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On New Approaches of Economic Cycles

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Abstract: This study presents a summary review of the basic causes of occurrence and nature of economic cycles. It considers the logistic curve as a valuable function for determination of the basic tendency of time series of economic variables and proposes new approaches to mathematical representation of economic waves. These approaches are based on introduction of the following parameters in the model: asymmetry, period and amplitude of oscillations. The proposed functions allowed to develop a system including seven comprehensive economic models. The study also provides descriptions of the new parameters introduced in the model, i.e. parameters of asymmetry, amplitude and contraction and expansion. Examples of real economic data approximated by the studied mathematical functions are provided. Then, based on the obtained results assertions are made about the consistency of the models and their application both on macro and micro levels.

Key words: Economic cycles, asymmetric cycles, equations of economic dynamics, asymmetry indicators, period shift of oscillatory waves, modeling of economic cycles

INTRODUCTION

Attempts of the human mind to lift the veil of the future and forecast the course of events leave behind its attempts to explain the world around it. It is obvious that such interest to forecast is based on sufficiently strong life motivations. Theoretical significance of forecast in regard to science development means that forecast serves as a method of checking scientific theories and hypotheses. And in terms of genesis, forecast (as any knowledge) meets pressing demands of human actions in the course of life and social struggle. It is clear that forecast becomes possible only in case of dynamic development of a system.

Economic reality is dynamic in its very essence. Most scholars consider economic phenomena as evolutionary processes of changes in their elements and their correlation and look for patterns of such changes. Evolutionary processes are changes towards a certain direction in the absence of dramatic outside perturbation and influences. However, dynamics of economic conjunctures is rhythmic. Everyone agrees that there are periods of high economic growth followed by periods of deceleration and in times of crisis-even negative economic trends. This situation is commonly known as cyclicality of economic processes, wavelike processes or oscillations. Nowadays the possibility of forecasting the unfolding of these oscillatory processes represents a key to development of modern economic thought. Therefore, forecast shall meet strict requirements of the new scientific and technical community. Unfortunately, the mathematical framework for cycle representation is currently very poor. Each economic situation has its own characteristics and we need different approaches to represent them. According to research practice, approaches and tools currently used do not take into account many tendencies of oscillatory processes, thus hindering interpretation of economic progress.

To resolve the problem a new mathematical framework for representation of cyclic processes was developed. The objective of our study is not only the proposal of various functioning mathematical functions, but also their comparison in practice, determination of advantages and disadvantages and proof of consistency. Practice orientation is an important integral part of the present work.

Character and modeling of economic cycles: Despite widespread interest of researchers in the issue of cyclicality of economic dynamics, up to the present day there is no unified concept of this phenomenon and the causes of its occurrence as well., The essence and

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structure of such oscillations are interpreted in different ways and some economists even deny the existence of regularities in the real economic picture.

Classicists and neoclassicists do not accept the cyclicality theory due to the idea of equilibrium proposed by Say. Here the question of distinction between statics and dynamics arises. The concept of balance between interrelated elements is particularly typical for the static point of view on economic reality. And, the concept of process of changes in economic elements and their correlation is the most typical for the dynamic point of view. While intended distinction between statics and dynamics cannot be found among the founders of the classical school, Mill (1848) provides such intended distinction and some considerations about causality of oscillations.

Marxists denied cyclicality in socialism, for absence of division into classes cannot lead to economy decline and, therefore, cyclic processes. According to this approach, development is represented by economy growth only. Some researchers tend to accept only some particular types of oscillatory processes. Among them we should mention those researchers who accept only the industrial cycles proposed by Marx.

Researchers that consider the theory of economic cycles consistent, have several different points of view on the issue of oscillations of economic conjuncture. Sismondi (1827) was one of the first who based his cycle theory on excess savings. His idea was that when savings are higher than consumer spending, it is observed the stagnation of economy, leading to disproportions.

According to Jevons (1871), economy is influenced by external factors, i.e., solar cycles (influence of sun spots on economic development) and he drew conclusions on the influence of the considered external factors on agriculture and trade. Later his ideas began to apply to economy in general thanks to his followers.

Trotsky proposed his own theory of long waves, based on historical periods of capitalism dynamics. Hawtrey (1915, 1928), in his turn, held the opinion that only changes in cash flow can influence and/or cause oscillations in economy. Other factors have only indirect effect on economy through decrease of these flows. Thus, economic cycles represent an exact copy of oscillations in money supply. Given stabilization in money supply, economic cycles would disappear. However, it seems impossible due to instability of the monetary system.

Another theory to be considered is the theory of oversaving. Differences in development of various sectors of economy cause misbalance leading to crises events, from where evidently the picture of a cyclic economy arises (Hansen, 1951).

We also shall mention the psychological theory of the cycle which can be divided into three lines of thought, depending on the causes of cycle occurrence. The first line is related to speculative motives for inflationary expectations and expectations for increase in the rate of securities. The second line is related to particular characteristics of manufacturing investment of capital. The third line is represented by the “balance theory of economic cycle” of Lucas (1980, 1983), characterized by analysis of information acquisition by business units and their behavior.

The English economist Solomou paid much attention to economic cycles, analyzing GDP changes. In fact, we can state that with the development of economic theory, the issue of considering differences between static and dynamic behavior was studied more thoroughly by classical economists, among them we can mention K. Marx, Jevons (1871), L. Valras, J. M. Clark, V. Pareto, J. Schumpeter (1939), A. Amonn and G. Cassel.

Let us agree from this point on that “cyclicality” characterizes the development of economy as a whole, by virtue of which disproportionality and misbalance occur in socioeconomic unfolding.

