

ECG Signal Smoothing Based on Combining Wavelet Denoising Levels

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Abstract: We present in this study an algorithm of smoothing real noisy ECG signal based on the classical wavelet denoising theory. The key idea of our proposed algorithm consists on generating a constructed denoised ECG signal by extracting and combining the delimited QRS complexes from the 2nd level wavelet denoising and the P and T waves from the 4th or 5th level wavelet denoising outputs. The used classical denoising algorithm utilizes the 'VisuShrink' calculus rule and the 'soft' thresholding strategy. On the other hand, the best suitable wavelet function and decomposition DWT level, for the denoising process, are determined by the means of the mean square error value. Two synthesis parameters have been utilized: the output SNR and the MSE values. We have applied our proposed algorithm to a set of MIT-BIH Arrhythmia Database ECG records added to a simulated 5dB and 0 dB SNR white Gaussian noise where it has been noticed an improvement of the input SNR (5 dB) to an output value of, generally, around 10 dB. To evaluate our algorithm, a comparative study was carried out referred to the low pass Butterworth filter and the 4th and 5th level classical wavelet denoising process. The obtained results demonstrate the superior performance of our proposed algorithm regarded to the tested filtering techniques where the output SNR remains in the most of cases less than 7 dB in the case of the input 5 dB WGN.

Key words: ECG, wavelet denoising, denoising level, QRS detection

INTRODUCTION

As the major part of real signals the Electrocardiogram (ECG) signal, the electrical interpretation of the cardiac muscle activity, is corrupted by different types of noise: the 50/60 power line interference, the Electromyogram (EMG) signal, muscle artifacts, 50/60 Hz power line interference and the baseline wandering. The EMG, a high frequency component, is due to the random contraction of muscles which generates millivolt-level potentials while the abrupt transients of the baseline are due to sudden movement of the body. The baseline wandering, a low frequency component is due to the rhythmic inhalation and exhalation during respiration. These different sources of noise prevent considerably the accurate analysis of the ECG signal and the eventual cardiac anomalies diagnosis. As a usual pre-processing phase, the real ECG is band pass filtered in order to remove the corrupted noise and to recover the signal waves (P, QRS and T)^[1-3]. However, J.

Pan *et al.*^[4] showed that the QRS complex power spectrum density-PSD-(5-15 Hz) overlaps with the muscle noise while the P and T waves PSD overlap with the respiration action and blood pressure at low frequency band (usually from 0.1 to 1 Hz)^[5]. Furthermore, the non-stationary behavior of the ECG signal, that becomes severe in the cardiac anomaly case^[6], incites researchers to analyze the ECG in both time and frequency plans simultaneously. The ability of the wavelet transform to explore signals into different frequency bands with adjustable time-frequency resolution makes it a suitable tool for the ECG signal analysis and processing^[1-10]. The wavelet denoising process based on the great work of Donoho and Johnstone has been widely utilized by several researchers as an alternative to band pass filtering^[11-12]. Different works have been established for ECG noise removing based on the wavelet denoising technique involving different parameters of the thresholding process: the wavelet function, threshold calculus and level decomposition ... Ronan La Page utilized the wavelet

denoising ECG signal as the input reference signal for a matching filter^[13]. A. Benazza *et al.* introduced a smoothing process to the approximation sequence which improved the denoised signal estimation^[14]. Xu and Yan introduced a level-dependent threshold by using a modified factor related to the power of corrupted noise^[15]. Erçelebi presented an ECG denoising algorithm based on the second generation wavelet transform: lifting based discrete wavelet transform and level dependent threshold estimation^[16]. Cherkassy and Kilts compared the denoising accuracy and robustness of the different techniques of wavelet based denoising process based on Vapnik-Chervonenkis (VC) learning theory whose the basic idea was to order the wavelet coefficients according to their magnitudes penalized by their corresponding frequencies (scales)^[17]. In this context, we develop an algorithm to remove the noise corrupting the ECG signal using the wavelet denoising theory based on a different approach. Our key idea was to construct a denoised ECG signal by combining the QRS complexes and the P and T waves localized at well determined wavelet denoising levels. Our approach is to delimit the QRS complexes of the 2nd level classical wavelet denoising and combine them with the P and T waves of the 4th or the 5th level wavelet denoising. The constructed denoised ECG signal offers an improved output Signal-to-Noise (SNR) and reduced Mean Square Error (MSE) values compared to a set of tested classical filtering techniques (the low pass Butterworth filter and the classical wavelet denoising algorithm).

Wavelet denoising: The wavelet transform is characterized by its adjustable time-frequency resolution^[18-20].

Continuous wavelet transform CWT: The CWT - $Wf(s, \tau)$ - is the inner product of a time-varying signal $f(t)$ and the set of wavelets $\psi_{s, \tau}(t)$; it is given by^[21-24]:

$$Wf(s, \tau) = \langle f, \psi_{s, \tau} \rangle = \frac{1}{\sqrt{s}} \int f(t) \psi\left(\frac{t - \tau}{s}\right) dt \quad (1)$$

The scaling and shifting the mother wavelet (ψ) with a factor of s and τ , respectively, generate a family of functions called wavelets given by:

$$\psi_{s, \tau}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t - \tau}{s}\right) \quad (2)$$

with $s > 0$

Discrete wavelet transform DWT: The DWT consists of applying the discrete signal to a bank of octave band filters based on low and high pass filters $h(n)$ and $g(n)$

respectively. The original time-varying signal $f(t)$ would be expanded using the following formula^[25]:

$$f(t) = \sum_{k \in \mathbb{Z}} \alpha_L(k) \phi_{L,k}(t) + \sum_{j=1}^L \sum_{k \in \mathbb{Z}} d_j(k) \psi_{j,k}(t) \quad (3)$$

