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Piecewise Linear Goal-Directed CPPI Strategy

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Abstract: Traditional Portfolio Insurance (PI) strategy, such as CPPI, only considers the floor constraint but not the goal aspect. This paper proposes a Goal-Directed (GD) strategy to express an investor's goal-directed trading behavior and combines this floor-less GD strategy with the goal-less CPPI strategy to form a piecewise linear goal-directed CPPI (GDCPPI) strategy. This paper applies Genetic Algorithm (GA) technique to find better piecewise linear GDCPPI strategy parameters than those under the Brownian motion assumption. The statistical tests show that the GA strategy can outperform the Brownian strategy.

Key words: Portfolio insurance, goal-directed strategy, piecewise linear GDCPPI strategy

INTRODUCTION

Portfolio insurance is a way of investment with the constraint that the wealth can never fall below a pre-assigned protecting wealth floor. The optimal trading strategy for a constant floor turns out to be the popular Constant Proportion Portfolio Insurance (CPPI) strategy[1,2] and can be expressed as $x_t = m_1(W_t\text{-}F)$ where x_t is the amount invested in the risky asset at time t, W_t is the wealth at time t, m_1 is a constant risk multiplier and F is the floor. This optimal strategy states that one should invest more in the risky asset when the wealth increases. In practice, a mutual fund manager generally sets up a performance objective in terms of wealth or return at the beginning of an investment period. If a fund manager follows the CPPI strategy, he will have a greater chance of failing his almost reached goal when current wealth is closed to the goal. The major reason is that CPPI strategy only considers the floor but does not take the goal state into account, while fund managers do have the goal state in mind during the investment process.

Evidences show that an investor will change his risk-attitude under different wealth levels. CPPI strategy demonstrates this phenomenon. In addition, some studies showed that fund managers change their risk-attitudes based on their performance compared to the benchmark. However, there are contradictory observations among these studies. Some studies observed that fund managers take risk-seeking behavior when their performance is worse than the benchmark while some other studies observed that fund managers take risk-averse behavior when their performance is worse than the benchmark.

These contradictions in fact can be explained by portfolio insurance perspective and goal-directed perspective, respectively. Goal-directed perspective proposes that an investor in financial markets will consider certain investment goal. A goal-directed investor will take risk-seeking behavior when the distance from current wealth to the goal is large and will take risk-averse behavior when the distance from current wealth to the goal is small. Obviously, a CPPI investor's risk-attitude changing direction is opposite to a goal-directed investor's.

We therefore construct a goal-directed (GD) strategy $x_t = m_2(G\text{-}W_t)$under constraint $W_t \le G$, where G is the goal and m_2 is a constant. The concept of GD strategy can also be supported by Browne's study[3]. We further combine the portfolio insurance constraint and goal-directed constraint as $F \le W_t \le G$ to construct a piecewise linear goal-directed CPPI (GDCPPI) strategy, $x_t = m_1(W_t\text{-}F)$, $F \le W_t < M$ and $x_t = m_2(G\text{-}W_t)$, $M \le W_t \le G$. The $M = (m_1F + m_2G)/(m_1 + m_2)$ is a wealth position at the intersection of GD and CPPI strategies. This M position guides investors to apply CPPI strategy or GD strategy depending on whether the current wealth is less or greater than M, respectively. In addition, if $m_1 \to \infty$, the piecewise linear GDCPPI strategy reduces to the GD strategy and if $m_2 \to \infty$, the piecewise linear GDCPPI strategy becomes the CPPI strategy. That is, the piecewise linear GDCPPI strategy is a generalization of both CPPI and GD strategies.

Moreover, the optimal m_1 and m_2 can be theoretically derived based on the Brownian motion assumption for stock prices as in traditional CPPI strategy[1,2] and

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Browne's study[3], respectively. There are some parameters in the optimal formulas of m_1 and m_2 that need to be estimated using historical data. However, the stock prices in real financial markets might not follow the Brownian motion assumption, especially in short-term periods. We therefore apply Genetic Algorithms (GAs) to find good parameter values in piecewise linear GDCPPI strategy based on historical data to improve its performance. The statistical test shows that the parameters found by GA are better than those calculated from the technique under Brownian motion assumption.