We may distinguish three basic approaches to description of reasons and nature of economic waves. The exogenous approach is based on influence of external factors: natural calamities, sunspot activity, politics, discoveries and inventions, etc. The endogenous approach attribute cyclic oscillations up to the influence of internal factors: savings, consumption, etc. The semi-exogenous approach combines the exogenous and endogenous approaches. Such factors as changes in volume of money supply, capital renewal, depletion of autonomous technologies, appearance of new technologies, force-majeure (such as wars, natural calamities), sunspots, etc., proposed by great economists at different times, represent causes of occurrence and development of cyclic processes.

Economy moves from one balanced state to another one, going through certain development stages: recessions, crises, slowdown of economic development which is followed by recovery and growth. In the simplest terms, the cyclic structure can be divided into two components: an ascending wave (growth) ending with a peak and a descending wave (recession) ending with a valley. As a rule, in the study of cyclic development of economy it is common practice to distinguish two basic cycle types:

- First type: Cycles encompassing long-term periods (about several decades). These cycles are characterized by the development of economy from growth to recession and vice versa
- Second type: Short industrial cycles characterizing the motion of capitalist economy from crisis to crisis

It is important to distinguish between causes of occurrence of these types of economic cycles: Long-term cycles depend on behavior of social and economic processes in society; industrial cycles rely only on production characteristics.

As a natural result, the question of analytic diagnosis of cycles and their mathematical representation arises. Until quite recently just one-sided vision of cycle type prevailed: the consideration of wave-shaped processes in the form of the classical symmetric sine curve. However, even waves do not have a perfect shape. The necessary mathematical framework should describe real processes with maximum accuracy representing the various features of oscillations. Further forecast of economic cycles will benefit from improvement and extension of this mathematical framework, in particular, for account of such factors as asymmetry, amplitude and contraction. We observe asymmetry of economic waves caused by various factors: breakthrough innovations, geopolitical phenomena, product life cycle, etc. Pioneer researches on the theory of asymmetric cycles were performed by Aoki (1996, 1998, 2001). Our study is based on the use of a simpler mathematical framework which represents the nature and dynamics of wave-like economic processes pretty exactly.

Description of mathematical tools: For the mathematical description of economic dynamics one of the fundamental steps is the determination of the exact superposition nature. All models considered here are functions of time. These functions with probability = 1 such that we learn and use using the theory of random functions.

Therefore, it is common practice to distinguish between additive and multiplicative models, depending on the basis of the correlation between the components.

Determination of the trend: The second step in studies of these processes is the determination of the basic tendency of the temporal series (main trend). Depending on economic situation, the trend line may be different. The most common trend models are based on linear, power, exponential and polynomial functions. However, as with the increase of the argument, these functions grow fast 'ad infinitum' (which is not typical of real economic situation due to resource limitedness), we should consider the logistic curve as shown in Fig. 1, represented by Eq. 1 as the most appropriate function to describe real situations:

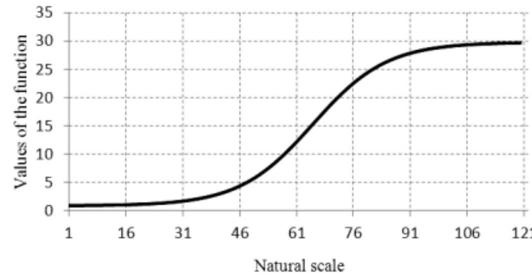

Fig. 1: Logistic curve representing the trend expressed by Eq. 1

$$\tilde{y} = \frac{a}{1 + b \cdot e^{-c \cdot (t - t_0)}} + d \qquad (1)$$

Upon choice of correlation type and further trend line deletion, we obtain the picture of oscillatory waves which shall be accompanied by an appropriate mathematical framework (model). A fundamentally new mathematical framework and improvement of the existing ones can solve the problem of inaccurate representation of the various economic cycles types with further possibility of forecasting.

Logistic trend reflects the overall picture of origin, development and reaching the maximum possible value of many economic, technical and natural processes. Further movement of the process is the gradual extinction with access to a minimum level of symmetry with respect to the curve:

$$y_0 = \frac{[\tilde{y}_{max} - \tilde{y}_{min}]}{2}$$

The mention of Verhulst, Lotka, Volterra makes the reader once again to note that the studied functions are solutions of differential equations. Here a differential equation of Verhulst, Lotka and Volterra. Especially, the properties of the "models" are defined precisely by the properties of differential equations.

The logistic model can be also applied on those intervals where the slowdown $\tilde{y}_t$ is small. In such situation can be used the more simple model of exponential growth. In the case where after the level $\tilde{y}_{max}$ (Fig. 1) begins a new ascent, the trend is the sum of two logistic functions as shown in Fig. 2. Local variations in the parameters of real economic processes lead to the need of estimation of the deviations of the statistical data from smooth mathematical curves.