Where:

$$\psi_{j,k}(t) = 2^{-j/2} \psi(2^{-j}t - k) \quad (4)$$

With:

$$\begin{cases} d_j(n) = \langle f, \psi_{j,n} \rangle = \sum_k g(2n-k) a_{j-1}(n) \\ a_L(n) = \langle f, \phi_{L,n} \rangle = \sum_k h(2n-k) a_{L-1}(n) \end{cases} \quad (5)$$

Where $\phi(t)$ is called the scaling function associated with the wavelet function $\psi(t)$ and is governed by the following condition:

$$\int \phi(t) dt = 1 \quad (6)$$

Denoising: The purpose of wavelet based denoising process is to estimate the signal of interest $s(i)$ from the composite one $f(i)$ by discarding the corrupted noise $e(i)$ ^[26-28]:

$$f(i) = s(i) + e(i) \quad (7)$$

The underlying model for the noisy signal is the superposition of the signal- $s(i)$ - and a zero mean Gaussian white noise with a variance of σ^2 , i.e., $\eta(0, \sigma^2)$.

The power spectral density-DSP- of a white noise is, theoretically, constant with amplitude of σ^2 along the whole axis of frequencies; thus the detail sequences ($d_i, i = 1..L$) are a white noise too. Therefore, the idea was to recover the signal of interest- $s(t)$ - by discarding the noise detail coefficients using a suitable threshold which is related to the noise variance.

Donoho and Jonhstone proposed the universal threshold, called by them 'VisuShrink', given by^[29,30]:

$$\text{Thr} = \sigma \sqrt{2 \cdot \log(N)} \quad (8)$$

In the case of white noise, its standard deviation can be estimated from the median of its lower resolution detail sequence (d_L) and is computed as follows:

$$\sigma = \frac{\text{MAD}(|d_L|)}{0.6745} \quad (9)$$

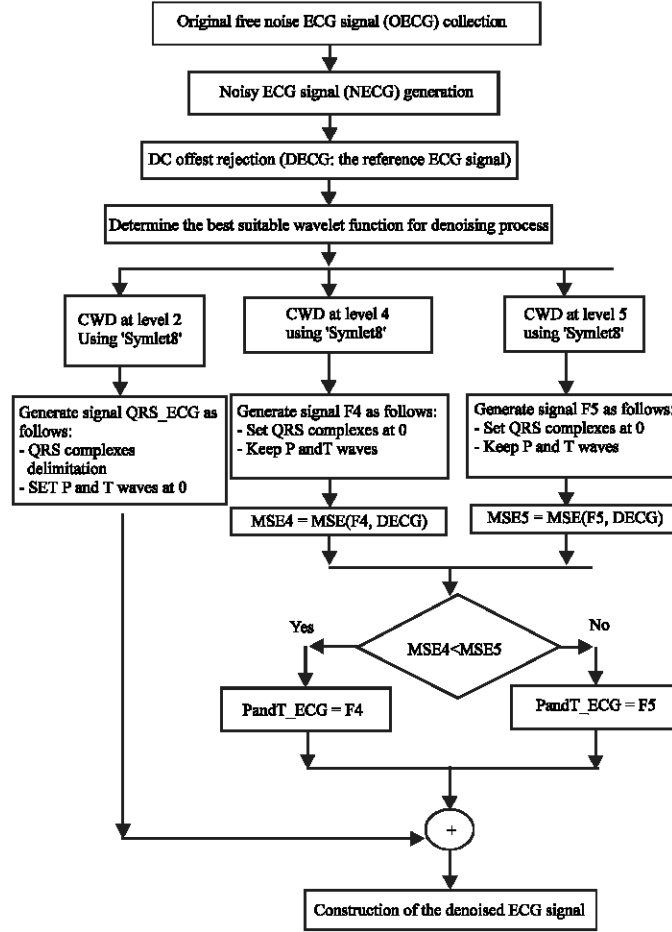


Fig. 1: The schematic of the denoising algorithm. CWD: classical wavelet denoising

where MAD is the median absolute deviation of the corresponding sequence.

However, there exist two approaches of thresholding process: ‘Soft’ and ‘Hard’ thresholding algorithms (T_{soft} and T_{hard} , respectively) expressed as follows, respectively:

$$T_{soft} = \begin{cases} 0 & \text{if } |x| \leq Thr \\ x + \text{sgn}(x) \cdot |Thr| & \text{if } |x| > Thr \end{cases} \quad (10)$$

$$T_{hard} = \begin{cases} 0 & \text{if } |x| \leq Thr \\ x & \text{if } |x| > Thr \end{cases} \quad (11)$$

MATERIALS AND METHODS

Our proposed denoising algorithm is illustrated on the Fig. 1 and is summarized by the execution of the following two phases:

Pre-processing phase including: 1) ECG data collection and noisy ECG signal generation with different SNR levels; 2) high pass filtering to suppress the DC component; 3) determining the best suitable wavelet function for the denoising process,

Smoothing phase including: 1) classical wavelet denoising ECG signal using the ‘Universal’ threshold and ‘Soft’ strategy; 2) delimiting the QRS complexes of the 2nd level (L_{qrs}) wavelet denoising; 3) decide which level (L_{pt}) (4th or 5th) wavelet denoising will be used to localize the P and T waves; 4) combining the determined QRS complexes with the P and T waves of the L_{pt} (4th or 5th) level wavelet denoising to generate the constructed denoised ECG signal.