MATERIALS AND METHODS

CPPI strategy: The formulation and solution of optimal portfolio insurance problem will be described following Grossman and Zhou's work[4]. Assume there are two assets: A risk-free asset such as a T-bill and a risky asset such as a stock. Let the stock price dynamic be $dp_t/p_t = \mu dt + \sigma dz_t$, where μ is the mean of return rates, σ is the standard deviation of return rates and dz_t is a Brownian motion at time t. The portfolio wealth dynamic then is $dW_t = rW_t dt + x_t(\mu dt + \sigma dz_t)$, where r is the risky-free rate of return and x_t is the dollar amount invested in the risky asset. Suppose an investor tries to maximize the growth rate of expected utility of the final wealth under the portfolio insurance constraint. The problem becomes:

$$\sup_{X} \lim_{T\to\infty} \frac{1}{\gamma T} \ln E[\gamma U(W_T)] \quad \text{s.t.} W_t \geq F, \forall t \leq T, \tag{1}$$

where x denotes the set of admissible trading strategies, $0 \leq \gamma \leq 1$ and $F > 0$ is the floor. If F is fixed, the optimal strategy to the above optimization problem is:

$$x_t = \frac{\mu}{\sigma^2(1-\gamma)}(W_t - F) \tag{2}$$

Eq. 2 can be simplified as:

$$\varsigma_t \equiv x_t = m_1(W_t - F), W_t \geq F, \tag{3}$$

where $m_1 = \mu/\sigma^2(1-\gamma)$ can be regarded as the investor's risk multiplier, F is the protecting floor. This ς_t is the popular CPPI strategy.

Risk attitudes: Evidences show that an investor will change his risk-attitude under different wealth levels. In particular, studies showed that fund managers change their risk-attitudes based on their performance compared to the benchmark. However, there are contradictory observations among these studies. Some studies[5-7] observed that fund managers take risk-averse behavior when their performance is worse than the benchmark (low wealth risk aversion) while some other studies[8,9] observed that fund managers take risk-seeking behavior when their performance is worse than the benchmark (high wealth risk aversion). These two types of risk-attitude are described as follows.

Low wealth risk aversion: An investor will become risk-averse when his current wealth is low and will become risk-seeking when his current wealth is high.

High wealth risk aversion: An investor will become risk-averse when his current wealth is high and will become risk-seeking when his current wealth is low.

The goal-less CPPI strategies demonstrate the low wealth risk aversion phenomenon. Goal-directed perspective proposes that an investor in financial markets will consider certain investment goal. A goal-directed investor will take risk-seeking behavior when the distance from current wealth to the goal (goal distance) is large and will take risk-averse behavior when the goal distance is small. Although low wealth risk aversion can be explained by the CPPI strategy, high wealth risk aversion can not be explained by CPPI. We argue that these contradictions can be explained from two perspectives: the portfolio insurance perspective and the goal-directed (or goal-seeking) perspective. That is, low wealth risk aversion can be explained by portfolio insurance perspective. High wealth risk aversion can be explained by goal-directed perspective and will be exploited as follows.