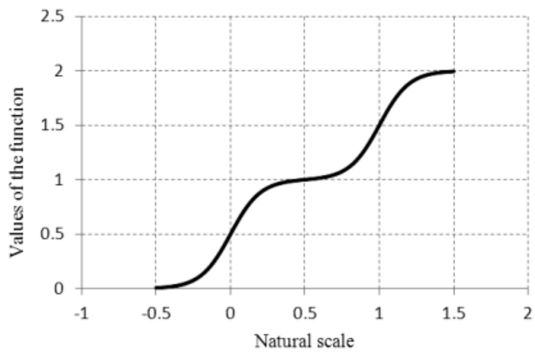

Fig. 2: The sum of two logistic functions

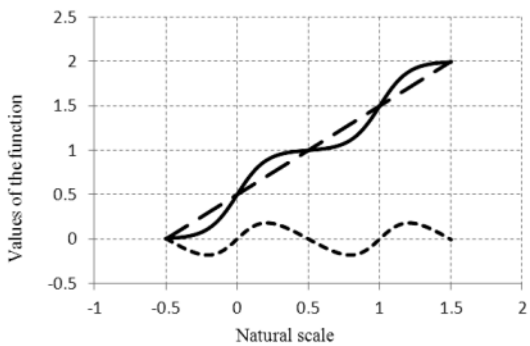

Fig. 3: The linear approximation of the sum of two logistic functions

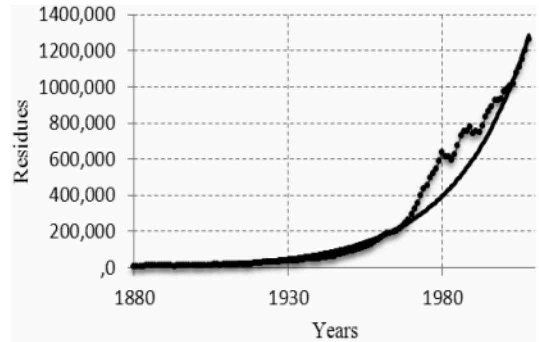

Fig. 4: Exponential trend of the GDP growth of the Netherlands

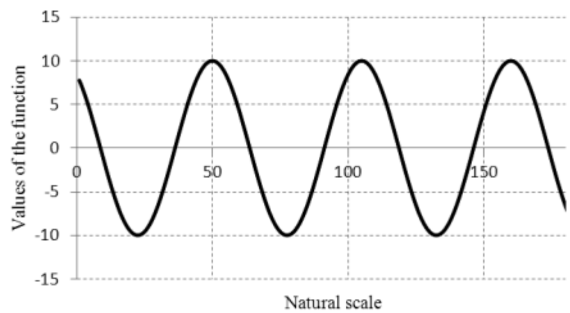

Fig. 5: Graph of harmonic oscillations as expressed by Eq. 2

This is the main source of the appearance of fluctuations against the trend. Another reason for the fluctuations are errors due to measurement inaccuracies and deliberate misstatement of the financial data. Therefore, we have to use the methods of data smoothing and filtering errors.

Figure 2 and 3 try to exemplify schematicallythe appearance of cyclic processes as deviations from the trend. Figure 2 shows the sum of the two logistic functions and Fig. 3 exhibits the linear approximation of this sum on the extreme points of the interval and residues (bottom curve), representing a typical cyclical process. Figure 4 shows the real data of the resulting residues for the GDP growth of the Netherlands (GDP data from Maddison) by subtracting the trend from the exponential model (or vice versa).

(Of course, it's not leftovers. Continuous-trend, Points-implementation of the GDP in the Netherlands. To another country point will be others but the trend remains exponential but the parameters will change. a, b-parameters are different for each country). Residues show a typical correlated random fluctuation process with a time-dependent amplitude. As will be shown ahead in this study, a good model of such residues is a linear second-order dynamic system with white noise on the input. The simplest model of the fluctuation process is known as harmonic function $\tilde{y}_t$ = A sin (ωt+φ). Various modifications of $\tilde{y}_t$ for example φ = π/2: $\tilde{y}_t$ = A cos (ωt) or $\tilde{y}_t$ = A sin (ωt+φ) or $\tilde{y}_t$ = A sin [ω(t)t+φ)] or $\tilde{y}_t$ = A cos [ω(A)t] are amazingly interesting models of processes in all branches of science and practice. We should keep on mind that it is necessary to pay attention to symmetric and non symmetric models as will be shown in the next sections.

Classical symmetric model and its modifications: It is assumed that the classical, common model of representation of economic cycles is Eq. 2 of harmonic oscillations, where parameter p is the function period, parameter a is the function amplitude, parameter f is the shift along the X-axis and parameter h is the shift along the Y-axis (when parameter h is absent then it amounts to zero):

$$\hat{z} = a \cdot \sin\left(f - \frac{2 \cdot \pi \cdot t}{p}\right) + h \tag{2}$$

This function is represented by the graph in Fig. 5. This model may reasonably describe residues from

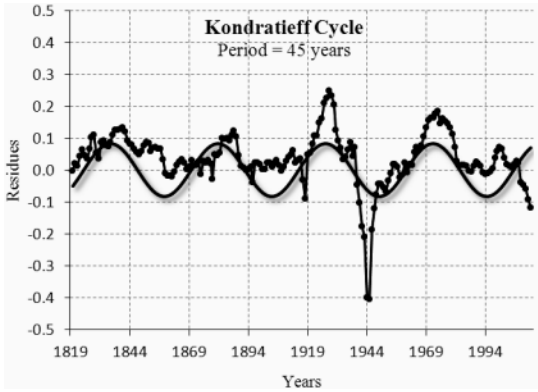

Fig. 6: Series of residues for the GDP of the Netherlands (1990 Int. GK$) converted in 2013 U$) with superposition of the graph of harmonic oscillations given by Eq. 2

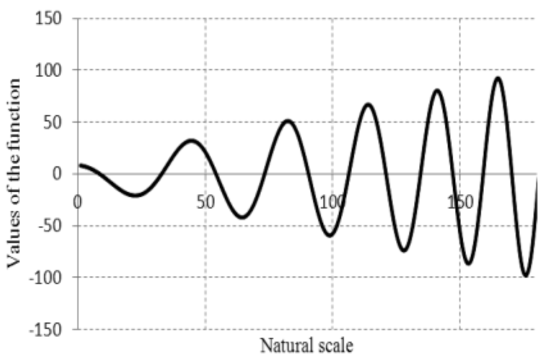

Fig. 7: The function of harmonic oscillations with variable amplitude and period as expressed by Eq. 3

economic indicators with symmetric behavior as for instance the unfolding of Kondratieff waves with model parameters; p =45.30; α = -0.08 as shown in Fig. 6 for the GDP of the Netherlands.