Pre-processing phase

Data collection and noisy ECG signal generation: We have opted, in our study, for the MIT-BIH Arrhythmia

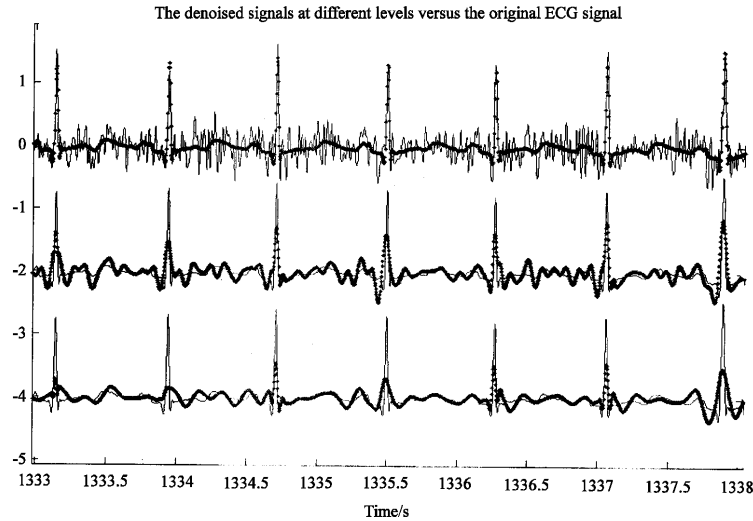


Fig. 2: The comparison between the Classical Wavelet Denoising (CWD) of the noisy 0 dB ECG signal, at different levels (2, 4 and 5) and the original ECG signal. a) In the top: the CWD at the 2nd level in continuous line; b) in the middle: the CWD at the 4th level in discontinuous line; c) in the bottom: the CWD at the 5th level in discontinuous line

Database a commonly so-used free downloadable manually annotated database. The database consists of 48 annotated records, obtained from 47 subjects studied by the BIH Arrhythmia Laboratory between 1975 and 1979. Each record consists of dual channel ECG data and is around 30 min long. The ECG signal is sampled at rate of 360 Hz with 11-bit resolution over a 10 mV range^[31].

The next step will consist on generating the noisy version of the collected ECG data. This is achieved by, simply, adding a white Gaussian noise with different SNR levels.

Rejection of the DC component of the ECG signal: The DC offset present at the original ECG signal grows along the approximation sequences at successive levels due to the low pass filtering that distorts considerably the denoising process. Thus we have used the classical high pass Butterworth filter, with a very low cut-off frequency of around 1.8 Hz, to reject the DC component of the ECG signal. The resulting signal (the original ECG with the DC component rejected: DECG) (Fig. 1) is referred, along the work, to the ECG reference signal.

Determine the best suitable wavelet function: Determining the best suitable wavelet function, for ECG denoising purpose, is achieved on the basis of the Mean Square Error (MSE) value between the denoised ECG signal ' $f_d(i)$ ' and the original with DC component rejected signal ' $f_c(i)$ ' given by:

$$MSE(w, l) = \frac{1}{N} \sum_{i=1}^N (f_c(i) - f_d(i))^2 \quad (12)$$

with w is the wavelet function, l is the level wavelet denoising and N is the length if the ECG segment.

The signal f_c is the original MIT-BIH ECG data signal with DC component rejected (the reference ECG signal) and the signal f_d is the denoised ECG signal obtained by the means of the classical wavelet denoising using the 'Universal' threshold and 'Soft' strategy.

Smoothing phase

Classical wavelet denoising: In our design, the computations carried out on the MIT-BIH ECG data records, with a length of 650 000 samples each, are executed on segment by segment until the end of the record. The segments are perfectly adjacent, i.e. with null overlapping, with fixed size of 4096 samples ($4096 = 2^{12}$), which in turns corresponds to a duration of around 11.38 sec.

Based on the visual inspection, we determine the denoising level (e.g., level Lqrs) which best matches to the QRS complexes and that (e.g., level Lpt) which best correlates the P and T waves. The empirical studies show that the 2nd Level wavelet denoising (Lqrs) corresponds best to the QRS complexes while the 4th or 5th Level wavelet denoising (Lpt) correlates best the P and T waves as it is illustrated on the Fig. 2.

The QRS complexes detection: A routine designed especially for the QRS complexes delimitation is used^[32].

This routine is applied to the 2nd Level (Lqrs) which captures best the QRS complex of the original free noise ECG signal. Our QRS complex delimitation routine is summarized as follows^[32]:

- Detect the maximum amplitude-MAX-in each sub-segment of 1024 (4096/4) samples and next calculate the corresponding threshold-THRSQRS-given by: $THRSQRS = 0.6 * MAX$

Referring to this adapted calculated threshold the R wave's peaks are detected.

We should mention that searching the R peaks using the adapted QRS threshold, within a sub-segment of 1024 samples, is aimed to better detect the R waves in the case of huge Baseline Wandering (BLW) circumstance.

- Based on the QRS complex morphology template^[32]; a backwards and forwards scanning with respect to the detected R wave peak allows the localization of the QRS onset and offset respectively which permits automatically to delimit the different QRS complexes of the 2nd level ECG wavelet denoising.

Determining the specific wavelet denoising levels: To decide which level (4 or 5) wavelet denoising captures best the P and T waves (Lpt) we execute the following procedure (Fig. 1): zeroing, first, the QRS complexes (delimited in the previous step) of the 4th and the 5th level wavelet denoising ECG signal which generates the two signals F4 and F5, respectively. Calculating, next, the Mean Square Error (MSE) values between the signals (F4 and F5) and the reference ECG signal (DECG) that leads, based on the minimum value of the MSE, choosing which level (4 or 5) wavelet denoising will serve to localize the P and T waves.

