

Computation of Electromagnetic Problem of a Linear Induction MHD Pump by the Finite Volume Method

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Abstract: This study presents a finite volume method to investigate the electromagnetic problem in the linear induction pump taking into account the movement of the liquid metal. The study focuses on the analysis of eddy currents and the electromagnetic thrust according to the frequency variations.

Key words: Magnetohydrodynamics (MHD), channel, electromagnetic problem, Finite Volume Method (F.V.M), eddy currents, electromagnetic thrust

INTRODUCTION

The problem of magnetohydrodynamic flow through channels has become important because of several engineering applications such as design of nuclear reactor coolings systems, astronomy and geophysics electromagnetic pumps, MHD flowmeters, MHD generators, blood measurements, etc. (Berton, 1991; Verardi *et al.*, 1998).

The interaction of moving conducting fluids with electric and magnetic fields provides for a rich variety of phenomena associated with electro-fluid-mechanical energy conversion. Effects from such interactions can be observed in liquids, gases, The pumping of liquid metal may use an electromagnetic device, which induces eddy currents in the metal. These induced currents and their associated magnetic field generate the Lorentz force whose effect can be actually the pumping of the liquid metal (Vinsard *et al.*, 1998). They have the advantage of no contact with the liquid metal which is at high temperature. Linear induction MHD pumps are electromagnetic devices using the principle of induction motors to move liquid metal by the action of a sliding field (Takorabet, 2006).

The finite element method was used in many electromagnetic fields computational (Vinsard *et al.*, 1998; Takorabet, 2006; Kadid *et al.*, 2003, 2004) and it excels in handling complex geometry. The finite volume method is widely applied to compute the Navier Stokes equations. The discretization of the partial differential Equations is done from a conservative form for each control volume by a technique which resembles to the finite differences method. Therefore, the principle of conservation is imposed over each volume of control contrary to the finite element method where the principle of conservation is

checked overall. This shows a limitation of this technique. Moreover, this method is simpler to develop and less expensive than the finite element method.

In the present research, a modelisation of the electromagnetic phenomena is analysed by the finite volume method in a transient state. The distributions of magnetic induction field, eddy currents and electromagnetic thrust in a mercury liquid metal have been carried out for induction pump and sinusoidal supplied currents. The calculation of these parameters is necessary for the study of the hydrodynamic phenomena.

MATHEMATICAL MODEL

A schematic view of the pump is shown in Fig. 1. It consists on 2 inductors, the air gap and the external area while the channel contains the liquid metal.

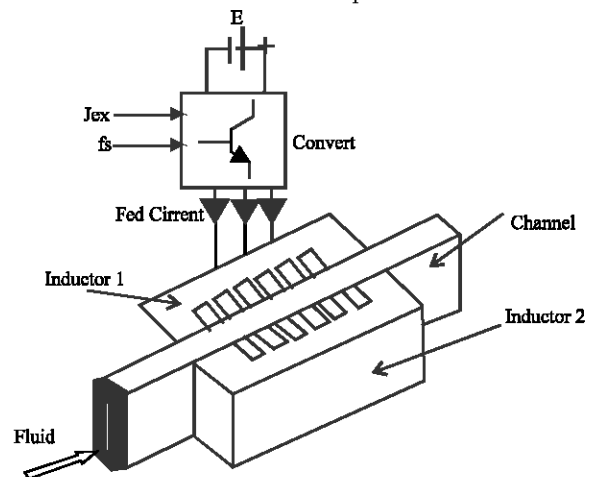


Fig. 1: The prototype of the NHD Pump

A three balanced system of currents supplies the windings.

$$J_1 = J_0 \sin(\omega t) \quad (1)$$

$$J_2 = J_0 \sin\left(\omega t - \frac{2\pi}{3}\right) \quad (2)$$

$$J_3 = J_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \quad (3)$$

The currents of the windings generate the travelling magnetic field which produce a current in the liquid metal. As a consequence a Laplace thrust acting on the fluid is obtained.

