

## Providing a Conceptual Model in Dynamical System Game Theory in the Study of Cognitive Behavior Competitors in an Oligopoly Market

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**Abstract:** The main idea of this study is to provide a conceptual model of a game in conditions of oligopoly based on feedback loops and system dynamics approach and explore the dynamics prevailing in a game in the real world and solve problems that conventional methods of game theory cant study them. During this study attempt to compare the complex method of game theory and system dynamics to explore the possibility of using the combination method of these two to consider oligopoly markets. The search method of problem is the combination of qualitative methods include using interviews with industry experts to prepare the models and quantitative methods of system dynamics, methodologies and simulation and methods of solving game theory search innovation consisted of representing a model in which considering the impact of behavior of the players on the environment and vice versa, also checking. The transition period a game in oligopoly and taking into account lag of information can be observed.

**Key words:** Games theory, dynamics system, oligopoly, dynamical system game theory, dynamics prevailing, feedback loops

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### INTRODUCTION

In the world of trade and business, the game is always going on among the activists, a kind of market structure in which a few vendors will be offer a product that is oligopoly. Companies to survive in the market, often use strategies, stable conditions, reasons and motivation and a possible change in most of them and utilize games theory. The main topic of this study is provide a theoretical framework of a conceptual model to use the combination of game theory and dynamics system in considering the behavior of firms in an oligopoly market. In this study, games based oligopoly markets attempt with the help of dynamics system approach has been evaluated and a conceptual framework that able to describe the dynamic aspects of the game environment and factors affecting on itImportance of subject is in this regard that what happens in an oligopoly market. Cause player uncertainty to the result of decisions.

Because of the intense bond between the system cast their actions can be traced to them. In Fig. 1, the initial loop feedback is displayed in game.

The main reason for using the combination method of game theory and dynamics system is some cases which

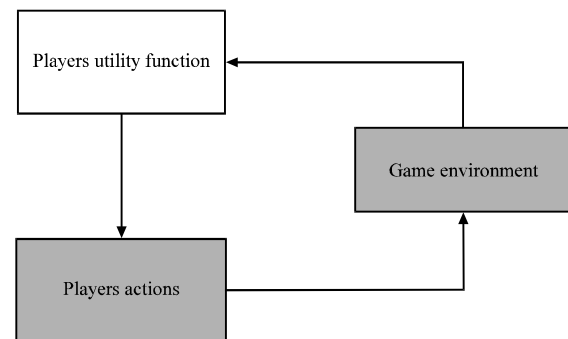


Fig. 1: Loop structure of feedback in game theory

cause synergistic combination method. At first glance what is clear is that systematical dynamics are soft tools that consider continuous and dynamic systems. If game theory often considers discrete systems and ISNA in hard space and only in some games are entered into continuous fields.

The basic theory in game conditions is players behave reasonably in the strategic environment here logic means that each player tries to maximize ones profits. What is clear is that in the real world assumption of

players being reasonable are not logical with reality and this assumption is one of the weaknesses games.

The major advantage of the games is several simultaneous decisions by some decision maker. With all these differences, system dynamics and game theory are fundamental similarity which is feedback display. Game theory will show this feedback in retaliation, malice and revenge way and show system dynamics of it in recursive relations manner.

System dynamics often used in most cases for considering systems which has decision maker or single politician while many system has a number of politician.

In general, we can say that Game theory generally with a combination of system dynamics leads to finding system stable states (equilibrium) and also helps to understand the system (games) that have balance or not and most importantly what policies will lead us to our expected state. The main problem in many games, find the balance point game (its balance) and system dynamics will help in solving this problem tremendously. In addition it must be said that in fact, system is a game if all players act base on other interests effectively. Similarly, use both of them in a hybrid system help to make more and better check behavior of the system and to adopt the necessary policies.

The main question of this research is that in terms of system dynamics in a game with mixed strategies with oligopoly condition what relation exists? And the dynamics is in structure of a game in the multi-state monopoly? And how is the effect of and selection decision by each of the actors on the environment of the game and the policies of other actors over time?

Also, the main objective of this research is conceptual modeling of a complex strategy of a game in an oligopoly condition on the basis of feedback loops and system dynamics approach and explore the dynamics exists in a real-world game

The research method in this survey is mixed research and actually will be used a combination of qualitative methods including methods of descriptive study to identify the structure of the system. In the first part efforts with help of equilibrium mathematical methods identify an oligopoly game. In addition, part of research method is also analysis and modeling the system using dynamics system methods.