Constructing the combined denoised ECG signal: The last phase of our proposed algorithm consists of constructing the denoised ECG signal by combining the delimited QRS complexes of the 2nd Level (Lqrs) with the P and T waves of the 4th or 5th Level (Lpt). This is accomplished by substituting the QRS complexes of the Lpt level wavelet denoising signal by those of the 2nd Level (Lqrs) wavelet denoising and keeping the P and T waves.

RESULTS AND DISCUSSION

We discuss in this section the obtained results when applying our proposed smoothing algorithm to a set of ECG data records of the 'MIT-BIH Arrhythmia Database'

corrupted by White Gaussian Noise (WGN), with different SNR values and a set of real picked-up ECG data records with unknown, a priori, corrupted noise energy. The performance of the algorithm is evaluated based on the MSE and output SNR criteria values. On the other hand, the Butterworth low pass filtering and the 4th and 5th level classical wavelet denoising process are used to assess the performance of our algorithm.

Preliminaries

Evaluation criteria values: We utilize both the output SNR value and the MSE value between the constructed denoised ECG signal and the original ECG signal with DC offset rejected (the reference ECG signal) to evaluate our denoising algorithm.

The output SNR is given by:

$$SNR_{out} = 10 \log \left(\frac{\sum_{i=1}^N f_e^2(i)}{\sum_{i=1}^N (f_e - f_d)^2} \right) \quad (13)$$

where f_e denotes the reference ECG signal) and f_d represents the constructed denoised ECG signal whereas N is the length of the data segment (4096 in our approach).

Denoising parameters determination: The Table 1 presents the MSE values between the different levels classical wavelet denoising (using 'Universal' threshold and 'soft' strategy) of the ECG signal and the reference ECG signal in the case of an added 5dB WGN ECG. It is noticed obviously that the wavelet function 'Symlet 8' provides the reduced MSE values along the different levels. Thus, the wavelet function 'Symlet 8' will be used along the whole denoising work.

Simulation study

Synthesis study

Local analysis: We present in this paragraph the obtained results of our denoising algorithm application to short duration 5dB noisy ECG data segments representing, each, a particular feature. The figure 3 shows a segment of the ECG record '100.dat' representing a normal sinus rhythm. The figure 4 shows a segment of the same record representing a PVC (premature ventricular contraction) while the figure 5 shows a segment containing a huge baseline wandering (BLW).

The 3 Fig. 3-5 demonstrate obviously the high performance of our proposed algorithm for denoising the ECG signal in different circumstances. However, a more exhaustive analysis of our algorithm by applying it to a

Table 1: The Mean Square Error (MSE) values of the classical wavelet denoising of a 0 dB noisy ECG signal, at levels (2, 3, 4 and 5), referred to the original ECG signal with DC offset rejected

Wavelet level	'db5'	'db10'	'Coif5'	'sym6'	'sym8'	'bior5.5'
2	0.0118	0.0115	0.0115	0.0117	0.0116	0.0122
3	0.0115	0.0118	0.0118	0.0118	0.0113	0.0135
4	0.0156	0.0187	0.0155	0.0151	0.0142	0.0195
5	0.0220	0.0261	0.0211	0.0193	0.0185	0.0261

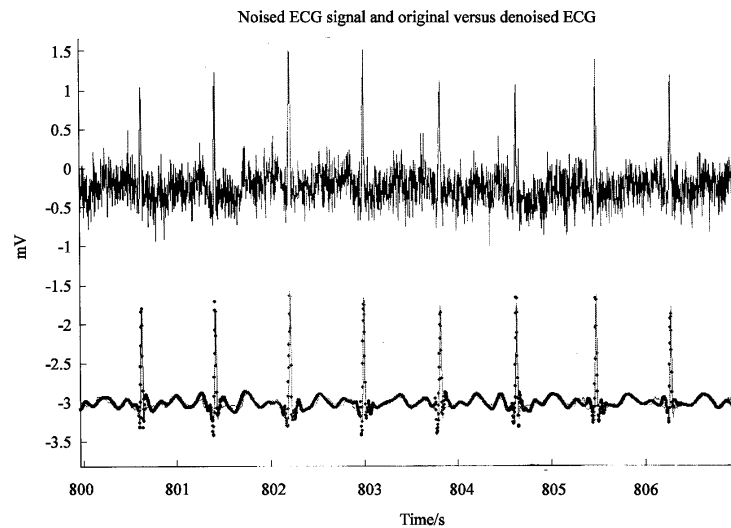


Fig. 3: Application of our denoising algorithm to a 5 dB noisy normal sinus rhythm ECG signal segment: at top) the noisy ECG signal; at bottom) the comparison between the original DC offset rejected (in continuous line) and denoised ECG signals (in discontinuous line)

