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Mining a Complete Set of Fuzzy Multiple-Level Coherent Rules

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Abstract: Data-mining techniques are developed to transform raw data into suitable knowledge-oriented data. The algorithms for mining association rules identify relationships among transactions using interesting measures like support and confidence at a single-concept level or multiple levels. Using support and confidence alone for mining associations would not give interesting rules both for quantitative as well as binary data. This study proposes a fuzzy coherent rule mining algorithm at multi-level hierarchies to discover the significant rules in quantitative transactions. The proposed method combines fuzzy coherent rules mining concept with that of taxonomical mining in a quantitative database. The algorithm works on a top down methodology in traversing the data that exists in a hierarchical form. An experimental comparison with the fuzzy coherent rule mining methodology conveys the significance of the proposed algorithm in finding the level-wise coherent rules.

Key words: Association rules, fuzzy coherent rule, quantitative database, membership, function

INTRODUCTION

Data mining is used to extract significant information from raw data. Association rule mining is used to derive itemset associations using interesting measures (Agrawal *et al*., 1993a, b; Agrawal and Srikant, 1994; Srikant and Agrawal, 1995; Webb and Zhang, 2005). Srikanth and Agarwal (1996) mined association rules from datasets using quantitative and categorical attributes. Han and Fu (1995) first proposed a method to find level-crossing association rules on multiple levels of taxonomy. Mining interesting level-wise rules at different thresholds at each level was proposed by Brin. FP growth algorithm was developed by Han *et al*. (2004) to mine frequent rules at two scans of the entire database. Fukuda *et al*. (1996) improved the efficiency of mining association rule which permitted single instantiated conditions on the left-hand side of association rules. Fuzzy set (Zadeh, 1965) has been increasingly used in logical and decision making systems for its easiness. Fuzzy learning algorithms for generating membership functions and inducing rules for a given dataset are proposed (Campos and Moral, 1993; Hong and Chen, 1999; Hong and Lee, 1996) and used in specific domains. Basically the current world data is of quantitative value. To mine the quantitative values, fuzzy mining has been proposed to derive the fuzzy rules from the quantitative transactions (Hong *et al*., 1999, 2004, 2011; Mangalampalli and Pudi, 2009). Kaya and Alhajj (2005) presented a methodology for fuzzy mining using genetic algorithm.

Hong *et al*. (2004, 2003) proposed FP growth for fuzzy mining (Lin *et al*., 2010). Lin *et al*. (2015) presented an algorithm for mining multiple fuzzy frequent pattern tree using less computations. Ma *et al*. (2011) suggested fuzzy interesting measures that generate interesting and significant fuzzy rules. The above discussed methodology is for mining at single level concept for a given support and confidence thresholds. In certain cases, if min-support threshold is set higher a large amount of possible rules are discarded or if the min-support threshold is set too low, a large amount of frequent rules are generated without any significance. To overcome the above problem (Liu *et al*., 1999) established a new methodology of using different minimum support to different items for mining rare itemsets. Lee *et al*. (2004) developed mining of multiple-level itemsets using maximum constraints and mining taxonomical fuzzy rules by specifying different support thresholds for fuzzy itemsets (Lee *et al*., 2008). Weng (2011) used both frequent and infrequent data to mine rare and interesting patterns in education data. Fuzzy association rules are used in wide range of applications including Intrusion detection (Tajbakhsh *et al*., 2009).

Sim *et al*. (2010) presented the concept of mining coherent rules based on the properties of propositional logic. These rules are used in deriving associations among items without using the concept of minimum support and confidence threshold. In this study, the concept of coherency is applied at multiple levels of taxonomical quantitative data set without minimum

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support and minimum confidence. Thus, the study proposes a multiple-level fuzzy coherent rule mining algorithm without specifying minimum support or confidence for different levels. Chen *et al.* (2014, 2013) introduced the concept of mining coherent rules by considering maximum and minimum boundary support measure and also deriving high utility fuzzy itemsets using coherent mining.

Review of related coherent rule miningalgorithms: This study reviews related works of mining coherent rules including fuzzy coherent rules.

