

## A Situational Model of Investment Portfolio

<sup>1</sup>Valeriy V. Davnis, <sup>2</sup>Many A. Ziroyan, <sup>1</sup>Marina V. Vladika,  
<sup>1</sup>Elena N. Kamyshanchenko and <sup>1,2</sup>Viktoriya I. Tinyakova  
<sup>1</sup>Belgorod State University, Belgorod, Russia  
<sup>2</sup>Russian State Social University, Moscow, Russia

**Abstract:** The study offers a new approach to building models of portfolio investments, in which the covariance matrix of Markowitz Model is replaced by a specially constructed matrix of additive interaction of financial assets. It is shown that additive interaction reflects more differentiated view on the situations that appears on the stock market than the multiplicative interaction which underlines the building of covariance matrix. Special regression models are constructed for the estimation of the additive interaction. With the help of these models the possible interaction options are defined. On the basis of the received options for each pair of assets a number of dependent variables are formed (which are corresponding to the regression models). The mathematical expectations of these models are considered as estimations of the additive interaction of the assets in the alleged situation and are used as the elements of the matrix of the Portfolio Investment Model. Empirical studies have confirmed the usefulness of the proposed approach in the problem of investment decisions in the stock market.

**Key words:** Portfolio investments, stock market, risk-effect, additive interaction of assets, matrix of additive interaction

### INTRODUCTION

The model which is suggested by Markowitz (1952, 1991, 1999) for formation of a portfolio of securities is based on the unit of statistical estimation. The versatility of this apparatus and the reliability of the results obtained by its help is not doubted. The model itself became a sort of an idea generator that underlines the efficient-market theory. At the same time, its practical use was very limited.

Among the most notable results in these trials, a special place, in our opinion is taken by the diagonal model of Sharpe. First regression models were used to for this model. Although, we can't disagree that Sharpe (1963, 1964, 1970) could use the potential of regression analysis very skillfully for construction of his model, this potential is bounded by simple-factored models. Modern theory and practice of econometrical modeling lets us to get rid of these boundaries. The research "Single-Component Model of Portfolio Investment" (Davnis *et al.*, 2012) could be an example in which a modified version of the diagonal model of Sharpe is presented. The presented model considers the connection of different markets in conditions of globalization (Rutkauskas and Kvietkauskienė, 2013).

Further the potential applications of combined regression models are under research to reveal the problems of portfolio investments. These models will let us reflect the nature of financial assets more precisely.

### MATERIALS AND METHODS

**Main approaches to modeling of dynamics of financial assets:** Processes connected with modeling of dynamic characteristics (cost, profitability, risk) of financial assets underline the justification of investment decisions. That's why, these models are always specified and corrected. Sharpe used this single-factored model:

$$r_{it} = \alpha_i + \beta_i r_{it} + \varepsilon_{it}, t = \overline{1, T}, i = \overline{1, n} \quad (1)$$

Where:

- $r_{it}$  = Profitability of the  $i$ th asset at time  $t$
- $r_{it}$  = Profitability of the market index at time  $t$
- $\alpha_i, \beta_i$  = Estimated coefficients of the regression model of the  $i$ th asset
- $\varepsilon_{it}$  = Unobserved random variable which characterizes the part of profitability change of the  $i$ th asset that can't be explained in terms of variation of market profitability

By using the only factor of Eq. 1 we can't get a full understanding of the mechanisms of the formation of the profitability of assets which are included in the portfolio. But, it provides correct output of the formulas which are needed for the diagonal model. The attempts of using of multifactorial models (k-factor model of the market of risky assets, arbitrage capital asset pricing model) did not give such a bright result as the diagonal Sharpe model.