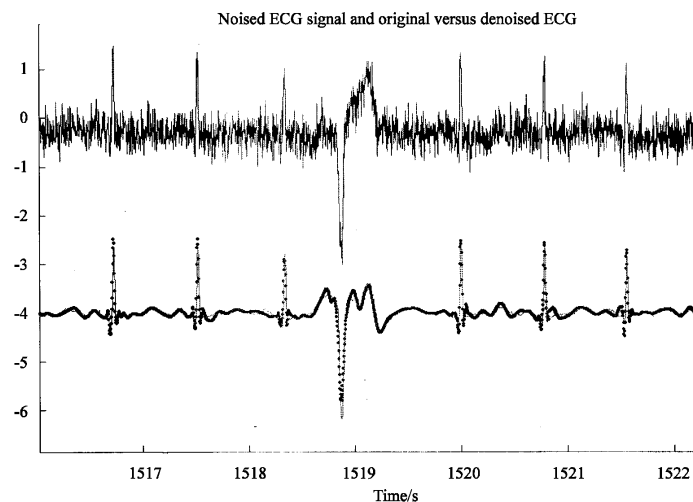


Fig. 4: Application of our denoising algorithm to a 5 dB noisy ECG signal with a PVC: at top) the noisy ECG signal; at bottom) the comparison between the original DC offset rejected (in continuous line) and denoised ECG signals (in discontinuous line)

set of the 'MIT-BIH Arrhythmia Database' ECG data records which, in turns, includes a large number of cardiac cycles would evaluate deeply our algorithm.

Overall analysis: We have treated a set of 3 MIT-BIH Arrhythmia Database ECG data records ('100.dat',

'101.dat' and '103.dat') added to the White Gaussian Noise (WGN) with a total duration of 90 min containing around 6222 annotated cardiac beats. We should note, again, that the algorithm is carried out on a set of successive, null overlapping, segments of 4096 samples (around 11.38 sec) for the different MIT-BIH Arrhythmia database ECG records.

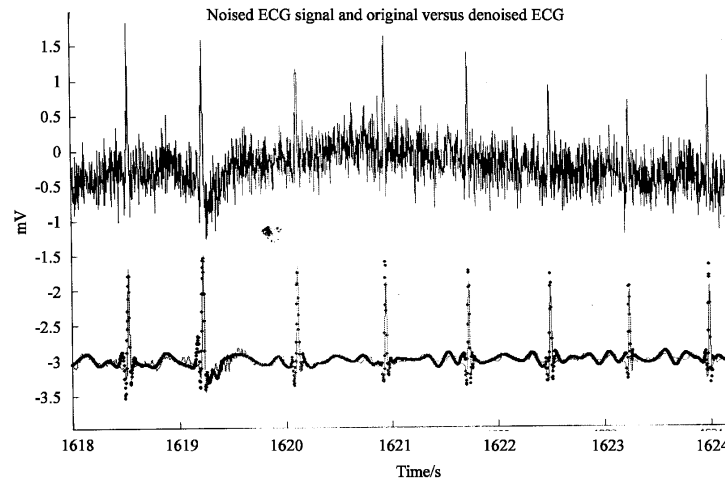


Fig. 5: Application of our algorithm to a 5 dB noisy ECG signal containing a huge BLW (base line wandering): a) the noisy ECG signal; b) the comparison between the original DC offset rejected (in continuous line) and denoised ECG signals (in discontinuous line)

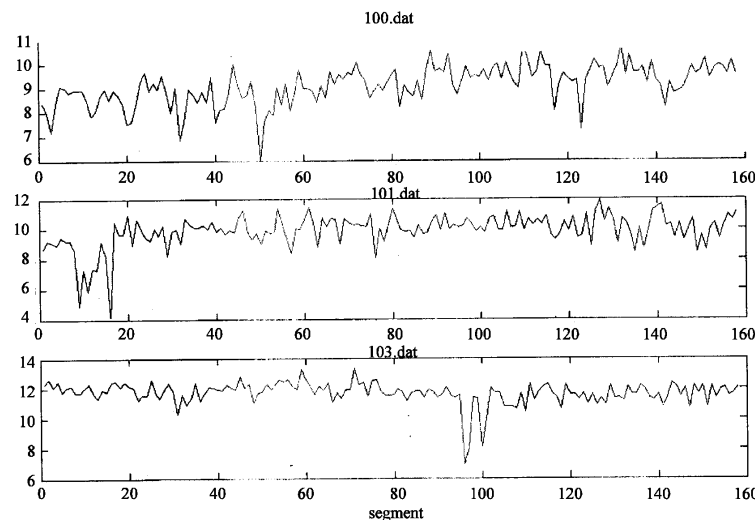


Fig. 6: The variation of the output SNRs (in dBs) in the case of the 5 dB noisy MIT-BIH Arrhythmia Database records computed along each ECG data segment of 4096 samples; a) at the top: the record '100.dat'; b) at the middle: the record '101.dat'; c) at the bottom: the record '103.dat'

The Figs. 6 and 7 illustrate the obtained output SNR and MSE values, for each 4096 samples ECG segment, respectively, when applying our algorithm to the three ECG data records corrupted by a 5 dB white Gaussian noise while the Figs. 8 and 9 show the obtained output SNR and MSE values, respectively, in the case of a 0 dB added white Gaussian noise.

The Figs. indicate some anomalies of our proposed algorithm (minimum values of the output SNRs and maximum values of the MSE); the analysis of the specified segments yields to that the major limitation of our algorithm is obviously the presence of huge Baseline Wandering (BLW). This is because, essentially, of the

limited performance of the delimited QRS complex routine that depends on local maxima and can not pursuit the huge variation of the baseline over a short duration of time. Nevertheless, our proposed algorithm shows performance superiority for noisy ECG signal filtering.

Comparative study: We have used the low pass Butterworth digital filter, with a cut-off frequency of 45 Hz^[33] and the 4th and 5th level classical wavelet denoising process to evaluate our proposed algorithm performance.