However, as can be inferred from the sequence of points (GDP data) of Fig. 6, real oscillatory processes may vary in amplitude and period. To solve this problem, it is sufficient to introduce certain parameters in Eq. 3 as shown below:

$$\hat{z} = (d + e \cdot t) \cdot \sin\left(f - \frac{2 \cdot \pi \cdot t \cdot k^t}{p}\right) + h \quad (3)$$

Where, d and e are parameters introduced in the model for solving the problem of variations in the amplitude and the parameter is the base of the exponential function, set into the model for allowing the changing frequency of cycles.

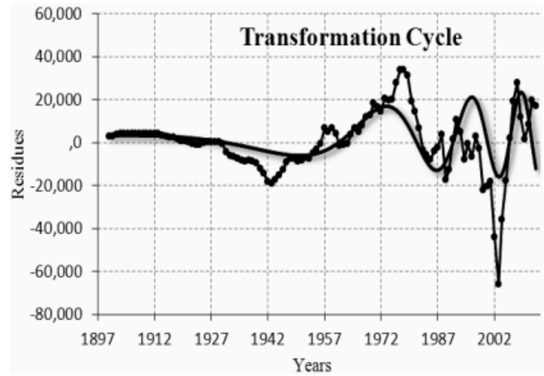

Fig. 8: Series of residues for the GDP of Venezuela (1990 Int. GK$, converted in 2013 U$) with superposition of the graph of the modified function of harmonic oscillations given by Eq. 3

The introduction of these parameters notably changes the form of the model, so we will designate the cycles, produced by such change in model as "transformation cycles". The graph for Eq. 3 is shown in Fig. 7. Note that upon change of amplitude any function may be used depending on the represented economic parameter (hereinafter the linear function of model amplitude change is used). A real economic condition that can be described by this function is represented in Fig. 8 for the case of Venezuela's GDP, with model parameters: p = -578.27; d = -3.39; e = -184.63; f = -9.59; k = -1.02.

Upon studies of other economic parameters we faced the problem of the asymmetry of the waves. This problem may be caused by several reasons: non-simultaneous implementation of innovations in economy, social disturbances, wars, etc. The classical model considered above does not allow to "pick up" asynchrony, thus leading to low quality of wave representation and arising the necessity of developing a new mathematical function.

Complex sine and its modifications: The next model presents the composition of two different see-saw-like sine functions, expressed by Eq. 4 and 5:

$$\hat{z} = d \cdot \sin\left[f - \frac{2 \cdot \pi \cdot t}{p} + g \cdot \sin\left(f - \frac{2 \cdot \pi \cdot t}{p}\right)\right] + h \quad (4)$$

$$\hat{z} = a \cdot \sin\left\{f - \frac{2 \cdot \pi \cdot t}{p} + b \cdot \sin\left[f - \frac{2 \cdot \pi \cdot t}{p} + c \cdot \sin\left(f - \frac{2 \cdot \pi \cdot t}{p}\right)\right]\right\} + h \quad (5)$$

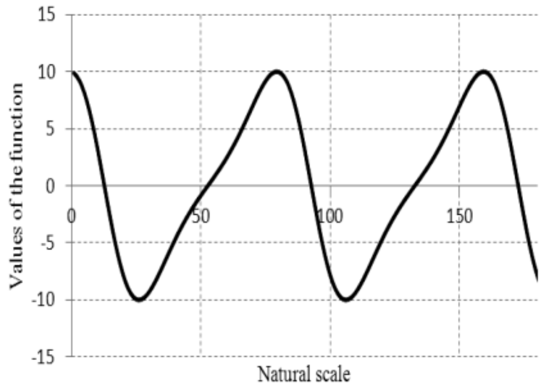

Fig. 9: The "complex sine" function expressed by Eq. 5

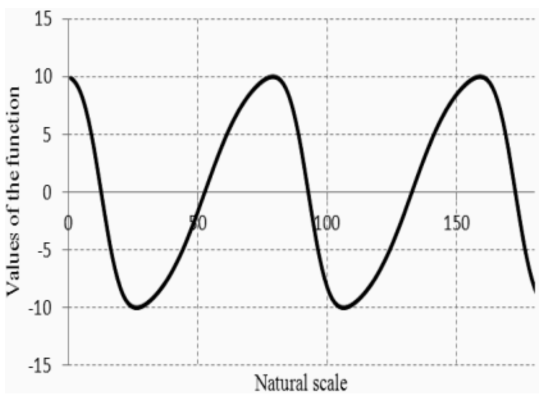

Fig. 10: The "complex sine" function expressed by Eq. 6

These models can characterize symmetric pictures when values of parameters g (in Eq. 4) and b and c (in Eq. 5) are equal to zero. Therefore, these models have substantial advantage over the models considered earlier. The graphs of both functions are represented in Fig. 9 and 10 and by Eq. 5. As in the preceding model, these models can be modified through the addition of parameters e and k resuting in the change of oscillation amplitude and cycle period as expressed by Eq. 6. The graph of one of the modified functions (combination of two sine functions) is represented in Fig. 11:

$$\hat{z} = (d + e \cdot t) \cdot \sin\left\{ f - \frac{2 \cdot \pi \cdot t \cdot k^t}{p} + g \cdot \sin\left(f - \frac{2 \cdot \pi \cdot t \cdot k^t}{p} \right) \right\} + h \quad (6)$$