The equations describing the pumping process taking into account the eddy currents in the channel are the Maxwell equations and Ohm's law (Anderson, 1984; Krzeminski *et al.*, 2000; Sadiku, 1992; Jin, 1993).

$$\text{div } D = 0 \quad (4)$$

$$\text{rot } E = -\frac{\partial B}{\partial t} \quad (5)$$

$$\text{div } B = 0 \quad (6)$$

$$\text{rot } H = J \quad (7)$$

$$J_i = \sigma(E + V \times B) \quad (8)$$

$$B = \mu H \quad (9)$$

$$D = \epsilon E \quad (10)$$

The current density vector c is up of by 2 components $J = J_{ex} + J_i$, where j_i is the eddy current density flowing in the fluid and J_{ex} is the current density in the windings. In (8) σ is the electrical conductivity and V is the velocity of the fluid. The other symbols are conventional.

For the calculation reported in the following, mercury is taken as liquid metal. The Maxwell's equations applied to a pump are characterised:

$$\text{rot}\left(\frac{1}{\mu}\text{rot}A\right) + \sigma\left(\frac{\partial A}{\partial t} - V \times \text{rot}A\right) = J_{ex} \quad (11)$$

where A is the magnetic vector potential. The eddy currents are computed by:

$$J_i = -\sigma\left(\frac{\partial A}{\partial t} - V \times \text{rot}A\right) \quad (12)$$

The thrust are given by :

$$F = J_i \times \text{rot}A \quad (13)$$

NUMERICAL METHOD

The differential Eq. 11, developed on cartesian coordinates, is integrated over each control volume delimited by the borders (E, W, N, S) as shown in Fig. 2 using the finite volume approach (Patankar, 1980).

$$\iint_{xy} \left[-\frac{1}{\mu} \left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right) \right] dx dy + \iint_{xy} \left[\sigma \left(\frac{\partial A}{\partial t} + V \frac{\partial A}{\partial x} \right) \right] dx dy = \iint_{xy} J_{ex} dx dy \quad (14)$$

Application of this procedure results in a series of discrete algebraic equations that take the form:

$$a_p A_p = \sum a_{nb} A_{nb} + b \quad (15)$$

in which a_p terms are the active coefficients on A , nb implies summation over the neighbouring nodes and b the source terms, such as:

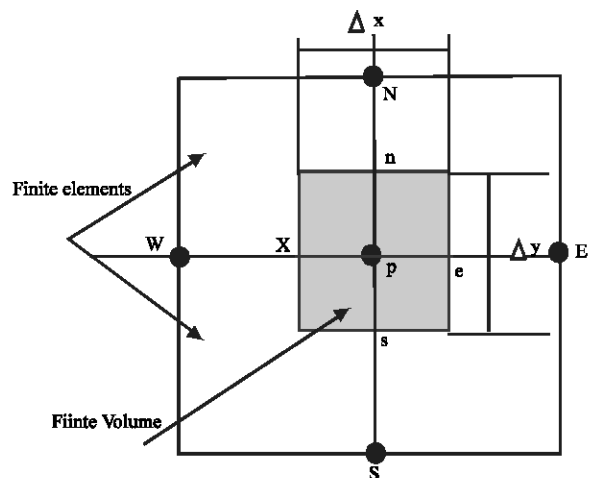


Fig. 2: Description of a finite volume

$$a_E = \frac{\Delta y v_e}{x_e (\delta x)_e},$$

$$a_W = \frac{\Delta y v_w}{x_w (\delta x)_w},$$

$$a_N = \frac{\Delta x v_n}{x_n (\delta y)_n},$$

$$a_s = \frac{\Delta x v_s}{x_s (\delta y)_s},$$

$$aa_E = \frac{1}{2} V \sigma_e \Delta y \Delta t$$

$$aa_W = \frac{1}{2} V \sigma_w \Delta y \Delta t$$

$$a_P = a_E + a_W + a_N + a_S - aa_E + aa_W,$$

$$b = J_{ex} \Delta x \Delta y + \sigma \Delta x \Delta y A_{p0}$$

A_{p0} : Will be taken as the value of from the previous iteration.

After evaluating the resulting integrals over the whole problem domain and then substituting the appropriate Dirichlet conditions, we obtain a set of simultaneous partial differential equations of the form:

$$[[M_1] + [M_2]] [A] = [F] \tag{16}$$

The materials $[M_1]$ and $[M_2]$ are calculated considering the elements matrices. The vector $[F]$ accounts for the currents J . Thus, the magnetic induction field can be calculated from (16) using $B = \text{rot } A$.

The resulting equations are solved using the iterative method (Gauss Seidel) until convergence is reached.

RESULTS AND DISCUSSION

To exploit the code generated, we consider a linear induction MHD pump of 6 slots.