We divide this comparative section into 2 sub-paragraphs: The first one emphasizes the evaluation of our proposed algorithm for a short segment of ECG data

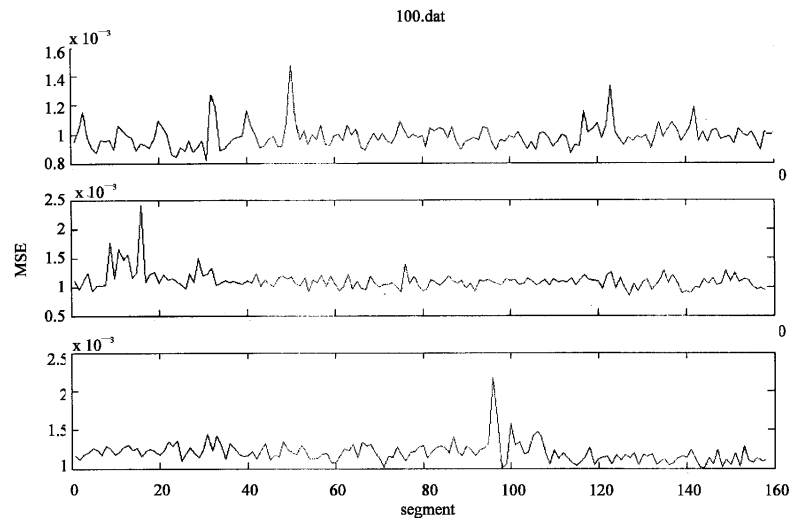


Fig. 7: The variation of the MSE values ($\times 10^{-3}$) in the case of the 5 dB noisy MIT-BIH Arrhythmia Database records computed along each ECG data segment of 4096 samples; a) at the top: the record '100.dat'; b) at the middle: the record '101.dat'; c) at the bottom: the record '103.dat'

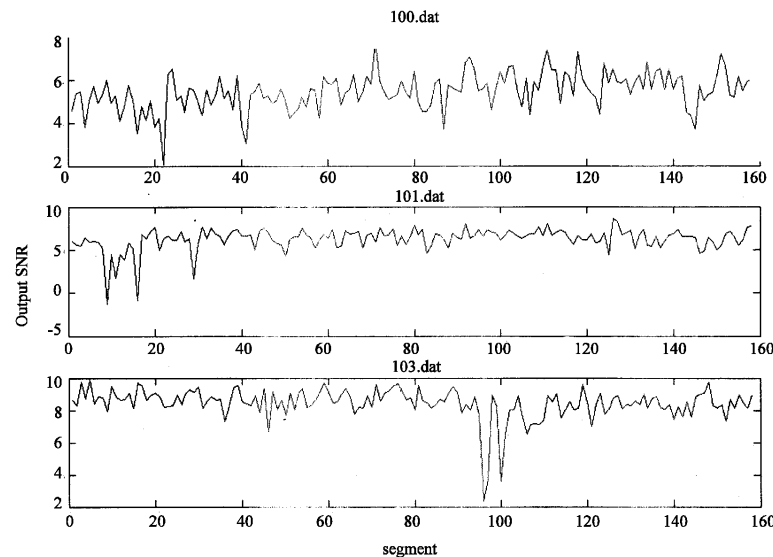


Fig. 8: The variation of the output SNRs (in dBs) in case of the 0 dB noisy MIT-BIH Arrhythmia Database records computed along each ECG data segments of 4096 samples; a) at the top: the record '100.dat'; b) at the middle: the record '101.dat'; c) at the bottom: the record '103.dat'

representing a normal sinus rhythm while in the second section a set of the MIT-BIH Arrhythmia Database ECG data records are analyzed.

Local comparative analysis: We emphasize our analysis in this paragraph on a short duration 0 dB noisy ECG data segment representing a normal sinus rhythm in order to assess the filtering effect of our proposed algorithm compared to the techniques stated previously. We have considered the temporal segment [1710..1715 sec] of the

MIT-BIH Arrhythmia Database ECG data record '100.dat'. The Fig. 10 illustrates the application of our algorithm as well as the low pass Butterworth and the 4th and 5th level classical wavelet denoising to the noisy 0 dB ECG signal. We should mention that the plots are shifted successively by -2.5 mV for better view.

Visually, it is obvious that our proposed denoising algorithm provides better results compared to the tested methods (low pass Butterworth filter and classical wavelet denoising). The Table 2 presents the output SNR

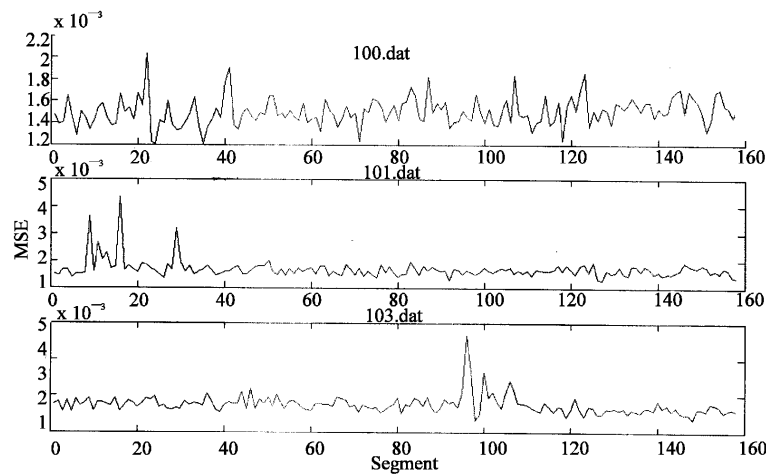


Fig. 9: The variation of the MSE values ($\times 10^{-3}$) in the case of the 0 dB noisy MIT-BIH Arrhythmia Database records computed along each ECG data segments of 4096 samples; a) at the top: the record '100.dat'; b) at the middle: the record '101.dat'; c) at the bottom: the record '103.dat'