A real economic situation represented by one of the proposed functions (Eq. 4) is shown in Fig. 12 for the

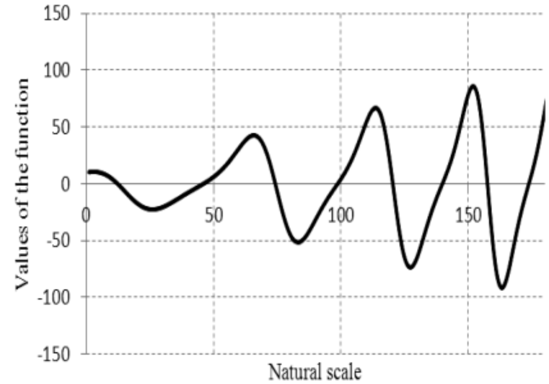

Fig. 11: "Complex sine" function consisting of two superposed harmonic oscillations with variable amplitude and period as expressed by Eq. 6

Fig. 12: Series of residues for the employment in USA (total nonfarm: all employees, thousands, USA) with superposition of the complex sine function graph given by Eq. 4

employment in USA with model parameters: p = -658.986; d = -6158.812; f = -20.638; g = -0.517. As can be seen, the using of models allowing to evaluate cycle asynchrony matches suitably some real economic variables. It is then reasonable to introduce coefficients of asymmetry in the models proposed in this study. The coefficients to be introduced may be considered as relative or absolute indicators. We have developed an analytic construction with the general aspect expressed by Eq. 7:

$$As = \frac{2 \cdot (t_{max} - t_{min})}{p} + 1 \quad (7)$$

Where, t_{max} and t_{min} are function extremums close to each other (extremums shall be chosen so that a maximum point is at the left from a minimum point in the coordinate system). Then the obtained correlation may be compared to 1. This coefficient form is convenient because it can

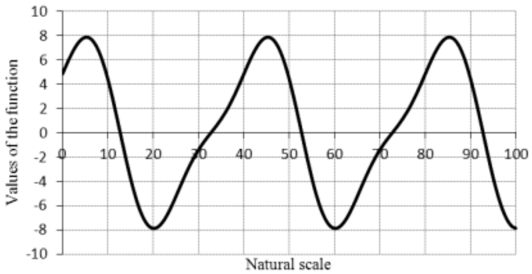

Fig. 13: The sum of two harmonic functions as expressed by Eq. 8

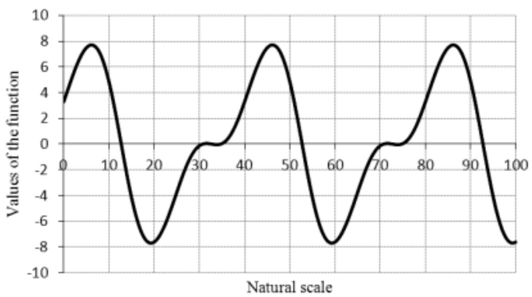

Fig. 14: The sum of three harmonic functions as expressed by Eq. 9

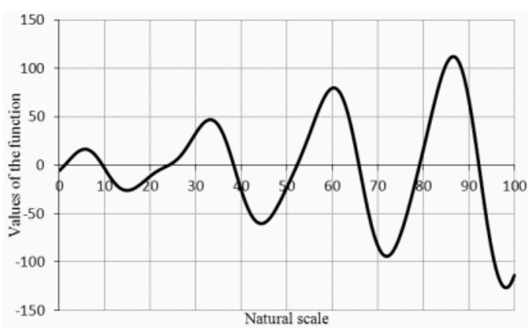

Fig. 15: The modified sum of three harmonic functions (with variable amplitude and period) expressed by Eq. 10

take strictly defined values ranging from 0-1: upon absolute symmetry the coefficient amounts to 0, upon high asynchrony it tends to 1. Such determination allows to compare asymmetry nature of various cycles (as the coefficient is relative). In Fig. 12, we have AS = 0.29.

Sum of harmonic functions and their modifications: The next model allows also to represent not only symmetric but also asymmetric economic variables. The model is characterized analytically by the sum of several Eq. of harmonic oscillations as expressed in Eq. 8 and 9. The graphs view of these functions are represented in Fig. 13-15.

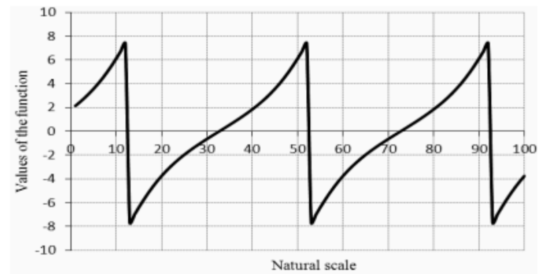

Fig. 16: "Sawtooth wave" function graph expressed by Eq. 11

$$\hat{z} = a \cdot \sin\left(f - \frac{2 \cdot \pi \cdot t}{p}\right) + b \cdot \sin\left[2 \cdot \left(f - \frac{2 \cdot \pi \cdot t}{p}\right)\right] + h \tag{8}$$

$$\hat{z} = a \cdot \sin\left(f - \frac{2 \cdot \pi \cdot t}{p}\right) + b \cdot \sin\left[2 \cdot \left(f - \frac{2 \cdot \pi \cdot t}{p}\right)\right] + c \cdot \sin\left[4 \cdot \left(f - \frac{2 \cdot \pi \cdot t}{p}\right)\right] + h \tag{9}$$