Coherent rule mining: A suitable minimum support is very tough to define in any dataset. To solve the above problem, Sim *et al.* (2010) derived coherent rules that map the association rules to equivalences. An association rule (m?n) is mapped to an equivalence a≡b implication if and only if m→n is true, ¬m→¬n is true, ¬m→n is false and m→¬n is false. The above process requires at least an individual transaction to determine each condition is true or false but there are four conditions for mapping an association rule to equivalence implication, it cannot be carried out by a single transaction record. For multiple transactions the mapping of association rule m→n to an equivalence implication a≡b is done if the following conditions are satisfied:

$$\text{Support}\ (m, n) > \text{support}\ (m, \neg n)$$

$$\text{Support}\ (m, n) > \text{support}\ (\neg m, n)$$

$$\text{Support}\ (m, n) > \text{support}\ (\neg m, \neg n)$$

In similar manner, the other association rules such as ¬m→n, m→¬n and ¬m→¬n is mapped to their corresponding equivalence implication. The process of mapping equivalence based support comparisons is called as pseudo implications. Using the concept of pseudo implications coherent rules are derived. The following four conditions are to be satisfied for generating coherent rules:

$$\text{Support}\ (m, n) > \text{support}\ (m, \neg n)$$

$$\text{Support}\ (m, n) > \text{support}\ (\neg m, n)$$

$$\text{Support}\ (\neg m, \neg n) > \text{support}\ (\neg m, \neg n)$$

The importance of any rule in a domain can be easily found out by using coherent logic without the prior knowledge of the domain. The above four conditions are represented using contingency table shown in Table 1.

Table 1: 2×2 contingency table for two items

TID	m	¬m
n	CO_1 = Support (n, m)	CO_2 = Support (n, ¬m)
¬n	CO_3 = Support (¬n, m)	CO_4 = Support (¬n, ¬m)

Fuzzy coherent rule mining: It integrates the concept of mining fuzzy association rules from quantitative data (Hong *et al.*, 1999) and coherent rule mining (Sim *et al.*, 2010). The quantitative transactions are converted into fuzzy set in linguistic terms using predefined membership functions. The complimentary values for each fuzzy region are calculated. The concept of coherency is applied by maintaining a single fuzzy region as antecedent and remaining regions as consequent. This approach mines fuzzy coherent rules (Chen *et al.*, 2013) and also solves the problem of setting appropriate minimum support in fuzzy data mining.

MATERIALS AND METHODS

Proposed Fuzzy Multi-Level Coherent Rule Mining algorithm (FMLCRM): The proposed approach integrates the concept of fuzzy coherent rule mining and mining association rules in multiple-level taxonomical quantitative data. The proposed fuzzy multiple-level coherent rule mining without predefined minimum support and confidence is described below.

Input: A body of n quantitative transaction data, a set of membership functions, a predefined taxonomy and a given item Y.

Output: A complete set of inter-cross fuzzy multiple level coherent rules.

Step 1: The symbol "*" is used to encode each item in the given predefined taxonomy using sequence of numbers. According to the encoding scheme the transaction item data are translated.

Step 2: The encoding starts from root and to the internal nodes from left to right.

Step 3: The notation k points to the level number (ranging from 1-t) which is set as one initially.

Step 4: Join the item name for level 1 representation in each transaction $D^{(i)}$ and add all the quantitative value of the item in the same level in the $D^{(i)}$. Where the number of the j-th group I_j^k for $D^{(i)}$ as $q_j^{(i)k}$

Step 5: Transform the quantitative value $q_j^{(i)k}$ of each transaction data $D^{(i)}$ i = 1 to n for each item I_j^k, j = 1 to m into fuzzy set $q_j^{(i)k}$ represented as:

$$\left(\frac{f_j^{(i)k}}{R_j^k}+\frac{f_j^{(i)k}}{R_j^k}+\frac{f_{jh}^{(i)k}}{R_{jh}^k}\right)$$

By using membership function for different items in each transaction $D_{(i)}$ will have a fuzzy membership function for I_j^k, R_j^k is the k-th fuzzy region of items I_j^k, $1 \le 1 \le h$ and $f_j^{(i)k}$ is $q_j^{(i)k}$'s fuzzy value in region R_j^k.