In this study, by using system dynamics and combined with game theory we consider the oligopoly markets. Qualitative methods including interviews with industry experts to prepare the models and quantitative methods, also including system dynamics methodologies and simulation and methods of solving game theory.

**Literature review:** According to studies, it seems that the first time Kima and Kimb (1997) stated the game theory

just like methods of system dynamics in the social sciences and economics are tools available to policy makers that with their help we can adopt appropriate policies on various issues.

Akiyamaa and Kaneko (2000, 2002) stated that any decision taken by the players led to changes in the environment game and then change in revenue will be played at a later stage. They attempted to measure the effect of the performance of a player playing on the environment.

West and Lebiere (2001) stated that changing in behavior of the players is due to the results in the previous round game. They mentioned that the ability to produce random behavior is crucial in game theory. Kavana and founder would be provided a combination of thinking systemic model and games theory to develop alternative strategies in New Zealand's education system. Formulated evolutionary game of prisoner's dilemma in a system dynamics model by Adamides took place. They used the results of system dynamics model for investigating the influence of pure and hybrid different strategies of collaboration and no co-starring in the cycle of the technology.

Elettrey and Hassan (2006) offered two different models of multi-team dynamic games. They seem each team consisting of two plants that are competing with each other. They tried in dynamic conditions of equilibrium point find the issue and determined stability conditions of model.

Shayne Gary (2008) stated that although traditional models of game theory has some weak points that use them on strategic decisions in practice are limited and In his study try to check the limitations and benefits of decisions based on system dynamics.

Elettrey and Mansour (2012) were introduced a Carnot games modeling by using system dynamics. In fact they introduce the use of system dynamics in salted games. They considered a dynamic model with incomplete information and introduce how to use it in standard salted games and were determined interactive and optimal solution in monopolistic markets.

$$G: (E(t), S(t)) \rightarrow (E(t+1), S(t+1)) \quad (1)$$

#### **Conceptual modeling in dynamical system game theory:**

We can show dynamical system game theory as follow: in the above equation, G represents dynamical system game, E represents the state of the environment and S represents the state of players. t is the time client. G in fact is changes mechanism for whole game over time. That is to determine the state of the environment and the players and also what mechanisms are their actions that have changed over time:

$$\begin{aligned} u: (E(t), S(t)) &\rightarrow (E(t'), S(t')) \\ v: (E(t'), S(t'), O(t)) &\rightarrow (E(t+1), S(t+1)) \\ G: u \circ v \end{aligned} \quad (2)$$

Where:

- u = Natural laws
- v = The effect of the decision of the players
- O = Player's preparation and decisions in relation to G order of u and v does not matter

Changes in the above equation and changing in state of environment and players can be according to natural laws or by the actions of the players. Therefore, we can correct game changes as follows: the above equations displayed this subject that changes in a game dynamics system can be caused by natural events market (u) or the effects of the decisions players make v. Equation 2 in fact represents a dynamic game that change over time and by the effect two factor, natural laws and the effects of the players' decision. But also need to add relation of cause and effect to the model to improve dynamical game system:

$$Z^i: (E(t), S(t)) \rightarrow (O^i(t)) \quad (3)$$

The next important component in dynamical games system is decision-making mechanisms of players. Each player i based on the state of the Environment (E), state and other players conditions and his situation will be decided to carry out an action that this decision is done based on other player one. This means that each player has a personal decision for himself that in fact determined decision-making mechanism. Finally, the mechanism of personal decision of each player will be defined as follows:

$$Z: (E(t), S(t)) \rightarrow (O(t)) \quad (4)$$

Where:

- I = The player numbers
- Z = Mechanism of decision-making
- O = The player's actions

In general, for all players, means when the decision-making mechanism is in the form of Z then the above equation will be written as follow: thus, similarly we can say that identifying the dynamics of each game need to know the main components of Z and G are that state of the environment, state players and their actions. In fact, game as a dynamic system, is displayed by placing decision making process Z in dynamics system G.

Total mechanism of players decision-making are his state and environment one and relations between these that led to the identification dynamical system of a game.