Moreover if the financial asset included in portfolio is considered as the base when justifying the risk-neutral price of an option with the CRR-model (Cox *et al.*, 1979), then in the description of price evolution on the (B, S) market the asset model is discrete (Shiryaev, 1998):

$$B_t = (1 + r_t) B_{t-1} \quad (2)$$

$$S_t = (1 + \rho_t) S_{t-1} \quad (3)$$

Where:

- $B_t$  = Value of the bank account at time  $t$
- $S_t$  = Value of the financial asset at time  $t$
- $r_t$  = Constant bank rate ( $r_t = r$ )
- $\rho_t$  = Changing in time profitability of the asset which is a Bernulli sequence of independent identically distributed random variables, taking two values with some probabilities

$$\rho_n = \begin{cases} b, & a < b \\ a, & \end{cases}$$

The most complete view on the nature of changes which are taking place in the stock market is provided by the known Bachelier model (Bachelier, 1964):

$$\frac{\Delta S}{S} = \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t} \quad (4)$$

Where:

- $S$  = Cost of the financial asset
- $\Delta S$  = The variation of  $S$  in time  $\Delta t$
- $\mu$  = Average profitability level of the financial asset
- $\Delta t$  = Small period of time
- $\sigma$  = The risk measured by the standard deviation of return
- $\varepsilon$  = Normally distributed random variable with zero mean and unit variance

By reflecting both continuous and discrete changes in dynamics of financial assets, the Bachelier Model emulates the nature of the assets more precisely. But, this emulation is possible only for a short period of time. The idea of considering both discrete and continuous changes in one single model in our opinion, should be used in the econometrical model.

**Econometrical model of financial assets:** The econometrical analogue of (Eq. 4) could be written as a discrete-continuous model in this way:

$$r_{it} = \alpha_i + \beta_i r_{t-1i} + d_i x_{it} + \varepsilon_{it}, t = \overline{1, T}, i = \overline{1, n} \quad (5)$$

Where:

- $r_{it}$  = Profitability of the  $i$ th asset at moment  $t$
- $r_{t-1i}$  = Profitability of the  $i$ th asset at time that precedes  $t$
- $x_{it}$  = Discrete random variable which can be -1 or +1. It characterizes the direction of the discrete jump of the profitability of the  $i$ th asset at time  $t$
- $\alpha_i, \beta_i$  = The estimated coefficients of the continuous part of the model of the  $i$ th asset
- $d_i$  = The estimated coefficient of the model which is considered as the average risk rate

The model is constructed in two stages. On the first stage the coefficients of the regression model ( $\alpha, \beta$ ) are estimated and in accordance with the hypothesis of alternative expectations (Davnis and Korotkikh, 2014), values of the discrete variable  $x_{it}$  are identified on a historical period:

$$x_{it} = \begin{cases} +1, & r_{it} - \hat{r}_{it} \geq 0 \\ -1, & r_{it} - \hat{r}_{it} < 0 \end{cases} \quad (6)$$

where,  $\hat{r}_{it}$  is rated profitability value of the  $i$ th asset at moment  $t$ . After the identification of  $x_{it}$  all the coefficients of Eq. 5 are estimated:

$$r_{it} = \hat{\alpha}_i + \hat{\beta}_i r_{t-1i} + \hat{d}_i x_{it}, t = \overline{1, T}, i = \overline{1, n} \quad (7)$$

With help of Eq. 7, we can recreate the whole historical period more precisely, than by using the autoregression model. This is a natural result but not the main one. Much more important is that the model reflects a different understanding of the nature of interaction of the profitability of the asset with the profitability of the market. This understanding opens a way to form another approach to modeling of a portfolio of securities. But, first we should expand the potential of the Eq. 7.

The main flaw of the discrete-continuous model is the lack of a mechanism which could let us use the discrete part in predictive calculations. Although, the opportunity of getting an estimation of the mean profitability of the market  $\hat{r}_{t+1}$  is obvious, obtaining of the expected value of the discrete variable  $\hat{x}_{t+1}$  is still problematic. One of the convenient ways to solve the problem is the model of

binary choice (Davnis and Tinyakova, 2005; Tinyakova, 2008). For example in the logit-model, the regressing dependence is described with these equations:

$$\hat{r}_{it} = \hat{\alpha}_i + \hat{\beta}_i r_{t-1i} + \hat{d}_i x_{it} \quad (8)$$

$$P_{it} = P(x_{it} = -1 | z_{it}) = \frac{e^{\hat{b}_{oi} + \hat{b}_{1i} z_{it}}}{1 + e^{\hat{b}_{oi} + \hat{b}_{1i} z_{it}}}, \quad t = \overline{1, T}, i = \overline{1, n} \quad (9)$$