Step 6: Calculate complement value for every fuzzy region R_j^k, according to the fuzzy concepts which is if $f_j^{(i)k}$ is the fuzzy value of k-th fuzzy region for item I_j^k then the fuzzy complement is calculated as $(1\text{-}f_j^{(i)k})$. The result is represented as follows:

$$\left(\frac{1\text{-}f_{j1}^{(i)k}}{R_{j1}^k}+\frac{1\text{-}f_{j2}^{(i)k}}{R_{j2}^k}+\ldots+\frac{1\text{-}f_{jl}^{(i)k}}{R_{ji}^k}\right)$$

Where:

i = The transaction id

$f_j^{(i)k}$ = $q_j^{(i)k}$'s membership value in fuzzy region R_j^k

Step 7: All k fuzzy regions are collected in the set ST_b^k.

Step 8: The item Y (consequent) is selected in any preferred taxonomical level by the user.

Step 9: The level k fuzzy regions of item Y is removed from the set ST_b^k, i.e., $C^K = St_b^k\text{-}Y$.

Step 10: For every item in the set C^K and consequent (Y) candidate coherent rules are generated and checked for coherencies as per the contingency table given in Table 1.

Step 11: The coherent rules derived in the level k are outputted in the set $MFCR^K$.

Step 12: The steps 3-11 are repeated for n coherent itemsets for a single consequent for k levels.

An example of proposed algorithm: In this study an example is proposed for mining fuzzy multi-level coherent rule algorithm. It shows the efficiency of proposed algorithm in a multi-level based. We considered the stationary sales transaction from a sales data set which is shown in Fig. 1.

Each transaction consists of an transaction-Id and items. Each of the items is represented with name and item amount. In Fig. 2 the stationary sales transaction taxonomy is divided into three classes namely Pen, Note, and Rubber. Each of the classes represents the category and brands. Assume that the fuzzy membership functions are used to bind the item amount by three fuzzy regions: low, middle and high. A triangular membership function is used for converting crisp values to fuzzy values is represented in Table 2.

Step 1: Each item in transaction data is first encoded using the predefined hierarchy tree based taxonomy.

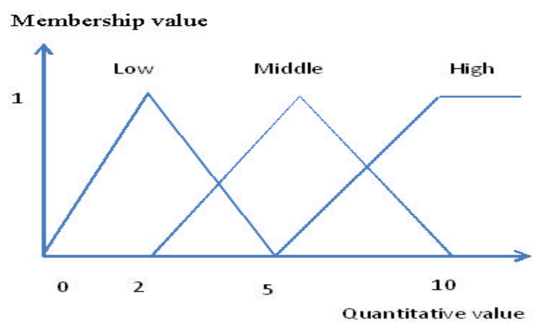

Fig. 1: The Membership function

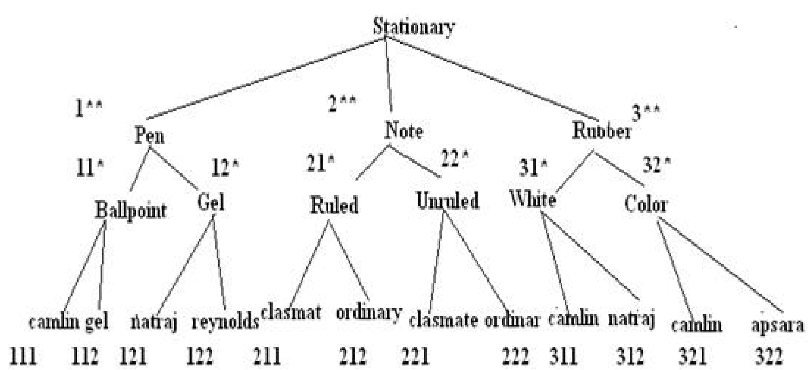

Fig. 2: Predefined taxonomy

Table 2: An example of six transactions

TID	Items
T1	(Camlin ballpoint pen,3) (cello ballpoint pen, 4) (classmate ruled note, 3) (ordinary ruled note, 6) (Camlin white rubber,4) (Nat raj white rubber,4)
T2	(Camlin ballpoint pen, 4) (cello, ballpoint pen, 4) (Nat raj gel pen, 2) (Reynolds gel pen, 5) (classmate ruled note, 4) (Camlin white rubber, 4)
T3	(Camlin white rubber, 3) (Nat raj white rubber, 3) (Camlin color rubber, 1) (Apsara color rubber, 2) (classmate ruled note, 4) (ordinary ruled note, 4)
T4	(Camlin white rubber, 2) (Camlin color rubber, 4) (classmate rule note, 2) (classmate unruled note, 3) (ordinary unruled note, 2)
T5	(Nat raj white rubber, 3) (classmate ruled note, 3) (ordinary ruled note, 1)
T6	(Camlin ballpoint pen, 2) (Camlin white rubber, 3) (Camlin color rubber, 3) (apsara color rubber, 3) (classmate ruled note, 3) (classmate unruled note, 3) (ordinary unruled note, 3)