The meaning of mechanism of decision-making players are process during each player players taking into account the environment and other competitors and oneself make decision to act. That is to say the mechanism depends on the state of the environment and players prediction of the future. Components and variable of a dynamic system game can be seen in the Table 1 below.

In general, the conceptual model of issue can be expressed as follow that a player has a mechanism of decision-making called Z that are used the state of the environment and the players as input. Finally, led to select an action by the player. Importantly, action will be affected as output of the mechanism to the states of players and the environment. The process is displayed in Fig. 2.

**Using of conceptual and dynamical system games theory in checking oligopoly markets:** In order to use the above conceptual model to explain oligopolistic market structure, current market agents are considered in the market as a player and as an environment in the above model. What seems obvious is that each firm in the market due to the assumption of rationality will follow the ultimate goal of maximizing the profits. In general, profit function can be presented as follows:

$$p = D \cdot P - C \quad (5)$$

Where:

- $\pi$  = The profit function of each firm
- D = Depending on demand or sales volume for each firm
- P = Product sales price
- C = Subject to the total cost of each firm

Table 1: Component and variables of a dynamic system game

Games	Sets
<b>Component of dynamic system game</b>	
Game climate	-
Player set	$N = \{1, 2, \dots, n\}$
<b>Variable of dynamic system game</b>	
State of game climate	E
Player set	$S = \{s^1, s^2, \dots, s^n\}$
Player action	$O = \{o^1, o^2, \dots, o^n\}$

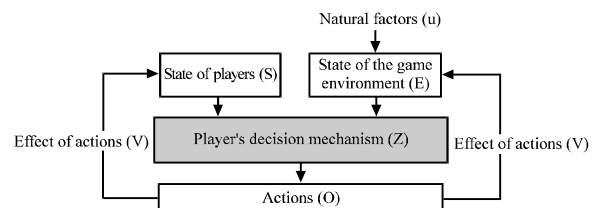


Fig. 2: Conceptual model of dynamic system game in total state

About the state of the players and the environment was also trying to consider criteria that firms in an oligopoly market as the stimulus their decisions and as the environment and the state of players.

Because of considering different companies and potential demand in the market consider as the state of environment and share market or profit rate of each firm to the expected interest as a criteria of player's state. In fact, an firm in any moment make decision due to the product's market share and market share of other competitors and potential market demand and given the current decision of competitors and predict their future decisions.

Market potential demand can swing without the involvement of firms in the market and by external reasons such as bank failures, community loss income and economic downturn. Of course, this demand can be changed by the firms' decisions. Means the behavior of firms and their decisions can also lead to changes in the market value and potential demand. With this interpretation in fact, the game environment will be display as follow:

$$E^t = \{D_0^t\} \quad (6)$$

As we have previously state game of environment is at the moment of t witch displayed as t with the potential demand of market d. Potential demand of market can be changed by the decisions of the players or natural factors. Also, what is as state firms (the players) place in issue the oligopoly markets is market share of each firm's profits to benefit their expected at any moment of time. Means players mode vector will be shown as follow:

$$S_i^t = \left\{ R_i^t, \frac{\pi_i^t}{\pi_i^e} \right\}, i = \{1, 2, \dots, n\} \quad (7)$$

In the above state of the firm is i at time t and market share of firm i at time t and total profit of firm i at time t and expected profit of firm is i. The total number of firms in the market is n because of each firm has goal of more profit in market. So, in any moment of time they make decision with considering market share and competitor and its profit and finally this issue is subordinate of goal means make maximize profit of all firms will be shown below:

$$D = f(O, D_0) \quad (8)$$

$$C = f(O, D, \alpha, \beta) \quad (9)$$

$$\pi = D.P - C = [D_{(O)} . P] - [C_{(O, D, \alpha, \beta)}] \quad (10)$$

Equation 8 shows that the demand is subordinate of each player's actions (O) and potential market demand D. That is in fact the players with making decisions cause to change in one's demand. For example, make decision on price, quality or amount of advertising. In Eq. 9 is considered each firm's total cost function where the variable cost of production per unit of output is cost of production function (Mohammadi and Moslehshirazi, 2016). According, to what was said, conceptual Fig. 3 of game can be presented as.