Where:

- $\hat{r}_{it}$  = The rated profitability value of the  $i$ th financial asset at moment  $t$
- $\alpha_i, \beta_i$  = Estimations of the coefficients of the continuous part of the model of the  $i$ th asset
- $\hat{d}_i$  = The estimation of the mean risk value of the  $i$ th asset
- $\hat{b}_{oi}, \hat{b}_{1i}$  = Estimations of the coefficients of the logit-model of the  $i$ th asset
- $z_{it}$  = Independent variable of the logit-model
- $P_{it}$  = Probability of having a low profitability of the  $i$ th asset at moment  $t$

It should be noted that sometimes, we can't pick a meaningfully interpreted independent variety of  $z_{it}$ . In these cases, it is formed according to special procedures and methods. In our case, we can use the deviation of the index from the current mean profitability of the market as values of the variable.

The hypothesis of proportionality underlines the right to use this approach (Davnis and Korotkikh, 2014). According to this hypothesis the deviation of profitability of an asset from the trend is proportional to the level of the influence of unforeseen factors from the set of the desultory and as usual, external factors. In other words, the profitability of an asset reacts on the changes that occur on the stock market that are engendered both by market and non market events.

With help of the mathematical expectation the construction of the final version of the regression model is made:

$$\hat{r}_{it} = \hat{\alpha}_i + \hat{\beta}_i r_{t-1i} + \hat{d}_i - 2\hat{d}_i P_{it} \quad (10)$$

This is a new type of econometrical model which realizes the mechanism of clarification of the calculated values with the expected values of risk-effect  $\hat{d}_i - 2\hat{d}_i P_{it}$ . We should note that the risk-effect may have negative values. It varies from  $-\hat{d}_i$  to  $+\hat{d}_i$  and depends on the probabilities determined by the external factors. It is a situational index not a mean risk. That's why, a portfolio built up with the accordance to risk-effect is oriented on an expected situation.

**Interaction-risk and the situational portfolio of securities:** The portfolio management which takes

risk-effects into account, demands preliminary identification of the main characteristics of the portfolio through, the parameters of Eq. 10. These characteristics differ from the mean values which are usually used in models of portfolio investments because they depend on the risk-effects, that actualize those mean values according to the expected situation. At first, we should define the profitability of a portfolio in case when risk-effects are taken into consideration.

If  $w = (w_1, w_2, \dots, w_n)'$  is a vector that characterizes the structure of the portfolio featuring the structure of the portfolio, then its expected profitability in our situation could be written as:

$$\begin{aligned} r_{tp} &= w_1 \hat{r}_{t1} + w_2 \hat{r}_{t2} + \dots + w_n \hat{r}_{tn} \\ &= w_1 \hat{\alpha}_1 + w_2 \hat{\alpha}_2 + \dots + w_n \hat{\alpha}_n + w_1 \hat{\beta}_1 r_{t-11} + \\ &\quad w_2 \hat{\beta}_2 r_{t-12} + \dots + w_n \hat{\beta}_n r_{t-1n} + w_1 (\hat{d}_1 - 2\hat{d}_1 P_{t1}) \\ &\quad w_2 (\hat{d}_2 - 2\hat{d}_2 P_{t2}) + \dots + w_n (\hat{d}_n - 2\hat{d}_n P_{tn}) \end{aligned} \quad (11)$$

In this Eq. 11, the profitability of an asset is divided into three parts: private profitability of the asset, profitability, formed from the results of evolutionary changes occurring in the market and profitability due to the risk-effects of current moment. Our aim is to form a portfolio with high profitability in other words, we must maximize (Eq. 11). The Markowitz model is one of the best to fit these demands. In the opinion, we could use it to build up the situational model of portfolio investments by substituting the covariance matrix with the matrix of interaction. To be able to form such a matrix, we must obtain an expression which will provide us with a mechanism of estimation of risk-effects in any situation.