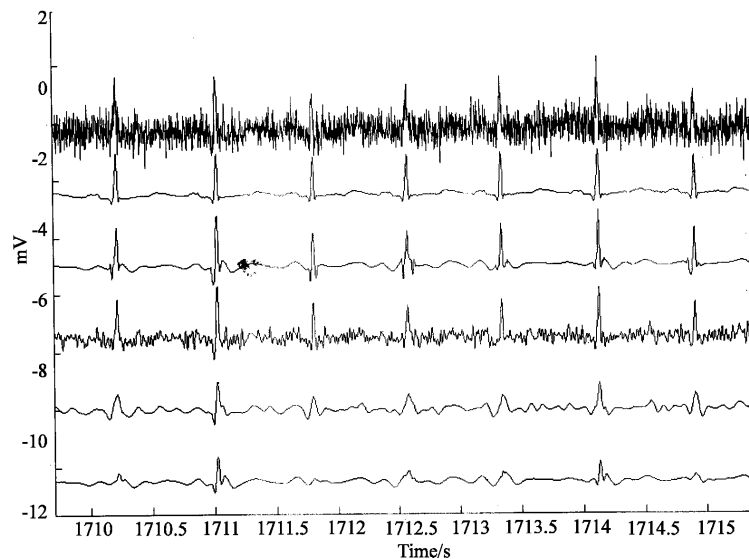


Fig. 10: The application of different denoising techniques to a 0 dB noisy ECG signal of the MIT-BIH Arrhythmia Database ECG data record '100.dat' (from up to down): a) the noisy ECG signal; b) the original DC offset rejected ECG signal (the reference ECG signal); c) our proposed denoising algorithm result using the wavelet function 'Symlet8'; d) the low pass Butterworth filtering, e) the classical wavelet denoising (CWD) at level 4; e) the classical wavelet denoising (CWD) at level 5

and MSE values, between the denoised and the reference ECG signals, obtained for each method. Again, the obtained results demonstrate the higher performance of our proposed algorithm for denoising ECG signal compared to the classical wavelet denoising method and the low pass Butterworth filtering.

The overall comparative analysis: The overall comparative analysis was carried out on the set of the 3

MIT-BIH Arrhythmia Database ECG data records ('100.dat', '101.dat' and '103.dat') treated previously.

The Tables (3 and 4) summarize the obtained output SNR and MSE values, respectively, for each approach applied to the 3 ECG data records, with a given input 5 and 0 dB WGN noise. The obtained results showed on Tables 3 and 4 coincide with those obtained in the case of short segment of ECG data (Table 2). This ensures, again, the performance superiority of our proposed denoising algorithm.

Table 2: The obtained Output SNR and MSE values for each tested filtering method applied to the 5dB noisy ECG data segment [1707..1718 sec] of the MIT-BIH Arrhythmia Database ECG data record '100.dat'

Denosing method	Our proposed algorithm	LP butterworth filter	4th level classical wavelet denosing
Output SNR (dB)	9.7489	6.6807	5.9663
MSE ($\times 10^{-3}$)	0.98377	1.4	1.5

Table 3: The obtained output SNR (in dBs) values (Mean \pm Standard Deviation -STD-) for each tested filtering method applied to the set of MIT-BIH arrhythmia Databases ECG records ('100.dat', '101.dat' and '103.dat')

Method record	Input SNR (dB)	Our proposed algorithm	LP butterworth filter	4th level classical wavelet denosing
100 dat	5	9.1024 \pm 0.7742	5.4718 \pm 0.8253	4.8539 \pm 0.7078
101 dat	5	9.8608 \pm 1.0835	6.7585 \pm 1.1487	6.8030 \pm 0.8668
103 dat	5	11.6784 \pm 0.7827	9.1075 \pm 1.0418	7.9001 \pm 0.7596
100 dat	0	5.3646 \pm 0.9009	0.4493 \pm 0.8367	1.7172 \pm 0.6288
101 dat	0	6.2985 \pm 1.3872	1.7625 \pm 1.2072	3.6197 \pm 0.8741
103 dat	0	8.3863 \pm 1.0676	4.0971 \pm 1.0606	4.5037 \pm 0.7581

Table 4: The obtained MSE values ($\times 10^{-3}$) (Mean \pm Standard Deviation -STD-) for each tested filtering method applied to the set of MIT-BIH arrhythmia Databases ECG records ('100.dat', '101.dat' and '103.dat')

Method record	Input SNR (dB)	Our proposed algorithm	LP butterworth filter	4th level classical wavelet denosing
100 dat	5	0.00098 \pm 8.38 $\times 10^{-5}$	0.0015 \pm 9.88 $\times 10^{-5}$	0.0016 \pm 9.7492 $\times 10^{-5}$
101 dat	5	0.0011 \pm 1.676 $\times 10^{-4}$	0.0016 \pm 2.57 $\times 10^{-4}$	0.0016 \pm 1.8102 $\times 10^{-4}$
103 dat	5	0.0012 \pm 1.308 $\times 10^{-4}$	0.0016 \pm 2.39 $\times 10^{-4}$	0.0019 \pm 1.8102 $\times 10^{-4}$
100 dat	0	0.0015 \pm 1.291 $\times 10^{-4}$	0.0027 \pm 1.76 $\times 10^{-4}$	0.0023 \pm 1.2331 $\times 10^{-4}$
101 dat	0	0.0017 \pm 3.614 $\times 10^{-4}$	0.0028 \pm 4.82 $\times 10^{-4}$	0.0023 \pm 2.6788 $\times 10^{-4}$
103 dat	0	0.0018 \pm 2.767 $\times 10^{-4}$	0.0029 \pm 4.40 $\times 10^{-4}$	0.0027 \pm 2.8431 $\times 10^{-4}$