In sum, we need to specify the mechanism of decision-players in a game dynamics system. In this

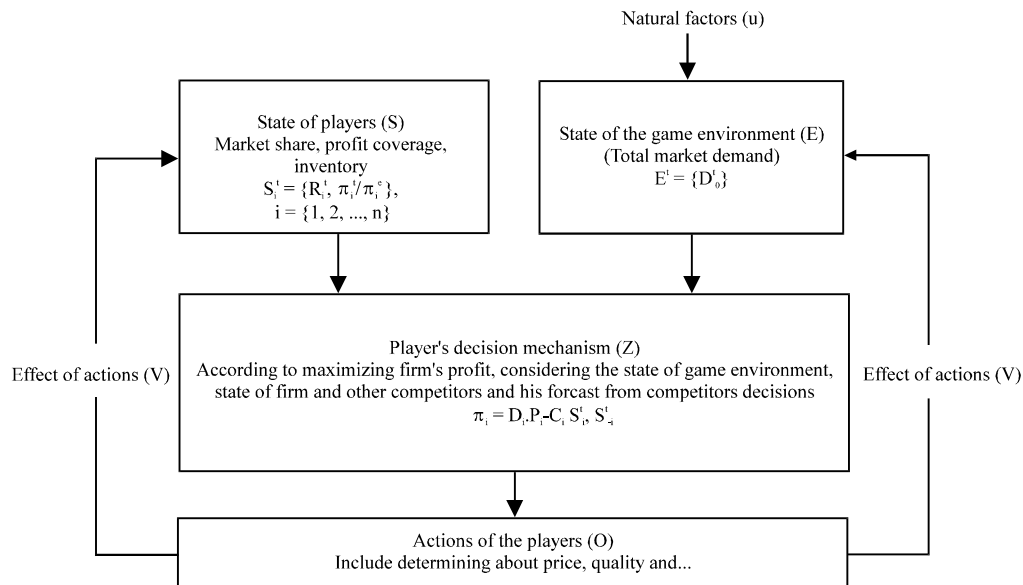


Fig. 3: Conceptual framework of decision-making in a multisided market based on game

regard oligopoly market is introduced as kind of game. State of firm, environment and purpose of any firm take into account as part of decision mechanism. We need to introduce each firm's target to make profit function that give us the total cost function and demand one.

## RESULTS AND DISCUSSION

**Results and interpretation:** Regardless of the type of demand, the usual method of finding equilibrium in oligopoly markets on the basis of the income gradient method is as follows:

$$\text{Max } \pi_i = D_i \cdot P_i - C_i \quad (11)$$

$$B_i(O_j^*) = O_i^* \Rightarrow \frac{d\pi_i}{dO_i} = \nabla \pi_i = 0 \quad (12)$$

In the above equation, the best firm's practice is  $i$ , and also represents the best decisions of player  $I$  and player against practice is  $j$ . The most popular non-linear functions in competitive models including two multiplicative model and competitive log are MCI models and MNL. In these models, the amount demand function in fact is equal to market share of each competitor in the total amount of market demand, the market share of each firms in the market are equal and the amount of each firms' sales relative to overall market demand that has the following function:

$$R_i = \frac{D_i}{\sum_{j=1}^n D_j} \Rightarrow D_i = R_i \times D_0, \sum_{i=1}^n R_i = 1, \sum_{i=1}^n D_i = D_0 \text{ if } \omega = 0 \quad (13)$$

$$\text{MCI Model: } R_i = \frac{M_i}{\sum_{j=1}^n M_j}, M_i = e^{\rho_i} \cdot \prod_{k=1}^K X_{ki}^{\nu_k} \cdot \varepsilon_i \quad (14)$$

$$\text{MNL Model: } R_i = \frac{M_i}{\sum_{j=1}^n M_j}, M_i = e^{\left(\rho_i + \sum_{k=1}^K \nu_k \cdot X_{ki} + \varepsilon_i\right)}$$