If we abandon, the preservation of specifics of risk-effects, the easiest solution is to consider risk-effects as deviations which occurred on some period of time and build up a covariance matrix based on them. We would get modified Markowitz Model in that case. This is an interesting approach to obtain a new model of portfolio investment but its realization will lead us to average solution which would not reflect the specifics of the situation which is described by the risk-effect. That's why, we will tend to replace the covariance matrix with a specially formed matrix of paired risk-interactions of financial assets, included in the portfolio. This approach does not consider specific risks which have small influence compared to the whole risk that is considered in the discrete-continuous model.

For simplicity, we will review a portfolio which consists of only two assets. The simplified example gives a good vision on the structure of the matrix of paired risk-interactions and the formation mechanism of the matrix for more number of assets. Considering risk-effect

of the portfolio as the result of paired risk-interactions of assets, we designate it as  $IA_p$ , let us write down its formulae in this particular situation:

$$\begin{aligned} IA_p(w_1r_1 + w_2r_2) &= IA(w_1r_1w_1r_1) + IA(w_2r_2w_2r_2) + \\ &\quad IA(w_1r_1w_2r_2) + IA(w_2r_2w_1r_1), \quad (12) \\ IA_p(w_1r_1 + w_2r_2) &= w_1^2IA(r_1r_1) + w_2^2IA(r_2r_2) + \\ &\quad w_1w_2IA(r_1r_2) + w_2w_1IA(r_2r_1) \end{aligned}$$

This formulae is constructed according to a similar logical scheme that was used to form the covariance matrix. To generalize this formula to any other number of assets the expression is conveniently written in matrix form:

$$IA_p = (w_1, w_2) \times \begin{pmatrix} IA(r_1r_1) & IA(r_1r_2) \\ IA(r_2r_1) & IA(r_2r_2) \end{pmatrix} \times \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \quad (13)$$

This matrix could be generalized for the case of many assets in the portfolio. The structure of this generalization does not differ from the one we did in covariance matrix. It is natural because, we consider at both matrixes the paired interactions of risk. The difference of the approaches is not in the structure of the matrix but in the definitions of the elements of the structure. The covariance matrix is built up on the mean results of multiplicative interactions while the matrix of paired interactions is based on the current results of additive interactions. In order to be able to see the difference between the approaches, we will represent all possible interactions in Table 1.

As we can see, the additive interaction model provides more detailed results of interactions. The problem of forming this matrix lays in the apparatus of statistical estimation which does not give a mechanism of rating the paired risk-interaction values. We offer using an econometrical approach. Its realization is based on an assumption that the effects of risk-interactions took part in the past. In order to identify them, we must review a discrete dependent random variable which can be described in this way:

$$\begin{aligned} -d_1 - d_2 &\leftrightarrow 0; -d_1 + d_2 \leftrightarrow 1, \\ d_1 - d_2 &\leftrightarrow 2; d_1 + d_2 \leftrightarrow 3 \end{aligned}$$

By using this dependent variable, the deviations of the profitability of the index from its mean value as the independent variable and the historical data, we can build up the multinomial logit-model of multiple choice:

$$P_j^{ik}(y_t = j) = \frac{e^{b_{0j}^{ik} + b_{1j}^{ik}z_d}}{1 + \sum_{j=0}^2 e^{b_{0j}^{ik} + b_{1j}^{ik}z_d}}, \quad j=0,1,2 \quad (14)$$

Table 1: Interactions of two assets in a portfolio

Options for changes in the profit		Options for interaction	Options for covariance
Asset 1	Asset 2		
$d_1$	$d_2$	$d_1 + d_2$	$d_1 d_2$
	$-d_2$	$d_1 - d_2$	$d_1(-d_2)$
$-d_1$	$d_2$	$-d_1 + d_2$	$(-d_1)d_2$
	$-d_2$	$-d_1 - d_2$	$-d_1(-d_2)$
The total number of options		4	2

$$P_3^{ik}(y_t = 3) = \frac{1}{1 + \sum_{j=0}^2 e^{b_{0j}^{ik} + b_{1j}^{ik}z_d}} = 1 - P_0^{ik} - P_1^{ik} - P_2^{ik} \quad (15)$$