These models can also be modified by the introduction of additional parameters and k, see Eq. 10 and 11:

$$\hat{z} = (d + e \cdot t) \cdot \left\langle a \cdot \sin\left(f - \frac{2 \cdot \pi \cdot t \cdot k^t}{p}\right) + b \cdot \sin\left[2 \cdot \left(f - \frac{2 \cdot \pi \cdot t \cdot k^t}{p}\right)\right] + c \cdot \sin\left[4 \cdot \left(f - \frac{2 \cdot \pi \cdot t \cdot k^t}{p}\right)\right]\right\rangle + h \tag{10}$$

A real application of one of the models (Eq. 10) is represented in Fig. 16 for the Juglar cycles observed in the USA Industrial prodution with model parameters: p = -112.16; α = -1462952.29; f = -0.15; b = -108845.93; c = -95915.22. AS = -0,11.

"Sawtooth wave": The next model presents a fundamentally different approach; it has the shape of the "Sawtooth Wave" widely used in physics, expressed by Eq. 11. Its graphical representation is shown in Fig. 16 and 17:

$$\hat{z} = a \cdot \operatorname{arctg}\left[\frac{b}{\operatorname{tg}\left(f - \frac{p \cdot t}{p}\right)}\right] + h \tag{11}$$

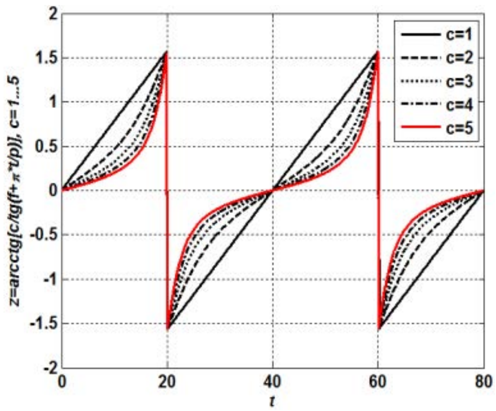

Fig. 17: Movement model

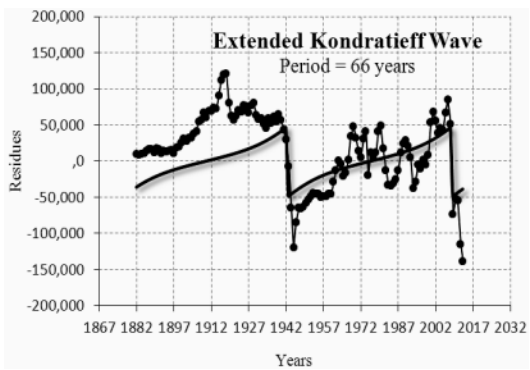

Fig. 18: Series of residues for the Italian GDP (1990 Int. GK$, converted in $U in 2013) with superposition of the “Sawtooth Wave” model

This model has certain disadvantages as for instance the fact that the “Sawtooth Wave” function has points of discontinuity. However, utilization of discrete data upon consideration of economic indicators allows to use this model. The coefficient of asymmetry for this model remains constant upon any change of parameters and this may be explained by the fact that the extremum points are fixed to some extent (due to the fact that the function has points of discontinuity). The coefficient of asymmetry amounts to 0.95, a significant asymmetry upon any change of parameters. However, this change influences only the region corresponding to economy growth. If coefficient bb (in Eq. 11) ranges from 0-1, peaks and crises become steeper; if coefficient bb is >1, peaks and crises become more flattened.

An example of a “sawtooth wave” function application is represented in Fig. 18 fot the case of the unfolding of the Italian GDP, corresponding clearly to a full Kondratieff wave. Mdel parameters: p = -66.08; b = -0.58; f = -3.75; a = -30825.73; AS = -0.95.

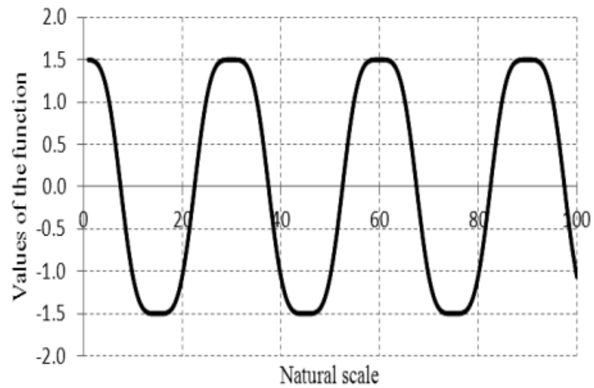

Fig. 19: The additive model of economic cycles expressed by Eq. 12

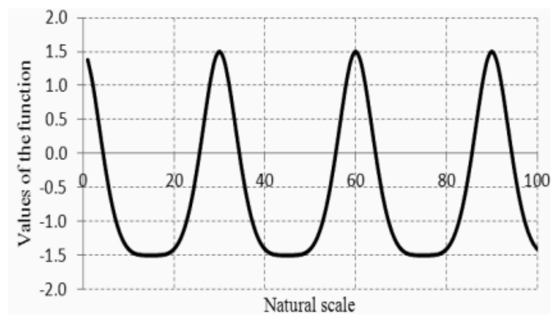

Fig. 20: The multiplicative model of economic cycles expressed by Eq. 13

More complicated trigonometric functions: The following functions were proposed by prof. Sokolov. These functions have more complicated form, however, they allow to represent various types of economic waves.