Goal-directed strategy: In Browne's study[3], one of the investment problems is to maximize the survival probability in danger zone or to maximize the probability of reaching the goal before reaching the bankruptcy point. The model can be described as follows.

$$\min_{X} P(\tau_a > \tau_b), \text{s.t.} a < W_t < b < S, \tag{4}$$

where x is the set of admissible strategies, $P(\bullet)$ is the probability function, a is the bankruptcy point, τ_a is the escape time when $W_t = a$, τ_b, is the escape time when $W_t = b$, S, is the safe point and is generally set up to be c/r, with c being the minimal consumption and r being the risk-free rate of return. This model tries to find an optimal trading strategy which minimizes the probability of reaching the bankruptcy point a before reaching the goal b. The optimal strategy turns out to be:

$$x_t = \frac{2r}{\mu - r}(S - W_t) \quad (5)$$

where μ is the mean of return rates for the risky asset. If $b \rightarrow S$ in fact can be regarded as the goal G that an investor wants to achieve. Then we define a goal-directed (GD) strategy as

$$\eta_t \equiv x_t = m_2(G - W_t), W_t \leq G, \quad (6)$$

where $m_2 = 2r/\mu\text{-}r$ is a constant.

The GD strategy shows that an investor should take a riskier action when goal distance (i.e., the distance from current wealth to the goal) is large and should take less risky activity when goal distance is small. This behavior is consistent with the high wealth risk aversion. In other words, the high wealth risk aversion can be explained by this GD strategy.

Piecewise linear goal-directed CPPI strategy: As we have noted that investors seem to have two different types of wealth risk aversion: the low wealth risk aversion and the high wealth risk aversion. Intuitively, investors will take different strategy when they posit different risk attitude. That is, if their risk attitude is low wealth risk aversion, they will adopt CPPI strategy. If their risk attitude is high wealth risk aversion, they will adopt GD strategy.

Recall that the constraint of CPPI strategy, $W_t \geq F$, is different from the constraint of GD strategy, $W_t \leq G$. In addition, the objective of CPPI, maximizing the growth rate of certain utility, is different from the objective of GD strategy, maximizing the possibility of reaching the goal first. Combining the two constraints $F \leq W_t$ and $W_t \leq G$, a new problem with constraint $F \leq W_t \leq G$ is derived. This new problem can be regarded as containing two objectives which are composed from the objectives of CPPI and GD strategies. The CPPI and GD strategies are depicted in Fig. 1.

We can see that CPPI strategy only considers the floor and GD strategy only considers the goal. In addition, there is a wealth position M projected from the intersection of these two strategies and the value of M can be calculated by

$$M = \frac{m_1 F + m_2 G}{m_1 + m_2} \quad (7)$$

M seems to be a natural dividing point for changing strategies. Since CPPI considers only the floor F but not the goal G, an investor can apply CPPI strategy

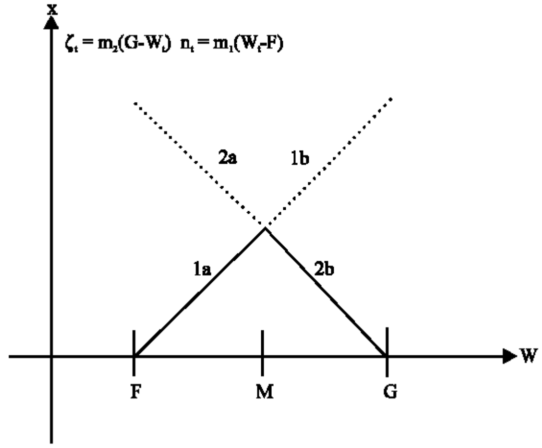

Fig. 1: The piecewise linear GDCPPI strategy

when $W_t < M$. On the other hand, since GD considers only the goal G but not the floor F, an investor can apply GD strategy when $W_t \geq M$. We therefore build a piecewise linear GDCPPI strategy as:

$$\theta_t \equiv x_t \begin{cases} 0, W_t < F \\ m_1(W_t - F), F \leq W_t < M \\ m_2(G - W_t), M \leq W_t \leq G. \end{cases} \quad (8)$$

It can be seen that the piecewise linear GDCPPI strategy θ_t combines portfolio insurance perspective and goal-directed perspective, as the segments 1a and 2b in Fig. 1. Note that θ_t is a generalization of both CPPI and GD strategies. In particular, if $m_1 \rightarrow \infty$, $M = (m_1F + m_2G)/(m_1 + m_2) = F$ and the constraint $M \leq W_t \leq G$ for GD segment will be $F \leq W_t \leq G$. Therefore, piecewise linear GDCPPI strategy reduces to GD strategy. If $m_2 \rightarrow \infty$, $M = (m_1F + m_2G)/(m_1 + m_2) = G$ and the constraint $F \leq W_t < M$ for CPPI segment will be $F \leq W_t < G$. Therefore, piecewise linear GDCPPI strategy reduces to CPPI strategy.