With help of this model, we can establish the probabilities of the expected effects of paired risk-interactions and evaluate the mathematical expectations of these effects:

$$\begin{aligned} IA(r_1r_k) &= d_1 + d_k - 2d_kP_2^{ik} - \\ &\quad 2d_1P_1^{ik} - 2(d_1 + d_k)P_0^{ik} \end{aligned} \quad (16)$$

All non-diagonal coefficients of risk-interactions are calculated with this formula. Matrix of the paired risk-interactions is more informative tool for reflecting the situation on the stock market than the covariance matrix. Table 1 illustrates it.

This matrix has another feature: its diagonal elements are calculated in another way. It is so due to the fact that the interaction of the asset with itself could be described only with two situations not four. That's why, the probabilities of these situations could be described through the econometrical model of binary choices (Eq. 10) and the numerical characteristics of paired risk-interactions is calculated with the known formulae:

$$IA(r_1r_1) = d_1 - 2d_1P_1 \quad (17)$$

This matrix will let us define the risk-effect of the portfolio with taking the expected situation into consideration. The model in which the risk will be defined by this matrix, we will call the situational model of investment portfolio. We write the equations down as it was written in Markowitz model:

$$w' \Sigma_{IAZ} w \rightarrow \min \quad (18)$$

$$w'r = u \quad (19)$$

$$w'i = 1 \quad (20)$$

where,  $\Sigma_{IAZ}$  matrix of paired risk-interactions of financial assets in the portfolio. Let us point at some features of this model. First feature is that the construction of the

matrix is fully underlined by econometrical approach without using statistical methods. The second is that the model could be tuned on an expected situation which is forecasted according to the activity of the market. The third is that the investment strategies, acquired through this model both positive and negative risk-effect. Additional opportunities could be obtained by using a model with combined criteria:

$$\tau w' r_z - w' \sum_{i \in A_z} w \rightarrow \max \quad (21)$$

$$w' i = 1 \quad (22)$$

Where:

$r_z$  = The vector of profitability of the assets which is actualized by risk-effects of the current moment  
 $\tau$  = Parameter of confidence of the investor to the forecasted values of profitability ( $\tau > 0$ )

There is no unique recommendation for definition of parameter  $\tau$ .

## RESULTS

The sample describes modeling results based on the matrix of pair additive interaction of financial assets and are calculated according to the updated situation values of their expected returns. The calculations used the weekly stock prices of “Gazprom”, “Norilsk Nickel”, “Lukoil”, “Savings Bank” and “Surgutneftegas” for the period of 04.01.2013 to 01.08.2014 where the situation has been evaluated as the most favorable ( $z = 99$ ).

Here are the discrete-continuous models of financial assets:

$$\begin{aligned} r_{1t} &= 0.272 + 0.266r_{1t-1} + 3.083x_{1t}; \\ r_{2t} &= 0.221 - 0.216r_{1t-1} + 2.523x_{2t}; \\ r_{3t} &= 0.020 - 0.181r_{3t-1} + 1.941x_{3t}; \\ r_{4t} &= -0.021 + 0.167r_{4t-1} + 3.060x_{4t}; \\ r_{5t} &= -0.076 - 0.096r_{5t-1} + 2.447x_{5t} \end{aligned}$$