TID	Items
T1	$\left(\frac{0.6}{1**.medium}+\frac{0.4}{1**.high}\right)\left(\frac{0.2}{2**.medium}+\frac{0.8}{2**.high}\right)\left(\frac{0.4}{3**.middle}+\frac{0.6}{3**.high}\right)$
T2	$\left(\frac{1}{1**.high}\right)\left(\frac{0.33}{2**.low}+\frac{0.67}{2**.middle}\right)\left(\frac{0.33}{3**.low}+\frac{0.67}{3**.middle}\right)$
T3	$\left(\frac{0.4}{2**.medium}+\frac{0.6}{2**.high}\right)\left(\frac{0.2}{3**.medium}+\frac{0.8}{3**.high}\right)$
T4	$\left(\frac{0.6}{2**.medium}+\frac{0.4}{2**.high}\right)\left(\frac{0.8}{3**.medium}+\frac{0.2}{3**.high}\right)$
T5	$\left(\frac{0.33}{2**.low}+\frac{0.67}{2**.medium}\right)\left(\frac{0.67}{3**.low}+\frac{0.33}{3**.middle}\right)$
T6	$\left(\frac{1}{1**.low}\right)\left(\frac{0.2}{2**.medium}+\frac{0.8}{high}\right)\left(\frac{0.2}{3**.middle}+\frac{0.8}{3**.high}\right)$

Fig. 3: Fuzzy set transformed from the level one data set

Table 3: Encoded transaction data

TID	Items
T1	(111, 3) (112, 4) (211, 3) (212, 3) (311, 4) (312, 4)
T2	(111, 4) (112, 3) (121, 2) (122, 5) (211, 4) (311, 4)
T3	(311, 3) (312, 3) (321, 1) (322, 2) (211, 4) (212, 4)
T4	(311, 2) (321, 4) (211, 2) (221, 3) (222, 2)
T5	(312, 3) (211, 3) (212, 1)
T6	(111, 2) (311, 3) (321, 3) (322, 3) (211, 3) (221, 3) (222, 3)

Table 4: Level one representation

TID	Items
T1	(1**, 7) (2**, 9) (3**, 8)
T2	(1**, 14) (2**, 4) (3**, 14)
T3	(3**, 9) (2**, 8)
T4	(3**, 6) (2**, 7)
T5	(3**, 3) (2**, 4)
T6	(1**, 2) (3**, 9) (2**, 9)

For example, the item “Natraj gel pen” is encoded as ‘121’ in which the first digit ‘1’ represents the ‘pen’ at level 1, and second digit ‘2’ represent flavors ‘gel’ at level2, and the third digit ‘1’ represent the brands ‘Natraj’ at level3 as shown in Fig. 1.

Step 2: Transfer of all the items in quantitative transaction data set in to encoded format are show in Table 3.

Step 3: Set k = 1 where k is used to point the level number being processed.

Step 4: Add the entire items amount in transaction to form a level one data set. For example in transaction T1 the item (111, 3) (112, 4) are join into (1**, 7). The level-one transaction results are shown in Table 4.

Step 5: Transformed the level one representation of quantitative transaction into fuzzy set by using predefined triangular membership function. Take the first item transaction T1 as an e.g., the amount (1**, 7) is converted into fuzzy set:

$$\left(\frac{0}{1^{**}.\text{low}}+\frac{0.6}{1^{**}.\text{medium}}+\frac{0.4}{1^{**}.\text{high}}\right)$$

using membership function. The result of all items is shown in Fig. 2 and 3, the notation Item. Item is called fuzzy region.