In the above equation firm's market share is  $i$  and condition means that assumed that customers have no option for not purchasing the product. The market share of MCI and MNL Model is calculated as follows: In these two equations firm marketing efforts is  $i$  and  $km$  parameter affecting on demand and is the impact factor or stretch each of the parameters on demand or marketing efforts.  $\rho$  also is defined as constant factor effect and impact of marketing efforts that in fact a sign of the impact of each firm's efforts in the community as well as is the impact of

random variables that are not placed among the model parameters. Also,  $e$  is estimated the display error. It will be achieved by multiplying each firm's market share in the total market demand, the demand of each firm calculated. Thus, we can consider the demand of each firm is as follows:

$$\begin{aligned} \text{MCI Model: } D_i &= \frac{M_i}{\sum_{j=1}^n M_j} \times D_0 = \frac{D_0 \cdot e^{\rho_i} \cdot \prod_{k=1}^K (X_{ki})^{\nu_k} \cdot \varepsilon_i}{\sum_{j=1}^n e^{\rho_j} \cdot \prod_{k=1}^K (X_{kj})^{\nu_k} \cdot \varepsilon_j} \\ \text{MNL Model: } D_i &= \frac{M_i}{\sum_{j=1}^n M_j} \times D_0 = \frac{D_0 \cdot e^{\left(\rho_i + \sum_{k=1}^K \nu_k \cdot X_{ki} + \varepsilon_i\right)}}{\sum_{j=1}^n e^{\left(\rho_j + \sum_{k=1}^K \nu_k \cdot X_{kj} + \varepsilon_j\right)}} \end{aligned} \quad (15)$$

According, to the non-linear above equations as well as the multiplication among variables, calculated equilibrium through the gradient will be hard to do, plus calculated the equivalent coefficients of the above conditions is also difficult and cannot do. So, we was tried to introduced conceptual model for the dynamics system in this conditions and identify relation among the competitors and behavior of players with this method.

For this purpose, we were trying by using method of conversion the log of axis and by making linear each equation can estimate coefficient values and determined how they impact on each of the parameters and on the demand for any firm. How to make linear equations of demand in the MCI Model with log-based method is as follows:

$$\begin{aligned} D_i &= \frac{M_i}{\sum_{j=1}^n M_j} \times D_0 = \frac{D_0 \cdot e^{\rho_i} \cdot \prod_{k=1}^K (X_{ki})^{\nu_k} \cdot \varepsilon_i}{\sum_{j=1}^n e^{\rho_j} \cdot \prod_{k=1}^K (X_{kj})^{\nu_k} \cdot \varepsilon_j} \\ \log(D_i) &= \log(R_i \times D_0) = \log(D_0) + \log(R_i) \\ \Rightarrow \log(D_i) &= \log(D_0) + \log\left(\frac{M_i}{\sum_{j=1}^n M_j}\right) \\ &= \log(D_0) + \log(M_i) - \log\left(\sum_{j=1}^n M_j\right) \end{aligned} \quad (16)$$

Now if value number 17 sum for all  $I$  values and divided into all number of firms we have:

$$\begin{aligned} \Rightarrow \log(D_i) &= \log(D_0) + \rho_i + \sum_{k=1}^K \nu_k \cdot \log(X_{ki}) + \\ &\log(\varepsilon_i) - \log\left(\sum_{j=1}^n e^{\rho_j} \cdot \prod_{k=1}^K X_{kj}^{\nu_k} \cdot \varepsilon_j\right) \end{aligned} \quad (17)$$