Models for calculating the average yield on the asset, expect the updated situation:

$$\begin{aligned} \hat{r}_{1t} &= 0.272 + 0.266r_{1t-1} + 3.083 - 2 \times 3.083 \frac{\exp(-0.232 + 0.008z)}{1 + \exp(-0.232 + 0.008z)}; \\ \hat{r}_{2t} &= 0.221 - 0.216r_{1t-1} + 2.522 - 2 \times 2.523 \frac{\exp(-0.148 + 0.002z)}{1 + \exp(-0.148 + 0.002z)}; \\ \hat{r}_{3t} &= 0.020 - 0.181r_{3t-1} + 1.941 - 2 \times 1.941 \frac{\exp(-0.292 + 0.006z)}{1 + \exp(-0.292 + 0.006z)}; \\ \hat{r}_{4t} &= -0.021 + 0.167r_{4t-1} + 3.060 - 2 \times 3.060 \frac{\exp(-0.304 + 0.030z)}{1 + \exp(-0.304 + 0.030z)}; \\ \hat{r}_{5t} &= -0.076 - 0.096r_{5t-1} + 2.447 - 2 \times 2.447 \frac{\exp(0.209 + 0.004z)}{1 + \exp(0.209 + 0.004z)} \end{aligned}$$

Formation of the interaction matrix assets:

$$\begin{aligned} IA(r_1 r_1) &= 3.083 - 2 \times 3.083 \frac{\exp(-0.232 + 0.008z)}{1 + \exp(-0.232 + 0.008z)}; \\ IA(r_1 r_2) &= (3.083 + 2.523) - 2 \times 2.523 P_2^{12} - 2 \times 3.083 P_1^{12} - 2 \times (3.083 + 2.523) P_0^{12}; \\ P_0^{12} &= \frac{\exp(-0.331 - 0.009z)}{1 + \sum^{12}}; P_1^{12} = \frac{\exp(-0.506 - 0.001z)}{1 + \sum^{12}}; P_2^{12} = \frac{\exp(-0.506 - 0.008z)}{1 + \sum^{12}}; \\ \sum^{12} &= \exp(-0.331 - 0.009z) + \exp(-0.407 - 0.001z) + \exp(-0.506 - 0.008z); \\ IA(r_1 r_3) &= (3.083 + 1.941) - 2 \times 1.941 P_2^{13} - 2 \times 3.083 P_1^{13} - 2 \times (3.083 + 1.941) P_0^{13}; \\ P_0^{13} &= \frac{\exp(-0.426 - 0.011z)}{1 + \sum^{13}}; P_1^{13} = \frac{\exp(-1.319 + 0.014z)}{1 + \sum^{13}}; P_2^{13} = \frac{\exp(-1.164 + 0.016z)}{1 + \sum^{13}}; \\ \sum^{13} &= \exp(-0.426 - 0.011z) + \exp(-2.319 + 0.014z) + \exp(-1.164 + 0.016z); \\ IA(r_3 r_4) &= (2.447 + 3.060) - 2 \times 3.060 P_2^{54} - 2 \times 2.447 P_1^{54} - 2 \times (2.447 + 3.060) P_0^{54}; \\ P_0^{54} &= \frac{\exp(0.366 - 0.005z)}{1 + \sum^{54}}; P_1^{54} = \frac{\exp(-0.782 + 0.0002z)}{1 + \sum^{54}}; P_2^{54} = \frac{\exp(-0.618 - 0.001z)}{1 + \sum^{54}}; \\ \sum^{54} &= \exp(0.366 - 0.005z) + \exp(-0.782 - 0.002z) + \exp(-0.618 + 0.001z); \\ IA(r_5 r_5) &= 2.447 - 2 \times 2.447 \frac{\exp(0.209 - 0.004z)}{1 + \exp(0.209 - 0.004z)} \end{aligned}$$

Matrix additive interaction of assets and the average yield, actualized by the expected situation:

$$\sum_{IAz} = \begin{pmatrix} -0.870 & 1.112 & 0.810 & 1.586 & 1.172 \\ 1.112 & -0.050 & 1.357 & 1.228 & 1.037 \\ 0.810 & 1.357 & -0.251 & 1.578 & 1.774 \\ 1.586 & 1.228 & 1.578 & -0.010 & 1.974 \\ 1.172 & 1.037 & 1.774 & 1.974 & 0.008 \end{pmatrix};$$

$$\hat{r} = \begin{pmatrix} 0.688 \\ -0.276 \\ -1.009 \\ 0.996 \\ -0.171 \end{pmatrix}$$

The resulting matrix and the vector are used in the calculations by Eq. 18-20 and 21-22.