As commented on in the first paragraph of section 2, we can have an additive formulation as expressed by Eq. 12 or a multiplicative formulation as expressed by Eq. 13. Both graphical representations are shown in Fig. 19 and 20:

$$\hat{z} = d \cdot \sin\left\langle \frac{\pi}{2 \cdot \mathrm{arctg}\left(\frac{\Delta}{2}\right)} \cdot \left\{ \mathrm{arctg}\left[\frac{1}{\mathrm{tg}\left(\frac{\pi \cdot t}{p} - f\right)} - \Delta\right] - \mathrm{arctg}\left[\frac{1}{\mathrm{tg}\left(\frac{\pi \cdot t}{p} - f\right)}\right]\right\} + \frac{\pi}{2} \right\rangle + h \qquad (12)$$

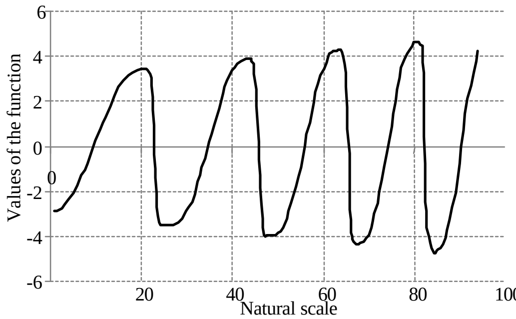

Fig. 21: The additive model of economic cycles with variable amplitude and period, expressed by Eq. 14

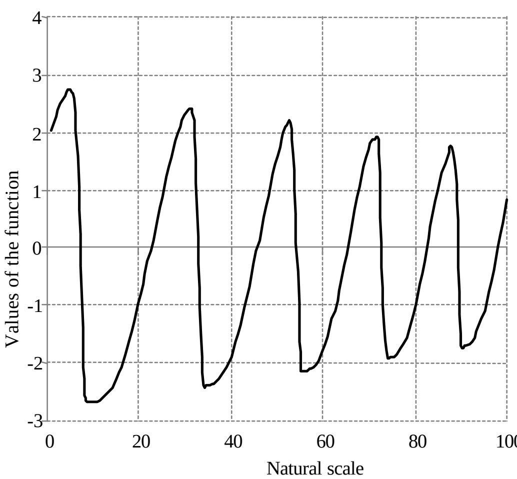

Fig. 22: The multiplicative model of economic cycles with variable amplitude and period, expressed by Eq. 15

$$\hat{z} = d\cdot\sin\left\langle \left\{ \frac{\pi}{\left[\mathrm{arctg}\left(\frac{\Delta}{2}\right)\right]^2+\frac{\pi^2}{4}} \right\} \left\{ \mathrm{arctg}\left[\frac{1}{\mathrm{tg}\left(\frac{\pi\cdot t}{p}-f\right)}-\Delta\right]\cdot \mathrm{arctg}\left[\frac{1}{\mathrm{tg}\left(\frac{\pi\cdot t}{p}-f\right)}\right]-\frac{\pi^2}{4}\right\}+\frac{\pi}{2}\right\rangle + h \quad (13)$$

In the same vein as in all preceding models described, these complex trigonometric functions models can also be modified to represent situations with different periods and amplitudes of oscillatory waves, by introduction of the parameters e and k. Equation 14 and 15 and their graphical representations in Fig. 21 and 22 and by Eq. 14-15:

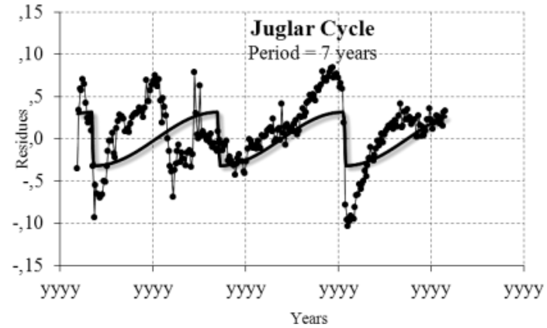

Fig. 23: Series of residues production of total industry in the Russian Federation: Index 2005 = 100: SA with superposition of the additive model

$$\hat{z}=(d+e\cdot t)\cdot\sin\left\langle \frac{\pi}{2\cdot \mathrm{arctg}\left(\frac{\Delta}{2}\right)}\cdot\left\{\mathrm{arctg}\left[\frac{1}{\mathrm{tg}\left(\frac{k^t\cdot\pi\cdot t}{p}-f\right)}-\Delta\right] - \mathrm{arctg}\left[\frac{1}{\mathrm{tg}\left(\frac{k^t\cdot\pi\cdot t}{p}-f\right)}\right]\right\}+\frac{\pi}{2}\right\rangle+h \quad (14)$$

$$\hat{z}=(d+e\cdot t)\cdot\sin\left\langle \left\{ \frac{\pi}{\left[\mathrm{arctg}\left(\frac{\Delta}{2}\right)\right]^2+\frac{\pi^2}{4}} \right\} \left\{ \mathrm{arctg}\left[\frac{1}{\mathrm{tg}\left(\frac{k^t\cdot\pi\cdot t}{p}-f\right)}-\Delta\right] \mathrm{arctg}\left[\frac{1}{\mathrm{tg}\left(\frac{k^t\cdot\pi\cdot t}{p}-f\right)}\right]-\frac{\pi^2}{4}\right\}+\frac{\pi}{2}\right\rangle + h \quad (15)$$

For additive and multiplicative models it is easier to calculate the coefficient of asymmetry than for other functions. This fact gives them one more advantage over other models of the presented mathematical framework. These coefficients can be represented analytically as follows (both for additive and multiplicative functions):

$$AS = 1-\frac{2\cdot \mathrm{arctg}\left(\frac{2}{\Delta}\right)}{\pi} \quad (16)$$

The coefficient of asymmetry is a number that measures the degree of asymentry of economic cycles and offers several opportunities for representation of cycles and their further forecast.