Step 6: Generate the complement set from the fuzzy set. Take the one item in transaction T1 using membership function as an example. The fuzzy value:

$$\left(\frac{0}{1^{**}.\text{low}}+\frac{0.6}{1^{**}.\text{medium}}+\frac{0.4}{1^{**}.\text{high}}\right)$$

of item is converted into complement fuzzy set:

$$\left(\frac{1}{1^{**}.\text{low}}+\frac{0.4}{1^{**}.\text{medium}}+\frac{0.6}{1^{**}.\text{high}}\right)$$

The result for all items as shown in Fig. 4.

TID	Items
TD1	$\left(\frac{1}{1**.low}+0.\frac{4}{1**.medium}+0.\frac{6}{1**.high}\right)\left(\frac{1}{2**.low}+0.\frac{8}{2**.medium}+0.\frac{2}{2**.high}\right)$ $\left(\frac{1}{3**.low}+0.\frac{6}{3**.middle}+0.\frac{4}{3**.high}\right)$
TD2	$\left(\frac{1}{1**.low}+\frac{1}{1**.medium}\right)\left(0.\frac{67}{2**.low}+0.\frac{33}{2**.medium}+\frac{1}{2**.high}\right)$ $\left(\frac{0.67}{3**.low}+0.\frac{33}{3**.medium}+\frac{1}{3**.high}\right)$
TD3	$\left(\frac{1}{2**.low}+0.\frac{6}{2**.medium}+0.\frac{4}{2**.high}\right)\left(\frac{1}{3**.low}+0.\frac{8}{3**.medium}+0.\frac{2}{3**.high}\right)$
TD4	$\left(\frac{1}{2**.low}+0.\frac{4}{2**.medium}+0.\frac{6}{2**.high}\right)\left(\frac{1}{3**.low}+0.\frac{2}{3**.medium}+0.\frac{8}{3**.high}\right)$
TD5	$\left(\frac{0.67}{2**.low}+0.\frac{33}{2**.medium}+\frac{1}{2**.high}\right)\left(0.\frac{33}{3**.low}+0.\frac{67}{3**.middle}+\frac{1}{3**.high}\right)$
TD6	$\left(\frac{1}{1**.medium}+\frac{1}{1**.high}\right)\left(\frac{1}{2**.low}+0.\frac{8}{2**.medium}+0.\frac{2}{2**.high}\right)$ $\left(\frac{1}{2**.low}+0.\frac{8}{3**.middle}+0.\frac{2}{3**.high}\right)$

Fig. 4: Complement set for level one transaction

Levels	Multi-level fuzzy coherent rules
1	$2**.high \rightarrow 3**.high, 2**.medium \rightarrow 3**.medium$
2	$21*.low \rightarrow 31.medium$
3	$211*.low \rightarrow 111.low$

Fig. 5: Result for example data set

Step 7: Fuzzy regions of level 1 are collected and stored in St_b^k = {1**.low, 1**.medium, 1**.high, 2**.low, 2**.high, 2**.medium, 3**.low, 3**.medium, 3**.high}.

Step 8: An item from any level can be selected as consequent for the above example we have not chosen any particular item as consequent instead we have mined all possible coherent rule for the given transactions.

Step 9: Every item is tested for consequent for a different antecedent.

Step 10: Calculate the contingency table for each and every candidate coherent rule with x.term and y.term. For example take the candidate coherent rule (2**.high→3**.high), since antecedent part is 2**.high and the consequence part is 3**.high. The contingency table of 2**.high 2**.high→3**.high is calculated with sup count value of count(2**.high→3**.high) is 2.22 (0.6+0+0.6+0.2+0+0.8). Similarly the count (2**.high→3**.high) is 1.4, count(~2**.high→3**.high) is 1.0 and count(~2**.high→3**.high is 3.0.

Step 11: Check for the coherent conditions for e.g., the (2**.high→3**.high) rule meets the following four condition, i.e., CO1>CO2, CO1>CO3, CO4>CO2, CO4>CO3 as per the contingency in Table 1. If yes put the satisfied coherence rule into Multi-level Fuzzy Coherent Rule (MFCR) set.

Step 12: For the first level two fuzzy coherent rules are found (2**.high→3**.high&2**.medium→3**.medium. For the example dataset no 2-item set coherent rules are formed. Simultaneously find the multi-level fuzzy coherent rules for level 2 and 3. The results are displayed in Fig. 5.