## DISCUSSION

Calculation experiments are held with three models and bonds of Russian and French Stock Markets. Five calculations for each market were held and consequently five portfolios are formed for each:

- Portfolio depending only on the matrix additive interaction of assets
- Self-financing portfolio
- A portfolio that optimizes the combined test pattern (Eq. 21-22)
- A portfolio that optimizes the criterion of the Eq. 18-20
- Markowitz portfolio

Yield formation for each portfolio was calculated, that provided profit on both the historical and the proactive periods of time.

For the 1st period of formation the portfolio is based on the data from 04.01.2013 to 01.08.2014 and tests portfolio of data-ahead points in time at 08.08.2014.

### Consequently:

- 2nd period portfolio s formed from 04.01.2013 to 08.08.2014, testing 08.15.2014
- 3rd period portfolio s formed from 04.01.2013 to 15.08.2014, testing 08.22.2014
- 4th period portfolio is formed from 04.01.2013 to 22.08.2014, testing 29.08.2014
- 5th period portfolio is formed from 4.01.2013 to 29.08.2014, testing 05.09.2014

Thus, the portfolio every time was updated with new observations when formed and the testing was carried out on the next observation. These results are shown in Table 2 and 3.

For the Russian stock market the portfolio was formed of five shares of the following companies: “Gazprom”, “Norilsk Nickel”, “Lukoil”, “Savings Bank” and “Surgutneftegaz”. We used data from weekly quotations for the periods indicated above.

For the French stock market the portfolio was formed of shares of the following companies: “Total”, “Axa”, “Danone”, “Societe general” and “Sanofi”. We used the data synchronized with the data of the Russian stock market.

For all cases, the test shows that preference should be given to the portfolio with combined criteria where the interaction of matrix assets is used. This is an expected result as far as the model of this portfolio provides two

Table 2: The model of calculation on portfolio investment (bonds of the Russian stock market)

Periods	Portfolio characteristics	Portfolio				
		$w_{min}$	$w_c$	$w_{min}$	$w_{IAz}$	$w$
1	The yield on the historical period	-0.131	0.335	0.204	-0.142	0.100
	The yield in proactive time	3.127	0.526	3.653	3.110	3.174
	Risks	1.204	-1.993	-0.788	1.202	2.160
2	The yield on the historical period	-0.106	0.384	0.278	-0.108	0.100
	The yield in proactive time	-4.362	4.364	0.002	-4.386	-1.507
	Risks	1.106	-0.977	0.129	1.106	2.170
3	The yield on the historical period	-0.012	-0.354	-0.367	-0.029	0.100
	The yield in proactive time	5.603	3.754	9.357	5.781	4.524
	Risks	1.689	-1.446	0.243	1.686	2.159
4	The yield on the historical period	0.067	0.189	0.256	0.082	0.100
	The yield in proactive time	-1.246	1.930	0.684	-1.085	-0.091
	Risks	1.599	-3.472	-1.873	1.575	2.248
5	The yield on the historical period	0.035	-0.198	-0.163	-0.016	0.100
	The yield in proactive time	-0.224	0.956	0.732	0.022	-2.076
	Risks	1.548	-0.516	1.032	1.514	2.230

Table 3: The model of calculation on portfolio investment (bonds of the French stock market)

Periods	Portfolio characteristics	Portfolio				
		$w_{min}$	$w_c$	$w_{min}+w_c$	$w_{bez}$	$w$
1	The yield on the historical period	1.070	-0.573	0.496	0.058	0.100
	The yield in proactive time	6.262	-5.351	0.910	-3.184	-0.973
	Risks	15.476	-5.530	9.945	-1.754	2.499
2	The yield on the historical period	0.161	0.447	0.607	0.128	0.100
	The yield in proactive time	-0.370	2.306	1.935	-0.541	-0.412
	Risks	0.030	-4.114	-4.084	0.008	1.904
3	The yield on the historical period	0.131	0.347	0.478	0.040	0.100
	The yield in proactive time	0.734	4.599	5.333	-0.476	0.588
	Risks	-0.032	-2.431	-2.463	-0.200	1.712
4	The yield on the historical period	0.133	0.332	0.466	-0.053	0.100
	The yield in proactive time	3.174	1.507	4.681	2.330	3.468
	Risks	-0.181	-1.839	-2.020	-0.758	1.669
5	The yield on the historical period	0.192	0.299	0.491	-0.059	0.100
	The yield in proactive time	2.838	1.723	4.560	1.392	2.371
	Risks	-0.254	-1.702	-1.956	-1.452	1.975