Examples of real economic situations with these cycles are presented in Fig. 23 and 24. Figure 23 exhibits

Fig. 24: Series of residues USA industrial production index: Total index; 2007 = 100; SA with superposition of the multiplicative model

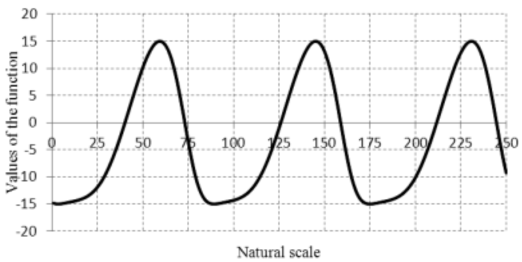

Fig. 25: The composite model of sine and exponential functions expressed by Eq. 17

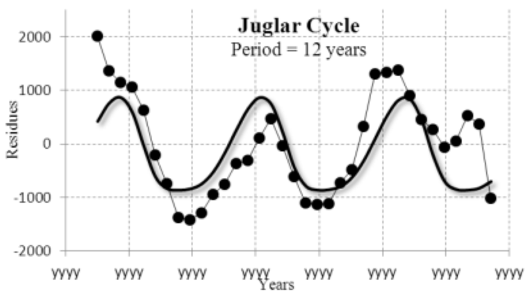

Fig. 26: Series of residues, USA median income Real 2008 dollars with superposition of the model of the composition of sine and exponential functions

the unfolding of Juglar cycles for the total industrial production of the Russian Federation (with model parameters: p = -89.01; d = -3.17; Δ = -50.22; f = -3.54; AS = -0.97) and Fig. 24 shows the typical behavior of Kuznets cycles for the USA industrial production (with model parameters p = -334.13; d = -3.18; Δ = -0; f = -3.03; AS = 0).

Composite model of sine and exponential functions: Finalizing our presentation of possible mathematical models, let's consider now the combination of exponential and sine functions as expressed by Eq. 17 whose graphical representation is shown in Fig. 25:

$$\tilde{y} = a\cdot\sin\left\{b\cdot\left[f-\frac{2\cdot\pi\cdot t}{p}\right]-c\cdot e^{\cos\left(b\cdot\left[f-\frac{2\cdot\pi\cdot t}{p}\right]\right)}\right\}+h \quad (17)$$

The application of this function for a real economic variable is shown in Fig. 26 which presents the unfolding of Juglar cycles for the residues of the median USA income (model parameters: p = -0.97; α = -868.43; b = -0.08; f = -41.97; c = -0.40, AS = -0.17).

CONCLUSION

For the description and thorough understanding of business cycles it is necessary the development of suitable mathematical tools that can provide a more accurate view of the nature and character of economic fluctuations, allowing to enhance the knowledge of real economic processes and forecasting of future economic scenarios as well.

The application of the largely used classical sine wave functions does not reflect the real course of economic development due to the lack of accuracy in the approximations and their incapacity of accounting for the variations in periodicity, amplitude and above all asymmetries. On the other hand, the also widely used harmonic functions can be useful for more realistic representation of economic cycles but are not effective in the consideration of large variations of amplitudes and cannot accommodate the observed real asymmetries. As solution for these drawbacks in existing models for describing business cycles we propose the usage of more complex mathematical functions, combining the logistic curve for the description of the main trend with other sinusoidal and exponential functions that can indeed match the real unfolding of economic fluctuations.

In this study, we performed an extensive analysis of possible complex wave-like functions that can be used for the purpose mentioned above, giving also some few examples of real economic variables that can be fitted with these functions. The following functions were analyzed: Classical sine model which is often used in economic literature and does not provide enough accurate picture of the nature of economic processes, for it assumes a strict symmetry of the processes of growth and decline. We have proposed some modifications of this simple model by incorporating some new parameters which gives it the opportunity of usage in some few cases.

Sophisticated sinus functions which solve the problem of insufficiently accurate approximation in many cases and allow the addition of an asymmetry coefficient, allowing to expand the horizon of research. However, the restriction on the range of some of its constituent parameters makes its usage very limited.

Sum of harmonics and its modification, based on the expansion in Fourier series was also analyzed and seems to be a more reliable mathematical tool to the description of economic processes with asymmetrical and asynchronous character.

"Sawtooth wave" functions constitutes another mathematical function analyzed that exhibits some potential of applications, but has the drawback of presenting points of discontinuity. Such points, however, might not be a serious disadvantage when considering the discrete nature of economic data.

More complex models based on trigonometric functions seems to be the best approach, with great potential to overcome all the problems of the description of economic processes outlined above. The only negative point of this tool regards its long analytical presentation. This model describes very accurately many real economic variables, both with symmetrical and asymmetrical behavior.

Finally was analyzed a composite model of exponential and sinus functions which still requires further study and modifications.

Several cases of real economic variables were analyzed using the proposed mathematical functions which show the good performance of the proposed approach in capturing the peculiar oscillatory behavior of economic dynamics, including changes in the amplitude, periodicity and asymmetry of the cycles. Among all mathematical functions studied the most promising one is the combination of the logistic trend model with complex trigonometric functions.

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