RESULTS AND DISCUSSION

This study presents the evaluation results of the proposed approach. They are implemented in Java on a personal computer with Intel Core i7, 2.93 GHz and 4 GB RAM. The dataset is described. The performance results are given.

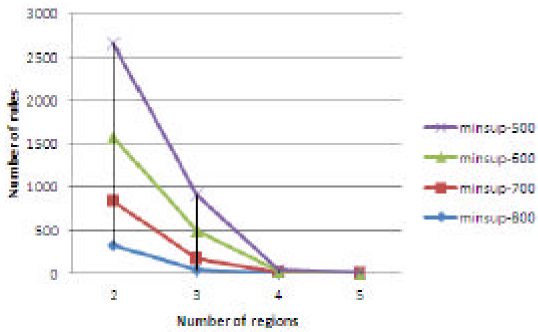

Fig. 6: The relationship between number of coherent rules and numbers of regions

Table 5: Comparison between FCRM and FMLCRM approaches

Variables	FMLCRM	FCRM
Number of rules (length of antecedent = 1)	1091	213
Number of rules (length of antecedent = 2)	63	27
Number of rules (length of antecedent = 3)	7	2
Number of rules (length of antecedent = 4)	1	0
Total number of rules	1162	242

Table 6: Execution time of proposed approach

Size of the dataset	10 k	5 k	3 k	1 k
Execution time (sec)	281.62	137.49	94.23	53.64

Dataset descriptions: There were 27 purchased items as terminal nodes on level 3, 9 generalized items in level 2 and 3 universal items in level 1. Each non terminal node would have 3 branches. Only terminal nodes would appear in transaction. In each data set, numbers of purchased items in transactions were randomly generated and then the quantities of each purchased items are generated. An item cannot be generated twice in a transaction. Twenty thousand transactions with an average of ten purchased items were run using the proposed algorithm. Membership functions are allocated dynamically for each taxonomical level. Three fuzzy regions used in the proposed approach are low, middle and high. Results are shown in the study 5.2.

Experimental results: the Fuzzy Coherent Rule Mining (FCRM) is compared with the proposed method. Experiments were performed to show the relationship between the total numbers of rules obtained for the given fuzzy regions. Table 5 and 6 displays the execution time required for FCRM and proposed multilevel approach. From Fig. 6, it is easily seen that the number of coherent rules mined decrease along with increase in the number of fuzzy regions. It is justified because the increase in number of regions would further distribute the quantity in different regions. The relationship between the number of coherent rules and number of fuzzy regions for different supports with a minimum confidence of 30% is shown in Fig. 2. From Table 5, it is easily identified that the total number of rules generated in proposed technique is more when compared to FCRM and the execution time of the proposed technique costs high compared to FCRM.

The comparison of results between FCRM and Fuzzy multiple level coherent rule mining (FMLCRM) are represented in Table 6. It can be observed that the number of rules derived by FCRM is 213 which is less than that by FMLCRM when the length of the Antecedent of the rule is 1. The time complexity of the proposed approach for a given item Y is given as:

$$o\left(\sum_{l=1}^{n}\sum_{h=2}^{w}|S_l|_{h-1}\times|F|\times D\times w\right)$$

Where:

$|S_l|_{h-1}$ = The number of fuzzy regions in the level l of set S and the length of each element is h-1
$|F|$ = Number of fuzzy regions in the set S
D = The size of the transaction
w = Average transaction width

When rule length increases, the number of rules generated by FCRM decreases rapidly compared to FMLCRM. It is due to inter cross fuzzy mining at multiple levels.

CONCLUSION

In this study, fuzzy multi-level coherent rule mining algorithm was proposed to mining the quantitative transaction without busing interesting measures for different level's to judge the importance. The proposed algorithm first converts a quantitative level data set into fuzzy set by using triangular membership function. Next the candidate coherent rules are generated and the contingency tables are calculated for candidate coherent to check the whether the coherent condition is satisfied or not. The proposed fuzzy multi-level coherent rule mining algorithm generates large item set level-by-level and then finds some interestingness rules highly coherent in nature from quantitative transaction data. An example is also derived for the proposed mining algorithm and generates the multi-level fuzzy coherent rule under multi-level minimum support. The main role of this approach is to find the highly fuzzy coherent rules. In future enhancement we will try using different membership functions across the taxonomies to mine interesting coherent rules.

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