options where you can take into account the effectiveness of the expected situation and confidence in estimates of expected assets returns included in the portfolio.

## CONCLUSION

The proposed model in this article, realizing the basic idea of Markowitz of forming an optimal portfolio of bond, at the same time is fundamentally different from the now-classic, Markowitz model. This difference is primarily in the method by which the interaction of portfolio assets is considered during the formation of portfolio.

Instead of multiplicative interaction, we suggest to consider the additive interaction, which expands the idea of the possible situations. In addition, the construction of a model based on additive interaction allows you to enter a different gauge of risk that characterizes not possible deviation of the mean value and the value of which depends on the probability with which the expected market situation in a proactive time. Moreover, the risk assessment here can be either positive or negative.

Within the framework of the theory of optimal portfolio investment relationship is established between return and risk and on the basis of this relationship it is also introduced the idea of the "front of efficient portfolios". The nature of the relationship of risk and return for the cases where the relationship of assets is described by the matrix of additive interaction is not considered here but there is much demand for the discussion. There is no visible analogy with the theory of Markowitz but the more interesting is the study of this problem within the framework of the proposed approach.

## REFERENCES

Bachelier, L., 1964. Theory of speculation. The Random Character of Stock Prices. Cambridge: MIT, pp: 71-78.

- Cox, J.C., S.A. Ross and M. Rubinstein, 1979. Option Pricing: A Simplified Approach. J. Financial Econ., 7: 229-263.
- Davnis, V.V., S.E. Kasatkin and A.A. Ardakov, 2012. Odnokomponentnaya model portfel'nogo investirovaniya. Sovremennaya Ekonomika: Problemy i Resheniya, 5 (29): 150-157.
- Davnis, V.V. and V.V. Korotkikh, 2014. Ob ispol'zovanii dvukh gipotez pri ekonometricheskom modelirovanii stokhasticheskikh protsessov. Sovremennaya Ekonomika: Problem i Resheniya, 7 (55): 30-40.
- Davnis, V.V. and V.I. Tinyakova, 2005. Profnoznii modeli ekspertnikh predpochtenii. Voronezh: Voronezhskii gosudarstvennii universitet.
- Markowitz, H.M., 1991. Portfolio selection. Efficient diversification of investments. Oxford; New York.: Blackwell.
- Markowitz, H.M., 1952. Portfolio selection. J. Finance, 7 (1): 77-91.
- Markowitz, H.M., 1999. The early history of portfolio theory: 1600-1960. Financial Analysts J., 55 (4): 5-16.
- Rutkauskas, A.V. and A. Kvietkauskienė, 2013. Investment decisions in global financial markets: The experience of Lithuania. Entrepreneurial Business and Economics Review, 1 (4).
- Sharpe, W.F., 1963. A simplified model for portfolio analysis. Manage. Sci., 9 (2): 277-293.
- Sharpe, W.F., 1964. Capital asset price: a theory of market equilibrium under conditions of risk. J. Finance, 19 (3): 425-442.
- Sharpe, W.F., 1970. Portfolio theory and capital Markets. New York: McGraw-Hill.
- Shiryaev, A.N., 1998. Osnovi stokhasticheskoi finansovoi matematiki. Moscow: Fazis.
- Tinyakova, V.I., 2008. Modeli adaptivno-ratsional'nogo prognozirovaniya ekonomicheskikh protsessov. Voronezh: Voronezhskii gosudarstvennii universitet.