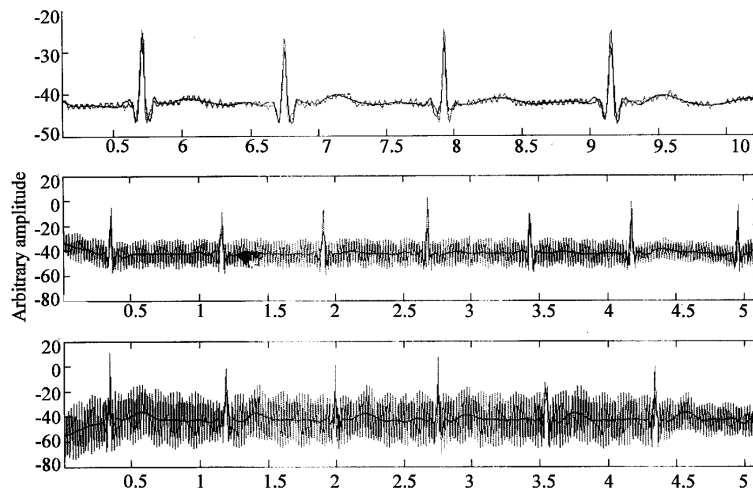


Fig. 11: Denoising of real noisy ECG signals with different SNR levels; original ECG (in red) and denoised signal (in black): High SNR (in the top), Medium SNR (in the middle) and Low SNR (in the bottom)

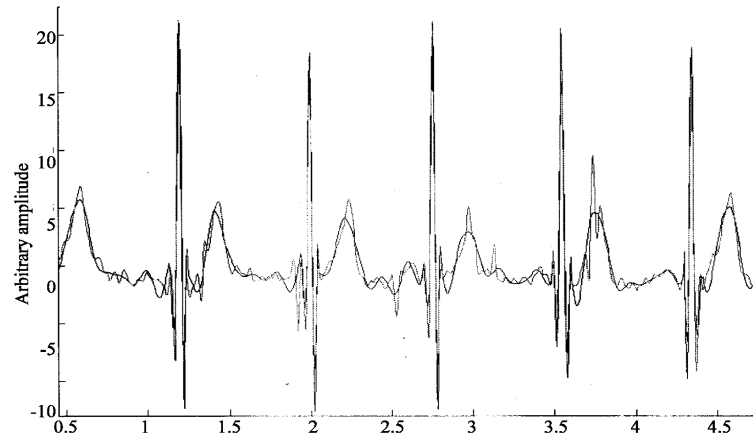


Fig. 12: Smoothed (low pass filtered) (in red) versus the combined denoised (using our algorithm) (in black) of the original low SNR ECG signal (of the Fig. 11-c)

Experimental study: We discuss in this section the application of our denoising algorithm, which was applied to a set of ECG data records added to a simulated white Gaussian noise, to the real ECG signals with unknown, a priori, noise energy. The analysed real noisy ECG signal is extracted from the GBM laboratory ECG data base^[32] sampled at rate of 200 Hz with 8 bits resolution. The figure 11 illustrates 3 ECG signals with three different SNR levels (high (HSNR) (in the top), medium (MSNR) (in the middle) and low (LSNR) (in the bottom)) compared to the corresponding denoised ECG signals using our algorithm.

To assess the performance of our proposed algorithm, the low pass Butterworth filter was used to smooth the low SNR (LSNR) ECG signal shown on the Fig. 11-c. The Fig. 12 shows the denoised ECG signal, using our denoising algorithm, (the black trace) compared to the low pass filtered version one (the red trace). Visually, the denoised ECG signal is smoother whereas the low pass filtered signal shows some undesirable peaks mainly on the 3rd, 4th and 5th T waves. Again, our denoising algorithm shows satisfactory results and provides better performance than the classical band pass Butterworth filter for the real picked-up noisy ECG signal.

CONCLUSIONS

We have developed an algorithm of filtering noisy ECG signal based on the classical wavelet denoising technique. Our key idea of the proposed algorithm is to construct a denoised version of the input noisy ECG signal by combining two specific levels wavelet denoising outputs. Our idea is argued by the fact that the different waves (P, QRS and T) spectra of a normal ECG signal occupy different frequency bandwidths. We have exploited this property by studying the different levels wavelet denoising outputs. Empirical studies, using a best suitable wavelet function ('Symlet 8'), showed that the QRS complexes are well preserved in the 2nd wavelet denoising level while the P and T waves, conjointly, are best preserved in the 4th or 5th level. We have utilized, in the study, a set of records of the 'MIT-BIH Arrhythmia Database' added to white Gaussian noise with different SNR levels. Our proposed algorithm provides a satisfactory efficiency even in the case of low SNR ECG signals although some limitations faced in detecting some QRS complexes in huge baseline wandering situations. A comparative study was carried out on the classical low pass Butterworth digital filter and the classical wavelet denoising at the 4th and 5th levels. This comparative study demonstrates the superior performance of our proposed algorithm for smoothing the noisy ECG signal. A more robust QRS detection routine might solve the practical

encountered problems mainly the presence of huge baseline wandering. Furthermore, our algorithm allows, in fact, the realization of a denoising model for smoothing real picked-up noisy ECG signals with unknown a-priori noise sources power and circumstances (EMG, muscle artifacts ...).

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