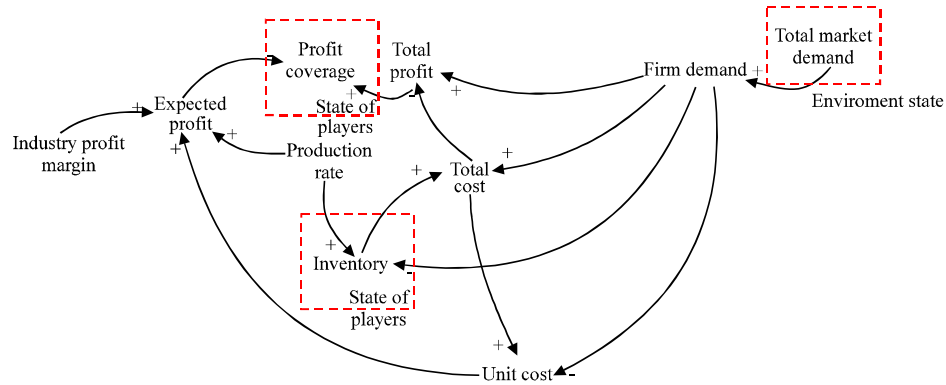


Fig. 4: Effect of enterprises demand on cost and benefit of each market

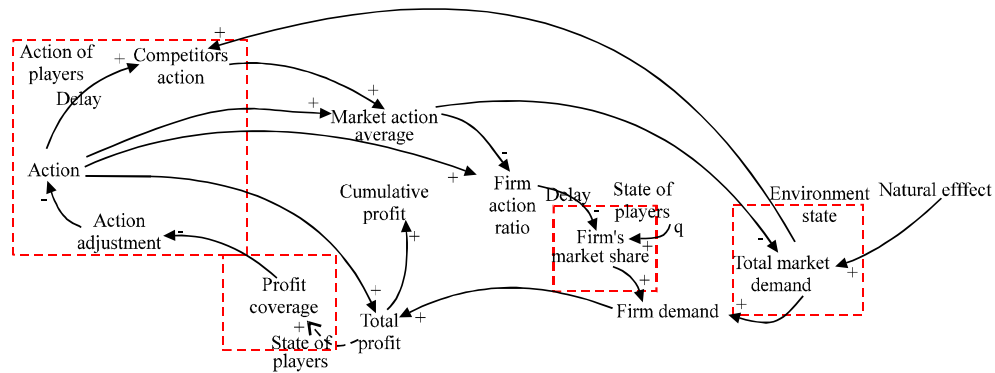


Fig. 5: Effect of player action on market share

Which in above equation  $\tilde{D}, \tilde{X}, \tilde{E}$  represent the average of related Geometric mean and  $\bar{p}$  show average related arithmetic mean then with minus 18 equations of 17 we will have:

$$\log\left(\frac{D_i}{\bar{D}_i}\right) = \rho_i^* + \sum_{k=1}^K v_k \cdot \log\left(\frac{\tilde{X}_{ki}}{\bar{X}_k}\right) + \varepsilon_i^* \quad (19)$$

In above equations we have:

$$\rho_i^* = \rho_i - \bar{\rho}, \quad \varepsilon_i^* = \log\left(\frac{\varepsilon_i}{\bar{\varepsilon}}\right)$$

We will have the following equations by converting to MNL model in which will show us related arithmetic mean:

$$\log\left(\frac{D_i}{\bar{D}_i}\right) = \rho_i^* + \sum_{k=1}^K v_k \cdot (X_{ki} - \bar{X}_k) + \varepsilon_i^* \quad (20)$$

Achieving Eq. 19 and 20 have several important advantages. First by making linear equations of demand we can estimate the possibility of the coefficients of each of the parameters. In addition, this relation suggests that what is actually affect to the sales firms the amount of

affected deviation parameters to the demand in compared to the market average. In fact, in these equations collection can be considered as a actions or players decisions:

$$O_i = \{X_{ki}\}, \quad k=1, 2, \dots, K$$

k is the number of variables that decision is in the hands of each player. Now With this regard to the conceptual model Fig. 3 can convert the Eq. 19 and 20 as a cause and effect model. At first to create cause and affect models we determine conceptual and dynamic system model of Oligopoly market with regard to profit function as income function can form cost function as Fig. 4.

Considering the overall shape of each firm action in the market we can provided model the player's cause and effect actions based on the conceptual model of Fig. 3 and related Eq. 19 and 20 as follows. In fact, in Fig. 4, stated that measure of each firm's will effecton actions of competition and ultimately it specifies each firm action over to the average of the total market share one in that specific variables. With combination of Fig. 4 and 5 we can achieve the figure of cause and effect of game oligopoly based on Eq. 18, 19 and 20.