Traditional CPPI strategy is based on the assumption of Brownian motion for stock prices. Browne's study[3] for goal seeking objective also made this assumption. When investors try to apply these above strategies, the parameter values are generally obtained by the long-term expectation method. That is, the mean and variance of return rates are the long-term expectations from historical data.

However, the historical data might not follow the Brownian motion[10]. Better m_1 and m_2 parameter values in piecewise linear GDCPPI strategy might be directly obtained using other data driven optimization methods with historical data. Genetic algorithm is the method chosen to search better m_1 and m_2 parameters values in this study due to its success in many applications.

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Procedure Basic GA
  Initialization: Generate a random population of n chromosomes
  While (termination condition is not satisfied)
    Evaluation: Evaulate the fitness of each chromosomes in the population
    loop (do genetic applications k times for each generation)
      Selection: Select two chromosomes according to their fitness
      Crossover: Crossover selected chromosomes to form new offsprings
              with a crossover rate
      Mutation: Mutate each position in each new offspring with a mutation rate
    endloop
    Replacement: Replace chromosomes in parent population with new offspring
  endwhile
    Report the best solution (chromosome) found
endprocedure
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Fig. 2: The basic genetic algorithm

Genetic algorithms: Genetic Algorithms (GAs) were proposed by Holland in 1975 from Darwin's theory of evolution: survival of the fittest[11]. Genetic algorithms use an evolutionary process resulting in a fittest solution to solve a problem. Genetic algorithms are computationally simple and powerful and show promising outputs in many applications.

The basic genetic algorithm: To solve a problem with genetic algorithms, an encoding mechanism must first be designed to represent each solution as a chromosome, e.g., a binary string. A fitness function is also required to measure the goodness of a chromosome. Genetic algorithms search the solution space using a population which is simply a set of chromosomes. During each generation, the three genetic operators: selection, crossover and mutation, are applied to the population several times to form a new population. Selection picks two chromosomes according to their fitness: a fitter chromosome has a higher probability of being selected. Crossover recombines the two selected chromosomes to form new offsprings with a crossover rate. Mutation randomly alters each position in each offspring with a small mutation rate. New population is then generated by replacing some chromosomes of the population with new offsprings. This process is repeated until some termination condition, e.g., the given number of generations, is reached. Fig. 2 shows the pseudo code of the basic genetic algorithm.

Financial applications of genetic algorithms: Financial applications of genetic algorithms are starting to show promising results. Bauer used genetic algorithms to generate trading rules which are Boolean expressions with three of the ten allowed time series[12]. Colin applied genetic algorithms to find the lengths in the moving average crossover strategy[13]. Deboeck studied methods of using genetic algorithms to train a neural network trading system[14].

In this study, we will apply genetic algorithms to search satisfactory m_1 and m_2 strategy parameter values in piecewise linear GDCPPI strategy.

RESULTS AND DISCUSSION

The main experimental purpose in this study tries to justify that we can find out better piecewise linear GDCPPI strategy by GA technique (generates GA strategy) than Brownian technique (generates B strategy).

Some parameter values are derived by two pretests. The first pretest tries to decide a suitable pair of year length and γ values for Brownian technique, where γ is defined in Eq. (1). The year length is decided to calculate the expected values of return rate μ and variation σ^2. In turn, the μ,σ^2 and γ will be used to calculate the parameters m_1 and m_2 in piecewise linear GDCPPI strategy for Brownian technique, where m_1 is defined in Eq. (3) and m_2 is defined in Eq. (6). The pretest shows that the year length is 8 and γ is 0.1. The second pretest tries to decide the learning length for GA learning and it shows that the better learning length is 100 trading days.

