the whole problem domain (Ω) and finally substituting the appropriate boundary conditions as shown in Fig. 4, we obtain a partial differential equation

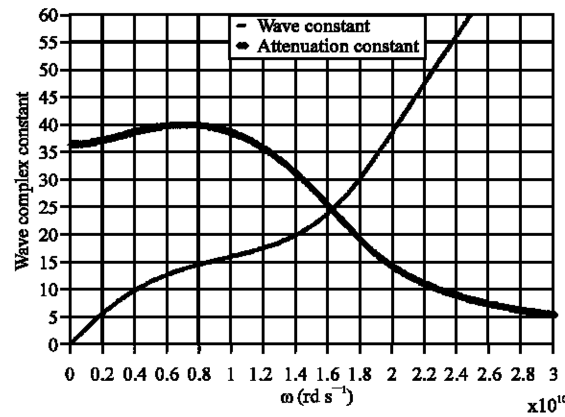

Fig. 3: Attenuation coefficient and wave constant versus angular frequency

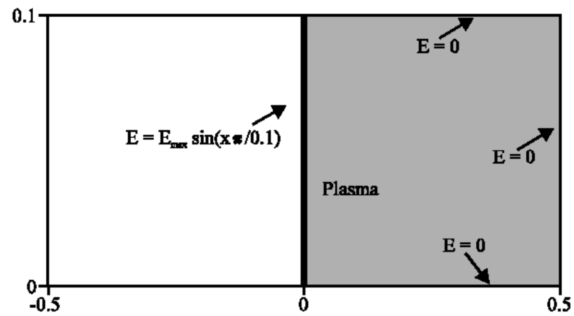

Fig. 4: Boundary conditions

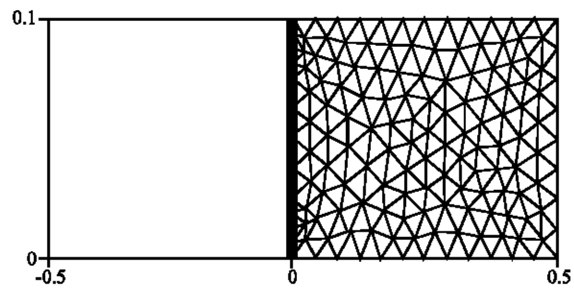

Fig 5: Mesh of the domain with triangular elements

of the form:

$$\left([C]+[M]\right)[E] = 0 \qquad (17)$$

with:

$$[C] = \int_\Omega \frac{\omega^2}{c^2}\varepsilon_r\,\psi_i\,\psi_j\,d\Omega$$

$$[M] = \int_\Omega \overrightarrow{grad}(\psi_i)\overrightarrow{grad}(\psi_j)\,d\Omega$$

The code generated is based on an unstructured mesh-generation. For accuracy in application of boundary conditions, triangular type mesh is utilized. The mesh of the domain is given in the Fig. 5.

Figures 6-8 shows a simulation of the electrical field, inside the plasma, respectively at an angular frequency of 25×10^9, 27×10^9 and 30×10^9rd s•^{1}, for $0 \leq x \leq 0.1$ m and 0• z • 0.5m. The attenuation coefficient is greatly reduced as the angular frequency is increased.

Power analysis: The power transmitted in the waveguide to the plasma, through its normal section (a,b), can be determined using the Poynting Theorem (Perez *et al.*, 1997):

$$P = \underset{(S)}{} \vec{P}..\vec{dS} \quad (18)$$

where $\vec{P}$ is the Poynting vector.

The energy of the wave is absorbed by the electrons near the interface, where the electric component of the wave is strong. Hence, the electrons are being heated here and transfer the energy into the plasma bulk by diffusion (Annemie *et al.*, 2002; Rusanov and Yakovenko, 2004).