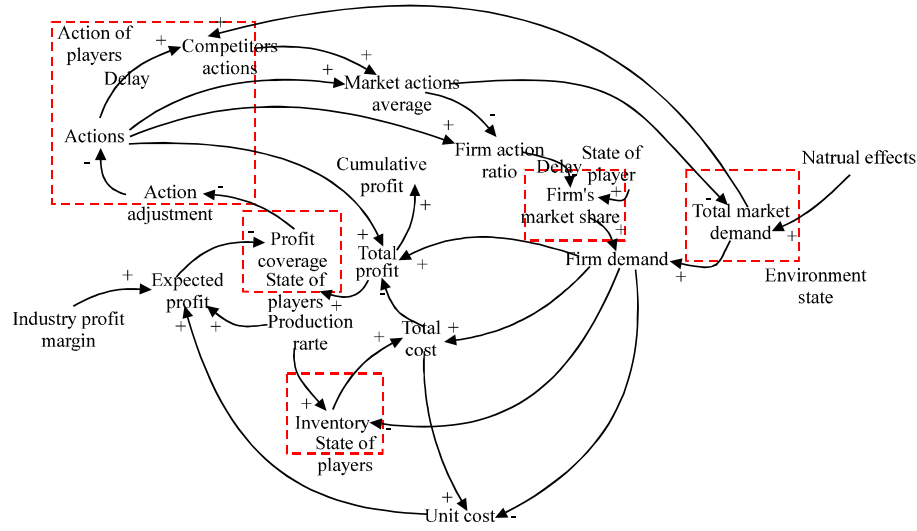


Fig. 6: Conceptual model of game

The important thing in this chart compliance with the conceptual model presented in Fig. 2 and 3. In fact, the cause and effect diagram provided by the totality of an oligopoly game, the total demand market can be described as a state of environment. In addition, market share, the amount of per firm's profit coverage over what is expected and even inventories can be considered as state players. Existing relation lead to the decisions of the players and their actions in fact form the same mechanism of decision-making firms. Diagram of all cause and effect market will be shown in Fig. 6.

Similarly, the model presented in Fig. 6 can be used as a conceptual model in the study of oligopoly markets by using of hybrid model dynamical system and game theory. With using of this conceptual model in simulation ones can determine game balance and also behavior of system in transient period over the existing models

## CONCLUSION

During this research was trying to help a hybrid method of game theory and dynamic systems to investigate the behavior of firms in an oligopoly market. Accordingly, at the first study we determine comparison of game theory and dynamic systems and stated strengths and weaknesses of each of them and benefits of combining these two methods. Then, by presenting a conceptual model of a dynamic system game we laid the foundations of the original issue.

Then with modeling mathematical market attempt game theory disadvantage examined in determining the

behavior of firms in these markets. For this purpose at the beginning of each firm's total profits function in an oligopoly market determined as a good function of the utility or revenue. According, to profit function as a good function of each firm, stated the necessary of demand function to maximize the benefits.

After taking into account non-linear multiplicative models demand and due to the difficulty of determining the equilibrium in such demand models were tried such as MCI and MNL Models. The behavior of firms and the balance of the game examined through modeling of dynamics system for that demand multiplicative competitor with help of log base method become linear and use as the base of modeling dynamics system. Finally, the conceptual model of an oligopoly market structure introduced based on hybrid game theory and dynamics system. We can refer to this point via innovation in research in most provided past research and no pay attention to the effect of player to environment and vice versa while in this research evaluated all demand of market as a game environment and creating dynamic loops. Due to the combination of system dynamics of game theory has been provided to study the behavior of firms and other parameters of the model during the transition period.

Also, in most circumstances research and position players as stimulating of system has not been considered in the present study, the main engine firm decisions have been taken states of the players and the environment. In addition with regard of the delay in deciding on another model of innovation combined approach of game theory

and dynamics presented in this thesis. As further suggestions you can check the decision of variables including price, product quality and advertising oligopoly market model and related demand function as well as mentioned the other factors in the competition between monopolies.

## REFERENCES

- Akiyamaa, E. and K. Kaneko, 2000. Dynamical systems game theory and dynamics of games. *Physica*, 147: 221-258.
- Akiyamaa, E. and K. Kaneko, 2002. Dynamical systems game theory II, A new approach to the problem of the social dilemma. *Physica*, 167: 36-71.
- Elettrey, M. and M. Mansour, 2012. On cournot dynamic multi-team game using incomplete information dynamical system. *Appl. Math. Computation*, 218: 10691-10696.
- Elettrey, M. and S. Hassan, 2006. Dynamical multi-team cournot game. *Chaos Solitons Fractals*, 27: 666-672.
- Kima, D. and D. Kimb, 1997. A system dynamics model for a mixed-strategy game between police and driver. *Syst. Dynamics Rev.*, 13: 33-52.
- Mohammadi, A. and A. Moslehshirazi, 2016. The combination of system dynamics and game theory in analyzing oligopoly markets. *Manage. Sci. Letters*, 6: 265-274.
- West, R. and C. Lebiere, 2001. Simple games as dynamic, coupled systems: Randomness and other emergent properties. *J. Cognitive Syst. Res.*, 1: 221-239.