Five stocks are randomly selected as experimental targets from 30 components of Dow Jones Industrial Average (DJIA), namely, American International Group (AIG), IBM, Merck (MRK), HP (HPQ) and Exxon Mobil (XOM). We also randomly select 5 starting learning dates, which are 1999/12/13, 2001/6/6, 2002/2/27, 2003/4/28 and 2004/12/03. Three different floors in the experiments for piecewise linear GDCPPI strategies are pre-assigned and calculated by the ratios of floor to initial wealth, which are 0.7, 0.8 and 0.9. Also 3 different goals in the experiments for piecewise linear GDCPPI strategies are pre-assigned and calculated by the ratios of goal to initial wealth, which are 1.1, 1.2 and 1.3. The testing length is always 30 days. The risk-free rate of return is 0.0001 per day. There are (5*5*3*3=) 225 cases and then generates 225 samples for statistical tests.

GA learning design: The purpose of applying GA technique in this optimization process is to search satisfactory strategy parameter values m_1 and m_2 to attain better investment performance, i.e, the rate of return in this experiment. In order to show GA's capability, we execute GA searchings for 225 circumstances by 5 stocks, 5 testing dates, 3 floors calculated by ratios of floor to initial wealth and 3 goals calculated by ratios of goal to initial wealth as defined above. The training length is 100 days as derived from the above GA pretest.

In addition, each m_1 and m_2 strategy parameter will both be encoded as a 7-bit long gene in a GA chromosome. Therefore, the length of each chromosome is 14-bit long. If the decimal value of each gene is D, each gene will be decoded as values within [1.0,13.7] calculated by (10+D)/10. Moreover, better m_1 and m_2 values implies better investment performance of piecewise linear GDCPPI strategy. The fitness function is to maximize the investment rate of return. The other important GA parameters are as follows: the population size is 40, each run executes 20 generations, crossover is two-point, mutation rate is 0.001 per bit and selection method is integral roulette wheel selection.

GA statistical testing results: We use the paired-samples t test to validate whether GA strategy can outperform B strategy. The null hypothesis is H_0: $roi_B(\theta) \geq roi_{GA}(\theta)$. The testing results are described as follows. The t value for the whole 225 samples in testing period is -2.303. The significance value (p value) is 0.011, which is statistically significant. Then, we can reasonably reject the null hypothesis. That is, the GA strategy can outperform the Brownian strategy.

CONCLUSION

Traditional portfolio insurance strategy such as CPPI does not consider the goal perspective and may fail an almost reached goal as the result. Although current Browne's study[3] considers the similar goal-seeking objective, it still does not consider both the objectives of floor protecting and goal seeking. This study combines the concept of CPPI strategy and the goal-directed strategy derived from Browne's study to form a piecewise linear goal-directed CPPI (GDCPPI) strategy under constraint $F \leq W_t \leq G$. This new strategy in fact extends the strategy solution space and can satisfy those two objectives. In addition, piecewise linear GDCPPI strategy reduces to the GD strategy when $m_1 \rightarrow \infty$ and reduces to the CPPI strategy when $m_2 \rightarrow \infty$. This study also make some experiments to show that the GA technique can outperform the Brownian technique significantly in order to find out better piecewise linear GDCPPI strategies.

Our future work will be on the generalization of our piecewise linear GDCPPI strategy to a piecewise nonlinear one. This would allow us to search for better strategies from a larger strategy space. In addition, the piecewise framework can be used in formulating different objectives for different conditions. This would allow the model to adapt to user needs. We are convinced that piecewise linear GDCPPI strategy represents an interesting perspective and the extension of piecewise strategy concept is worth exploiting in the future.

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