As a result Fig. 3 and 4 show the magnetic vector potential distribution and the magnetic induction in the MHD pump.

Figure 5 and 6 represent, respectively the eddy currents distribution and the variation of the eddy currents density in the channel for several frequencies. It is noticed that the eddy currents density is proportional to the frequency and increases with the increasing of the frequency.

Fig. 3: Magnetic vector potential distribution in the MHD pump

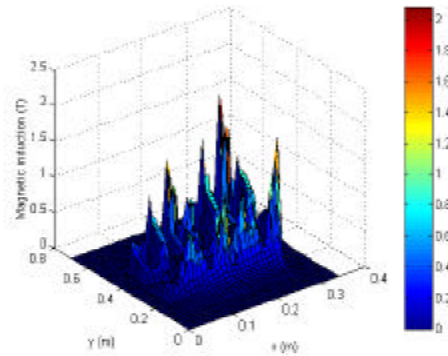


Fig. 4: The magnetic induction in the MHD pump

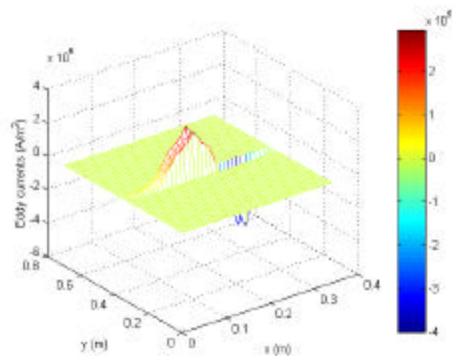


Fig. 5: The eddy currents distribution in the channels

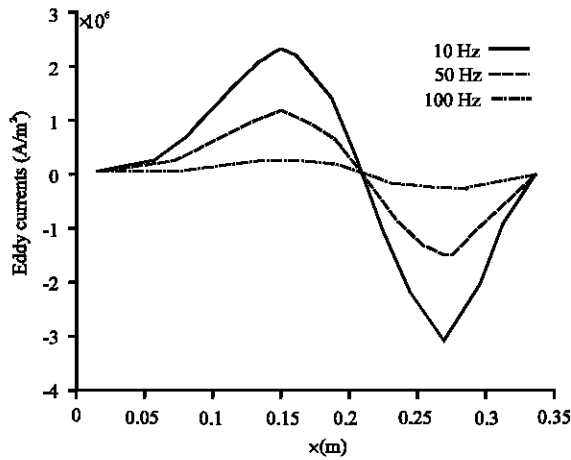


Fig. 6: Eddy current for different frequencies

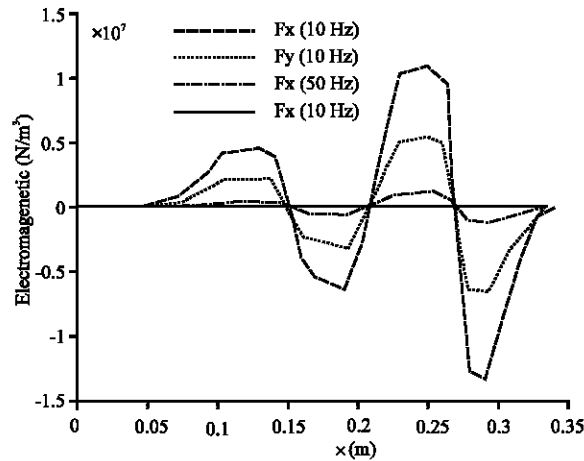


Fig. 7: Electromagnetic thrust distribution in the channel for several frequencies

Figure 7 shows the distribution of the thrust for several frequencies. It is shown that the value of the force at the entry and the outlet side of the machine is not the same; this is due to the ends effects. Moreover, it is noticed that the force according to x is more significant than that following y ; this is with the direction of the fluid flow.

CONCLUSION

A model of a mercury linear double induction pumps taking into account the movement of the fluid linear is developed and analysed in the transient state using 2D finite volume method. The eddy currents density and electromagnetic thrust and their amplitudes increase with the increasing of the frequency.

The visualisation of the fields leads to appreciate the effect of the frequency on the eddy currents density and electromagnetic thrust distribution inside the channel (Kadid *et al.*, 2003; Yamagushi *et al.*, 2001).

This study shows the advantage of using this method for a hydrodynamic coupling.

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