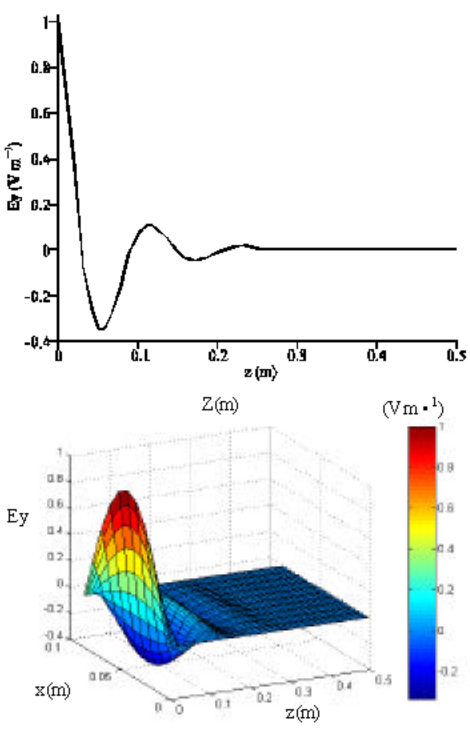

Fig. 6: *TE*10 Electrical wave in plasma 25×10^9 rd s•^{1}

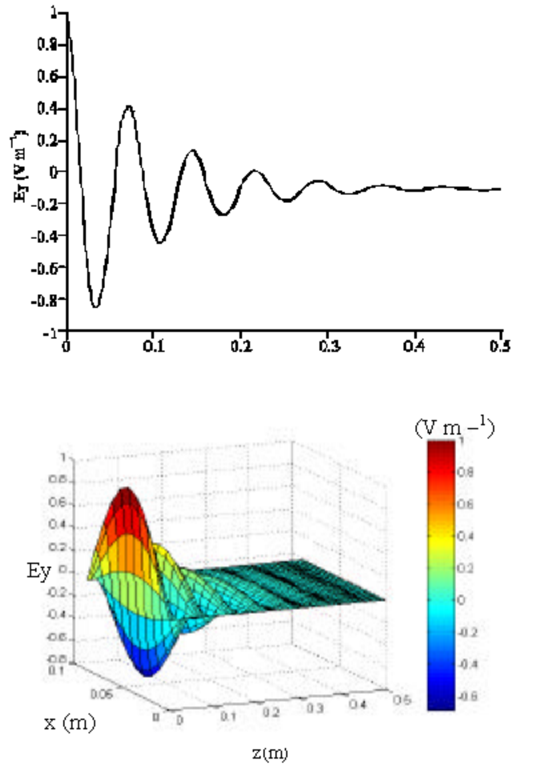

Fig. 7: TE_{10} Electrical wave in plasma 27×10^9 rd s•^{1}

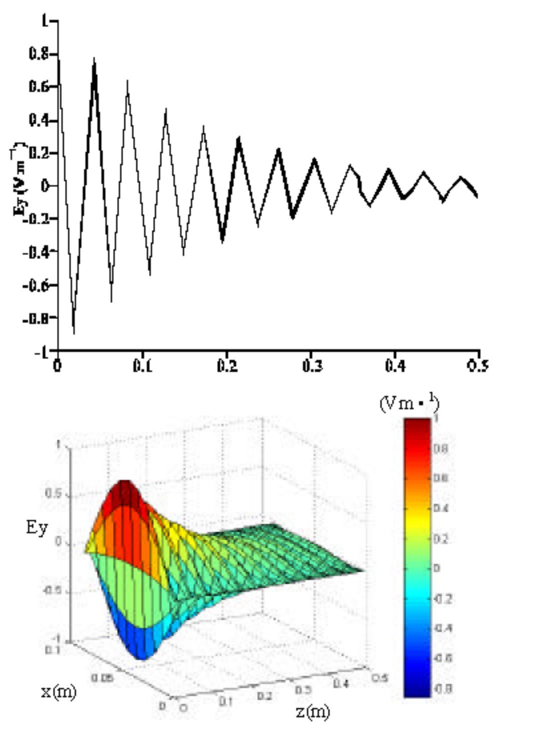

Fig. 8: TE_{10} Electrical wave in plasma 30×10^9 rd s•^{1}

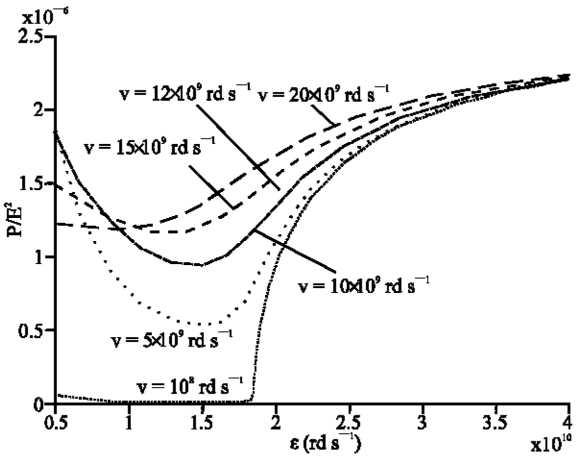

Fig. 9: Maximum power versus angular frequency

The mean value of the transmitted power inside the plasma decays by a factor of $2K''$ and its maximum is obtained for z = 0 as:

$$\langle P(0)\rangle = \frac{1}{4}c^2 a.b.\varepsilon_0 E^2.(\frac{K'}{\omega}) \tag{19}$$

Figure 9 shows that the maximum mean value power transmitted to the plasma reaches its minimum at the plasma frequency. For • > $\bullet_p$, the maximum power is quasi linearly dependant on the angular frequency.

DISCUSSION

The power analysis of the transmitted energy to a collisionnal plasma confined in a rectangular waveguide is carried out. The results show that, for an optimal usage of the waveguide, it is necessary to avoid the frequency band $\bullet_p^2 - \bullet^2 < \bullet^2 < \bullet_p^2$ where the attenuation constant reaches its maximum value and thus the power transmitted to the plasma reaches its minimum.

As the collision frequency is varied for different temperatures of the plasma, the power transmitted to the plasma increases proportionally to the plasma collision frequency for a fixed plasma density.

For an even higher increase in the angular frequency such that • >> $\bullet_p$, the results show that $0 < \bullet_r < 1$ and the plasma behaves as vacuum where the wave has no attenuation and all the energy is transmitted to the medium.

CONCLUSION

In this study, the power transmitted to argon plasma is studied for different collision frequencies. The analysis takes into account the variations of the complex permittivity of argon with frequency in the case where the three parameters $\bullet_p$, • and • have close values.

The analytical and numerical results show that the power absorbed by the plasma reaches its minimum for frequencies less than $\bullet_p$ where the wave is highly attenuated.

The main assumption is that the density of the plasma remains constant and do not change axially. This is not the case since the power absorption decays rapidly along the axis of the waveguide and suggests that the plasma parameters be also dependant on the z dimension. As hypothesized by (Motta *et al.*, 2001), the results remain valid when the thickness of the plasma is of the same magnitude as the skin depth.

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